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ARTICLE

PEDAGOGY STUDENTS' NARRATIVES ABOUT SCHOOL MATHEMATICS AND THE EDUCATOR CHALLENGE [1]

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ABSTRACT: The article presents parts of a research that aims to analyze how future teachers have learned mathematical concepts throughout their school and academic trajectory and how their representations related to school mathematics and its teaching can influence the elaboration of new representations. The text is the result of research-teacher education carried out by Hifopem research group, which used the biographical studies as the theoretical-methodological reference. Data were produced using memorials regarding education, pedagogical narratives, and transcription of the discussion-reflection group meetings. A narrative analysis was performed with these data. The research revealed evidence that students have often learned mathematical concepts painfully. Punishment and/or the feeling of impotence are the most visible and create representations of cold school mathematics, which is difficult to access, can only be learned by a few people, and only deals with calculations. Students have distanced themselves from Mathematics and believe they are not capable of teaching it. The clues given by the undergraduates have revealed the size of the challenge faced by educators: to make them aware of the obstacles arising from their school mathematics education to make these difficulties object of reflection, to be overcome and (re)signified during their academic education.

Keywords: school mathematics discipline, research-teacher education, teacher education, mathematics education, pedagogy course.

NARRATIVAS DE LICENCIANDOS EM PEDAGOGIA SOBRE A MATEMÁTICA ESCOLAR E O DESAFIO DO FORMADOR

RESUMO: O texto é resultado de uma pesquisa-formação, no âmbito do grupo de pesquisa Hifopem, tendo os estudos biográficos como referencial teórico-metodológico. O objetivo do artigo é analisar como os futuros docentes se apropriaram/apropriam de conceitos matemáticos ao longo de sua trajetória escolar e acadêmica, e como suas representações relacionadas à matemática escolar e a seu ensino podem

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influenciar a elaboração de novas representações. Os dados foram produzidos por memorial de formação, narrativas pedagógicas e transcrição das reuniões do grupo de discussão-reflexão. Realizou-se uma análise narrativa com esses dados. A pesquisa revelou indícios de que os estudantes se apropriaram/se apropriam de conceitos matemáticos, em muitos casos, de maneira sofrida. O castigo e/ou o sentimento de impotência são os mais visíveis, criando representações de uma matemática escolar gélida, de difícil acesso, que poucos poderiam aprender e que trata unicamente de cálculos. Os estudantes se distanciaram da Matemática e acreditam que não conseguem ensiná-la. As pistas deixadas pelos licenciandos revelaram o tamanho do desafio que os formadores precisam enfrentar: provocar neles a tomada de consciência dos obstáculos advindos de sua formação matemática escolar para que as dificuldades sejam objeto de reflexão, superação e (re)significação durante sua formação acadêmica.

Palavras-chave: disciplina matemática escolar, pesquisa-formação, formação de professores, educação matemática, curso de pedagogia.

NARRATIVAS DE ESTUDIANTES EN PEDAGOGÍA SOBRE MATEMÁTICAS ESCOLARES Y EL DESAFÍO DEL FORMADOR

RESUMEN: El texto es resultado de una investigación-formación, en el ámbito del grupo de investigación Hifopem, con estudios biográficos como referente teórico-metodológico. El objetivo del artículo es analizar cómo los futuros docentes se han apropiado/apropiado de conceptos matemáticos a lo largo de su trayectoria escolar y académica, y cómo sus representaciones relacionadas con las matemáticas escolares y su enseñanza pueden influir en la elaboración de nuevas representaciones. Los datos fueron producidos por memorial de entrenamiento, narrativas pedagógicas y transcripción de los encuentros de los grupos de discusión-reflexión. Con estos datos se realizó un análisis narrativo. La investigación reveló evidencias de que los estudiantes se apropiaron/se apropiaron de conceptos matemáticos, en muchos casos, de forma dolorosa. El castigo y/o el sentimiento de impotencia son los más visibles, creando representaciones de una matemática escolar fría, de difícil acceso, que pocos pueden aprender y que sólo se ocupa de cálculos. Los estudiantes se distanciaron de las Matemáticas y creen que no pueden enseñarlas. Las pistas dejadas por los estudiantes revelaron la magnitud del desafío que deben afrontar los formadores: sensibilizarlos sobre los obstáculos derivados de su formación matemática escolar para que las dificultades sean objeto de reflexión, superación y (re)significación durante su entrenamiento académico.

Palabras clave: asignatura matemática escolar, investigación-formación, formación de profesores, educación matemática, curso de pedagogía.

INTRODUCTION

This text is an excerpt from doctoral research (MOURA, 2019) that focused on school mathematics and the academic education[2] of future pedagogues at a federal university in the Brazilian Northeast. The objective of the article is to analyze how future teachers have learned mathematical concepts throughout their school and academic career, and how their representations related to school mathematics and its teaching can influence the elaboration of new representations.

The first author, named here as professor-researcher, and teacher of a Pedagogy course, produced and analyzed the data with the help of the second author. We carried out research-teacher education (JOSSO, 2010a, 2010b) that uses biographical studies as a theoretical-methodological reference. We used the formation memorial (PRADO; SOLIGO, 2007), the pedagogical narratives (PRADO; FERREIRA; FERNANDES, 2011), and the transcriptions of the discussion-reflection group meetings as devices for data production. Furthermore, we carry out a narrative analysis (BOLÍVAR, 2012).

In the sections that follow, we present the research methodology as well as a brief discussion about the narrative and the school mathematics discipline. Afterward, we analyzed the students' narratives about school mathematics and the teacher's challenge in front of them. Finally, we present our considerations about the investigation.

RESEARCH METHODOLOGY

The research-teacher education type[3], was the investigation that generated this article, which conceives that the subjects involved are learning and, through lived experience, become aware of who they are and the role they can play in society. In Josso's (2010a, 2010b) understanding, research-teacher education favors the awakening of the researcher and the subjects involved in the research concerning their weaknesses, their intentions, and the (in)constancies of desires and life projects.

In research-teacher education, experiences throughout people's lives help to build their identity and subjectivity, because life stories are seen as knowledge projects, as construction and invention of themselves since education presupposes transformation and change and implies the movement of looking at oneself through others and through one's writings, in individual and collective work. This scenario is understood by Josso (2010b, p. 71) as research-teacher education, "because the research activity contributes to the education of participants in terms of reflective and interpretative learning and takes place, in their life path, as a moment of retroactive and prospective questioning about their life project(s) and their current education demand(s)."

The data were produced with 11 students from a federal university in the Brazilian Northeast during the last 2 years of the Pedagogy course they were taking. The students were monitored in the subjects of *Fundamentals and Methodologies for Teaching Mathematics, Fundamentals and Methodologies for Teaching Science, Internship in Teaching Pedagogical Disciplines (in early childhood education),* and *Internship in Teaching for Initial Years I and II of Elementary School.* To produce the data, we used the formation memorial, the pedagogical narratives, and the transcription of the discussion-reflection group meetings.

Formation memorials are considered a textual genre included in the set of works in Social and Human Sciences that indicate life stories as an object of investigation in many areas since the 1970s. In our case, they are rich sources of testimonies from graduate students in Pedagogy inserted in situations in which they think and talk about their experiences during their academic course.

According to Prado and Soligo (2007), the formation memorial is a textual genre that includes narrative, descriptive, and argumentative discourse, although narrative discourse predominates. It is

[2] We believe that the term *initial education (pre-service)* is not plausible to determine the beginning of the formation of an education professional, given that their years of basic education are their first formation. Therefore, we understand that a higher education course is another step in the education of this professional, which is why we use the expression *academic education.*

[3] It is important to highlight that the research project was approved by the Research Ethics Committee of Universidade São Francisco (USF), with opinion number 1,564,633.

detailed and analytical and deals with the process of formation of the narrator's subject in each period. Therefore, the use of narratives to compose this material is essential.

Pedagogical narratives, closely related to formation memorials, have different aspects from these because they are dedicated to short themes about a place in the subject's education. They may only deal with the first years of education, a specific course taken, or, even, some point in the academic education that the subject experienced. For Prado, Ferreira, and Fernandes (2011, p. 145), pedagogical narratives are "predominantly narrative and autobiographical texts, written to share lessons learned from experience, reflection on experience, observation of peer practice, discussion collective, reading, study, and research."

After the data was produced, we carried out a narrative analysis. This type of analysis is in line with biographical research, as the researcher's job is to configure the data elements into a story that unifies them and gives meaning to them, to authentically express individual life, without manipulating the voice of the research subjects.

One of the main characteristics of this type of data analysis, according to Bolívar (2012, p. 42, our translation), is to present "concrete human experience as a description of intentions through a sequence of events in times and places in which biographical-narrative reports are the privileged means of knowledge/investigation."[4] When looking at the texts of the 11 students who agreed to participate in this research, we had the difficult task of (re)reading the narratives and analyzing the extent to which they are understandable to us, taking into account the temporal and spatial context of the narrators and focusing on the attitudes, feelings, and decisions that they express in the narratives.

Neutrality does not exist in narrative analysis. We had the difficult task of epistemological vigilance to not silence the voices of the narrators, much less use them as a simple transcription of data to justify or defend our ideas, because — as Bolívar, Domingo, and Fernández warn (2001, p. 114, translation) — "the point is not to sacralize the reports, nor to assimilate them to traditional paradigmatic ways of knowing."

Therefore, in narrative analysis, the interest is not in looking at the life stories and education of each of the 11 students and then generalizing them to create categories. The focus is on seeing the pertinence of the reports within the space-time and sociocultural environment of which the students are part, to then create a group narrative since the subjects of this research belong to a group with specific social conditions: students on a Pedagogy course at a federal university in northeastern Brazil.

To achieve this, we construct integrated units of meaning (BOLÍVAR, 2012), as what is important in narrative analysis are the worlds lived by the research subjects, the singular meanings they express, and their logic of argumentation printed in their narratives (BOLÍVAR; DOMINGO; FERNÁNDEZ, 2001). Among the integrated units of meaning, we bring to this article the one that deals with memories of school mathematics classes, *Fundamentals and Mathematics Teaching Methodologies* classes, and internships in academic education.

Next, we present a brief discussion about the narrative and the school mathematics discipline. Then, we analyzed the testimonies of the 11 students in the investigation about school mathematics and the teacher's challenge facing it.

NARRATIVE AND SCHOOL MATHEMATICS DISCIPLINE

Narrative is a rudimentary form of human communication; through it, people tell their stories, remember their experiences, and find possible explanations for them. With narrative, people remember what happened and attribute meaning to the experience, since narrating is the elementary form of human communication. Bolívar, Domingo, and Fernández (2001) assert that one of the tasks of narrative is to help humans improve what they know and not prescribe what they must do, contributing to the understanding, analysis, and interpretation of successes. Looking at teaching, the narrative breaks with the conception of technical rationality, addressing the contextual, peculiar, and complex nature of educational processes, with importance being given to what teachers think about this process, which always includes, in addition to technical aspects, moral dimensions and ethical, emotional, and political.

[4] "Concrete human experience as a description of intentions, through a sequence of events in times and places, where biographical-narrative reports are the privileged means of knowledge/investigation."

Dealing with the characteristics of a narrative, Bruner (1991) helps us think by saying that it is diachronic. This trait is governed by human time, and not by clock time, that is, "it is time whose significance is determined by the meaning attributed to events at their pace." (BRUNER, 1991, p. 6). In it, there are also particularity, links of intentional states; therefore, the focus is on interpretation, which "is related to the reasons why things happen and not their causes." (BRUNER, 1991, p. 7). Furthermore, in this construction, we see hermeneutic "compositionality", as Bruner (1991) says, of an interpretative way of processing knowledge when telling a story and understanding it as a story. We also noticed: canonicity and violation; referentiality, related to the verisimilitude that constitutes narratives, and not to verifiability, as the rationalist or empiricist tradition suggests; genericity; normativity, which is the founding condition of the narrative, as it violates the conventional expectation of discourse, as it "changes with the concerns of the moment and the circumstances surrounding its production" (BRUNER, 1991, p. 15); context sensitivity and negotiability; and narrative accretion, which creates "something quite varied called 'culture' or 'history' or, more loosely, 'tradition'." (BRUNER, 1991, p. 17).

Understanding these characteristics as an expressiveness of the human, Delory-Momberger (2006) states that the subject is established in discourse, understanding as a project of himself. This is possible due to the narrative, which gives shape to people's lives and experiences, which is why "it is the narrative that makes us the character of our life, it is ultimately what gives our life a story: *we do not make the narrative of our life because we have a story; we have a story because we make the narrative of our life."* (DELORY-MOMBERGER, 2006, p. 363, emphasis added).

Sharing the idea of Delory-Momberger (2006), we believe that the narrative brings together, organizes, and addresses the occurrences of human existence thematically, giving meaning to a multiform, heterogeneous, polysemic experience. Thus, we understand life and formation stories as a research-teacher education methodology, that is, as a methodology in which the person is, at the same time, the *object* and subject of formation, based on the discovery and appreciation of the singularity of the subject of formation, which is singular-plural, as Josso (2010a, 2010b) says. It is the subject who is always seeing himself from himself and others, with others, and in interaction with others. With this, a movement is constituted in which the narratives align and reveal the statements and meanings that the narrator gives to his life story and education. Furthermore, as Moura and Nacarato (2019) state, through narratives, we can understand people's future development or change projects. The subject's life story and education can reveal many marks; we highlight the mark of school mathematics, which can be revealed through the narrative.

School mathematics, as we know it today, emerged in Brazil with the reforms of Francisco Campos (nº 19,890, of April 18, 1931) and Gustavo Capanema (nº 4,244, of April 9, 1942). The figure who had a strong influence on the development of school mathematics programs in these reforms was the engineer, intellectual, mathematical educator, and principal of Colégio Pedro II, Euclides de Medeiros Guimarães Roxo (1890-1950), as Carvalho explains (2004, p.120):

> The position of Euclides Roxo, principal of Colégio Pedro II, *ex-officio*, gave him a prominent role, guaranteeing him the position of president of the committee in charge of preparing the mathematics programs in the Campos Reform. Later, although removed from the board of directors at Colégio Pedro II, he played a decisive role in the Capanema Reform.

The mathematical educator incorporated Arithmetic, Algebra, and Geometry into a single subject, which until then had been taught separately. These transformations were inspired by the international mathematics reform movement, led by the German mathematician Felix Klein (1849-1925), which was circulating in many European countries. Furthermore, Roxo took the ideas of the French mathematician and philosopher Henri Poincaré (1854-1912), defender of intuitionist philosophy, to support his proposals for renewing the teaching of Mathematics, adopted by the Francisco Campos reform (CARVALHO, 2004; DUARTE, 2019). The Francisco Campos reform — by incorporating aspects specific to the philosophical current of intuitionism, concerning the teaching of Mathematics, which were not present in traditional teaching, but were proposed by Professor Euclides Roxo, until then put into practice at Colégio Pedro II — gave rise to widespread discussions in Brazilian society. However, following the reform, "we now have only the mathematical discipline, instead of the classic separation into three branches (arithmetic, algebra and geometry)." (DUARTE, 2019, p. 314).

In the meantime, we noticed a relationship with time when dealing with school discipline. There is an increase in the quality of teaching when it is projected into the future. Associated with this, terms such as *evolution and progress* are recurrent in Euclides Roxo's defenses. However, we perceive the action of the subjects in the inscription of the discontinuities that mark the relationship between present and past, understanding the present as a reinvention of traditions, as historian Eric Hobsbawm (2020, p. 8) recalls:

> By "reinvented tradition" we mean a set of practices, normally regulated by tacit or openly accepted rules; such practices, of a ritual or symbolic nature, aim to inculcate certain values and norms of behavior through repetition, which implies, automatically; continuity concerning the past.

From this perspective, we openly find a set of norms and teaching practices still present in society in general that includes a vision, shared by many teachers who teach school mathematics, that this subject is knowledge for the few, for those who are born with the *gift* for Mathematics. These traditions — even affected by the idea of evolution, progress, and new pedagogical trends — do not cease to exist and do not lose their importance, like the old multiplication tables, a device constantly (re)invented by teachers who teach school mathematics in the early years of elementary Education, but which does not lose its invariability, being considered, by many teachers and guardians of students, necessary for them to learn the four basic operations. In other words, the use of the multiplication table is not a simple convention or pragmatic routine; as a tradition in teaching school mathematics, "it is essentially a process of formalization and ritualization, characterized by referring to the past, even if only by imposing repetition" (HOBSBAWM, 2020, p. 11). Therefore, it is important to know and understand the history of school subjects, "spontaneous and original creations of the school system" (CHERVEL, 1990, p. 184), which "forms not only individuals but also a culture that penetrates, shapes, and modifies the culture of global society" (CHERVEL, 1990, p. 184). There is another difference: school subjects have their content, which

> they are conceived as entities sui generis, typical of the school class, independent, to a certain extent, of any cultural reality outside the school, and enjoying an organization, an internal economy, and an effectiveness that they do not seem to owe to anything other than themselves, meaning their own story. (CHERVEL, 1990, p. 180)

Through their content, school subjects take shape and discipline bodies and minds; they doubly constitute the historical product of schoolwork and pedagogical work devices. In other words, every school subject "represents a combination, in varying proportions, of expository teaching, exercises of incitement and motivation practices and an apparatus of tests, exams, and examinations that give them legitimacy and conformation" (CHERVEL, 1990, p. 207). It is a true school culture.

For Viñao Frago (2007, p. 87), "school culture would be, in short [plural], something that remains and lasts; something that successive reforms only scratch lightly, that survives them, and that constitutes a sediment formed over time." Thus, we can talk about school mathematics class cultures, understanding that each teacher who teaches this subject has their culture and peculiar characteristics; however, they reveal verisimilitudes of a community of teachers who teach school mathematics. So, we understand how important it is to think about these cultures to write narratives, taking experience as a path that triggers remembrance, to stimulate the exercise of thinking to help narrate (orally or in writing), that narrating to trigger a reflection that shakes off our tensions, our most hidden feelings, and awareness. When we remember the characteristics of a narrative, proposed by Bruner (1991), we can say that its use is powerful to analyze how undergraduate students appropriated, or appropriate, mathematical concepts throughout their school and academic career and how their representations, related to school mathematics and its teaching, can influence the development of new representations.

In the narratives of the 11 students, subjects of this research, there are elements that we can identify as constituents of school mathematics class cultures. We say this because we also dialogue with Julia (2001, p. 10, emphasis added), who asserts that school culture is

> a set of *standards* that define knowledge to be taught and behaviors to be inculcated, and a set of *practices* that allow the transmission of this knowledge and the incorporation of these behaviors; norms and practices coordinated to purposes that may vary according to the times (religious, sociopolitical, or simply socialization purposes).

For school mathematics, these elements, since its existence, have made it a subject considered to be of great use in the lives of citizens; its curricular structure undergoes few changes; and the influence of pedagogical trends is almost nil, even with the impact of psychologization, considering the understanding of Mathematics as a dead, ahistorical, ready-made, and finished science. This impacts the crystallization that learning school mathematics is not the easiest or most enjoyable task. Thus, many expressions (*Mathematics is difficult, Mathematics is boring, I can't understand it, I have a horror of Mathematics, it is the school's monster*) frequently uttered at school or outside reveal a certain pessimism towards teaching and learning of this school subject. This can be a barrier not only for the work of the Basic Education teacher but also for the teacher education undergraduate courses in which academics will be able, upon taking up teaching, to teach school mathematics.

In the following section, we present the narratives of the subjects of this research. We reveal the hallmarks of school mathematics and the challenge faced by the trainer of future teachers who will teach this subject in Early Childhood Education and the early years of Elementary School.

NARRATIVES FROM PEDAGOGY GRADUATES ON SCHOOL MATHEMATICS AND THE TRAINER'S CHALLENGE

When referring to the hallmarks of school mathematics, we dialogue with Dominicé (2010), who understands them as those that we carry with us. Some are left by acquaintances, institutions, important figures in our lives, and/or family members; others are printed by ourselves since the formation process has similarities with socialization: both are based on relationships. Brands occupy an important place in narrative and formation. Thus, we understand that many representations of school mathematics that undergraduate students have come from brands printed, during their student careers, by professionals who teach this subject in basic schools. We also understand that these marks, when they are blockages, are challenges for trainers. Therefore, we see this statement by Nacarato, Passos, and Carvalho (2004, p. 10) as plausible:

> One of the great challenges for teacher educators who teach or will teach Mathematics – Pedagogy graduates – does not only lie in breaking down barriers and blockages that they bring from their Basic School mathematics education but, mainly, in raising awareness of these facts, bringing them to the surface so that they can be an object of reflection, overcoming and (re)signification.

This awareness about which Nacarato, Passos, and Carvalho (2004) write was the challenge that the teacher-researcher took on in the process of education 11 students on a Pedagogy course at a federal university in the Brazilian Northeast. One of the moments of interaction and discussion occurred about Fundamentals *and Methodology of Teaching Mathematics*, whose syllabus indicates what should be taught:

> Mathematics content in early childhood education curricula and the initial years of elementary school. Study and analysis of official curriculum proposals, at national, state, and municipal levels. The content-form relationship in mathematics teaching. Psychological foundation for teaching numbers and operations; space and shape, magnitudes and measurements, and processing of information based on mathematical language and its relationship with the student's daily life, when solving problems. Theoretical-methodological procedures based on trends, theories, and mathematical principles relevant to the development of interdisciplinary pedagogical practices. Analysis and use of teaching resources and specific assessment procedures in the field of mathematics education. (FEDERAL UNIVERSITY OF MARANHÃO, 2007, p. 63)

We try to use as a flagship the teaching of numbers and operations, space and shape, magnitudes and measurements, and processing of information to discuss the other themes imposed by the syllabus. We confess that it was not easy at all, but, based on the students' narratives, we reorganized ourselves, looking for paths and alternatives that would help us in classroom discussions about the topics on the syllabus.

The students who participated in this investigation gave their real names, through authorization in the Informed Consent Form. We highlight that, in biographical research, especially the one using formation memorials, it does not make sense to use pseudonyms to name the subjects unless revealing the names constitutes a real risk to their integrity, or if there is some legal aspect that prevents disclosure, which was not the case in this research. On the contrary, the characteristics are the result of the self-writings of the formation memorial that they built, their writings reveal their identities, their marks, and the way they want others to see them. In the type of research, we carry out human beings are subjects of culture, history, and knowledge. The subjects' identities are unique and non-transferable, their names identify what they want us to know about them (BOLÍVAR; DOMINGO; FERNÃNDEZ, 2001). Consequently, we cannot leave them absent, we must provide them with recognition in the text that is written about them and in their stories so that we can read the writing based on and from their narratives, which is why we use the real names of the 11 students.

The graduates are Camila, Cíntia, Daniel, Hilsaneide, Keiliane, Luciana C., Luciana N., Patrícia, Vanda, Raquel and Teresinha. Their life stories intersect with those of the Pedagogy course at the university where they studied. These university students write the narrative of their lives, so they have a story to tell, full of marks, inconsistencies, challenges, and victories. Such narratives show a body that feels, that looks, that creates a posture based on itself or others, that has actions and sayings, a body in formation.

Next, we organize the narratives into two subsections. The first reveals the marks left by basic education with an emphasis on multiplication tables. The second refers to the possible meanings produced in the Pedagogy course. In each of them, the challenges for the trainer are problematized.

Marks of basic education: the multiplication table in question

When we proposed to students that they write a formation memorial during the disciplines *Fundamentals and Methodologies for Teaching Mathematics*, Fundam*entals and Methodologies for Teaching Science, Internship in Teaching Pedagogical* Disciplines (in early childhood education), and *Internship in Early Years Teaching I and II of Elementary School*, we found many marks printed on their bodies that became challenges for the teacher. The first is narrated by Patrícia, a 23-year-old student at the time, who tries to associate her choice of the Pedagogy course with her escape from the field of Exact Sciences:

> Until then, my only experience with the exact area was a traditionalist experience. I remember that, when I was a child, I did small calculations using my fingers and I couldn't even do that in the classroom, the answers had to be on the tip of my tongue. Many of my current college classmates said that they also did not have a good experience and that is why they would not choose a course in the exact area, as they did not have an affinity with it. I also remember that the tasks were those that smelled of alcohol or tasks in the book, only counts and problems, which never interested me due to the mechanical way that was taught to me (PATRÍCIA, MF, 2017).

The prohibition of using the body or part of it to help with tasks was very common for many teachers who taught school mathematics, perhaps it still is for many others. Patrícia, who was born in the late 1990s, must have entered the first years of Elementary School in 2002, as she would have been 7 years old. It surprises us that, after so many curricular reforms that were suggested in the 1980s and 1990s, there was a ban on using fingers as an aid in school mathematics classes. At the same time, we remember the idea of tradition proposed by Hobsbawm (2020), associated with the school culture defended by Julia (2001), mainly the idea of norms that determine knowledge to teach and behaviors (not using fingers) to inculcate. Therefore, the tradition of teaching mathematics at school, expressed in the student's narrative, is resistant.

According to Patrícia, many colleagues also said they had no memories of a good experience with school mathematics and, therefore, would not choose a course in Exact Sciences. This is the case of Teresinha. In her memorial, she recounts two experiences that left a deep impression on her. Then, she reveals what she managed to do with what they did to her during elementary school, especially in school mathematics classes. In the following excerpt, she narrates an experience she had and her relationship with the teacher who taught her school mathematics:

> I remember that, in one of those tests [she mentions an oral assessment with the multiplication table], I got the multiplication 9 x 6 wrong; Unfortunately for me, the person who asked the question was my brother, whom I always got the better of at school and outside. So, my brother decided to take revenge; and in the end, I was still punished for not getting the question right. After the class ended, the teacher released all the students, and it was just him and me in the room. It was past noon, and I was still there, without lunch or snacks; he wanted me to study more and more, but he couldn't concentrate anymore, he was hungry; and he, to show that I needed to be punished, did not release me [...]. From that day on, I started to hate him, and my admiration for him went down the drain. As a result, the subject of Mathematics had the same status as him: I also started to hate it. For me, that punishment was completely unnecessary. I understand that making mistakes is human, and the role of the teacher is to teach and guide. (TERESINHA, MF, 2017)

Highlighting the figure of the teacher and culture of school mathematics classes narrated in the excerpt from Teresinha's formation memorial, we raise some questions: in academic teacher education courses for Early Childhood Education and the initial years of Elementary School, what has been made to deconstruct this type of feeling that Teresinha narrated and so many other course participants may have? Under what conditions do these same teachers find themselves with their school marks to problematize them or provide these students with moments and/or situations for individual or collective reflection? Looking at practicing teachers, we ask: how can teacher educators help them reframe their school brands? How can these brands be/present paths of awareness for teaching professionals?

The prominence of the figure of the teacher is associated with the importance he has in our life and education trajectory. The teacher that Teresinha mentions in the excerpt of her narrative made her hate the school mathematics subject because she hated his attitude towards assessment. She extended the disappointment she had in her teacher to school mathematics, as, before, the student felt great affection for him. Teresinha narrates that, at the time he arrived to be a teacher in the multigrade room where he studied, he was received with celebration and joy: "he was very loved by everyone, he was treated like a doctor. A man of few words, always serious, and I liked him a lot." (TERESINHA, MF, 2017).

But Teresinha started to hate him and built the same feeling for school mathematics: "I started to hate him, my admiration for him went down the drain. As a result, the subject of Mathematics had the same status as him: I also started to hate it" (TERESINHA, MF, 2017). How many of our attitudes can arise from the same *métier* as Teresinha's teacher or from something that is related to this situation? How can our attitudes favor the construction of a feeling in a school subject? In Teresinha's case, the feeling lasted until she found other teachers who could, even indirectly, encourage her to approach school mathematics: "[in high school] Mathematics classes were calm, I remember three teachers, each in their way, but all competent. I managed to get good grades, it was the subject that interested me most, I liked calculations; So, in the subjects that required calculations, I did well." (TERESINHA, MF, 2017).

The student does not reveal the reason, other than the teachers' competence, for her becoming interested in school mathematics in high school. Perhaps she realized that this discipline was not as horrible as she had imagined or discovered that she liked it, but maintained, for a while, a hateful relationship with it. It is visible in Teresinha's narrative that all the embarrassment she went through is linked to not knowing the multiplication tables on the *tip of her tongue*.

For Nürnberg (2008), the different meanings given to the multiplication table, which make up its status in the teaching of school mathematics, were established in the course of changes and the emergence of pedagogical trends, each contributing to the attribution of different meanings. In times of traditional and technical pedagogical tendency, the multiplication table was surrounded by stigmas and

constituted a small contribution to school mathematics, being indispensable for children to advance in arithmetic operations. In many cases, memorizing the multiplication table was an extra-class exercise or even synonymous with punishment.

> The focus given to the multiplication operation is the multiplication table. The fundamental thing was to teach how to do "times" calculations. Thus, it is understood that the first condition for successful learning was to memorize the multiplication table, that is, mere memorization. Furthermore, it was associated with punishments specific to mathematics teaching. When the student "disobeyed" the teacher or failed to fulfill obligations that the school determined, they were forced to write one, several, or all the multiplication tables from 1 to 10 several times. (NÜRNBERG, 2008, p. 32)

With the advent of the renewed progressive and non-directive trend, the multiplication table came to be seen as an empirical concept that, through experimentation or the associative process, must be memorized and learned. In Nürnberg's (2008) understanding, the golden material, the colorful Cuisenaire bars, and the language of ordered sets and pairs work on the well-known multiplication table, even with changes in the way teaching procedures were governed. However, its historical penetration into the ideas of teachers and the community did not disappear overnight. We suspect that, for this reason, many students in the Pedagogy course who studied the *Fundamentals and Teaching Methodology of Mathematics* subject resisted the idea of another device to deal with arithmetic operations.

This resistance is related to the invariability of the multiplication table considered by licensees necessary for children to learn the four basic operations. This leads us to understand the use of the multiplication table as a tradition, in the terms of Hobsbawm (2020), in teaching school mathematics, not being a convention or pragmatic routine of teachers. It is fundamentally a process of formalization and ritualization, marked by reference to the past, even if this is revealed only by the imposition of repetition.

Thus, over the years, the teaching of multiplication tables acquires a character of strong memorization, even precocious, to later be associated with the use of songs, rhymes, tongue twisters, games, and jokes, among other didactic resources conceived as engaging and playful, to alleviate the painful memorization (NÜRNBERG, 2008). This association causes some relief for teachers, who, criticized for teaching multiplication tables via strong memorization, now have devices that would slow down this process, which were characterized as alternatives by students. We analyze this issue in the following subsection.

Possible meanings produced in the Pedagogy course about the marks of school mathematics

In the Pedagogy course, Terezinha found school mathematics and made two highlights about the subject of *Fundamentals and Mathematics Teaching Methodology*. One is about manipulable materials, and the other is about the relationship she establishes with her teacher.

Regarding manipulable materials, the teacher-researcher offered three to be analyzed by the students, who, during the course, should reflect on these tools and use them. The instruments available were the golden material, which can also be called Montessori material or multibase 10; the abacus; and the Tangram. These and many other materials already have a long history in teaching Mathematics worldwide and in Brazil. Nacarato (2005, p. 1, emphasis added) provides us with some information that can historically situate the use of manipulable materials:

> The use of manipulable materials in teaching was highlighted for the first time by Pestalozzi, in the 19th century, when he argued that education should begin with the perception of concrete objects, with the performance of concrete and experimental actions. In Brazil, the discourse in defense of the use of teaching resources in Mathematics classes emerged in the 1920s. This period was marked by the emergence of a trend in Mathematics teaching that became known as empirical activism, arising from New School ideals that were opposed to the traditional teaching model in which the teacher was seen as a central element of the teaching process.

The author questions whether the use of manipulative materials for teaching school mathematics could be a facilitator or a complicator. The teacher-researcher, during classes in the subject

Fundamentals and Methodology of Teaching Mathematics, asked students questions about how to use the manipulative materials, what mathematical content could be mediated with them, whether their use would only be to visualize basic notions or whether it would be appropriate to use them to exhaust this or that mathematical content. These questions hovered throughout the course because the teacher avoided answering them. The aim was for us to find the answers together, talking and reviewing mathematical concepts, even with the limited time we had, as the subject lasted 60 class hours and was the only one that covered the foundations and methodologies of Mathematics.

Terezinha, like the other subjects of this research, found it difficult to understand the manipulation of materials, the context in which they could be used and the mathematical relationships to be represented by them. Terezinha's narrative (MF, 2017), highlighting her concerns about some discussions we had about the golden material, clearly reveals the difficulties:

> Some subjects caught my attention, Fundamentals and Methodology of Teaching Mathematics and Fundamentals and Methodology of Teaching Science, when I saw them offered for the course, I confess that, at first, I didn't understand much. I thought, "Am I going to see concepts or operations again?" [...] In mathematics, I learned from the teacher that it can be fun if you work with the game, avoiding the traditional, but I realized that the game that was presented to us is too complex to be able to make it happen in the classroom. For example, when we worked with the gold material, I hadn't thought about the mistakes in representing it in two-dimensional form, instead of the correct way which is three-dimensional. I didn't even notice this difference, so I said that what was in the book was correct. I didn't understand the relationship between the pieces of material. I keep imagining that you need to know mathematics well to be able to teach mathematics through playing.

The student's other colleagues report similar discussions. For them, it was amazing to discover these elements, which they called details. These can complicate the teaching of school mathematics if the teacher does not know the mathematical concepts and does not carry out his analysis correctly before proposing activities with manipulable materials that contribute to children's learning.

Two more cases caught our attention. One is the story of Raquel, who has worked as a daycare manager for many years. The other is the narrative of Cíntia, who is a young student. Both were unaware of the existence of the golden material. Raquel's narrative (MF, 2017, our emphasis) reveals her lack of knowledge about this resource and causes us concern:

> Recalling the moment of interaction between the materials, the practice in the toy library guided by the teacher led me to see that there is a universe of activities before our eyes that, if used correctly, allows the child to discover the knowledge of learning in an autonomously, and we will be more successful in teaching and learning. *The use of gold material was a big surprise for me, because, even with so many years of working in schools, I didn't know this material or how to use it*. The readings proposed by the teacher and the activities with the material left me distressed due to my lack of mathematical knowledge to be able to use it. I kept imagining a student asking me a question and I didn't know how to answer, because I couldn't, alone, explain the relationships between the elements of the material such as the plate and the cube, the idea of counting, operations, and recording, especially doing subtraction with reservation. Even to teach mathematics to young children, you need to know the mathematics content well. I'm glad I didn't miss this class.

Cíntia went through school and learned school mathematics which allowed her to learn and be promoted to the following years of studies until she reached Higher Education, but she says she was unaware of the existence of the golden material. Furthermore, even though she had some kind of knowledge about the abacus, she had never handled one. She also highlights her concerns about knowing the mathematical content and making use of these manipulative materials:

> Among the classes throughout this subject, the one that "marked" me the most was the class that took place on January 14th in the toy library, where we discussed the pedagogical materials used in teaching mathematics (abacus and golden material). Through this class, I was able to visualize and mainly handle the abacus, a material that I already knew because it was discussed a lot in classes, but I had not touched. I didn't even understand the relationship between orders and classes; for me, it was just seeing the colors and knowing that each one has a different value, 10 out of 10. I started to understand the importance of the position that the ring occupies to

> represent a value, the so-called relative value. As for the golden material, I did not know; For me, it was new. Maybe because I hadn't learned any of this at school, I didn't even know that it was necessary to do these difficult analyses to teach with the pedagogical material. (CÍNTIA, NP, 2016)

There is evidence, in the students' narratives, that, through mediation, they understood that the inappropriate use of all manipulative materials will not contribute to the mathematical learning of Basic Education students. On the contrary, conceptual mistakes can be made by teachers if they are unable to carry out the analysis narrated by Cíntia, Raquel, and Terezinha in the excerpts above. This statement has already been made by Nacarato (2005, p. 4), who alerts us to how manipulative materials are used: "An inappropriate or little explored use of any manipulative material will contribute little or nothing to mathematical learning. The problem is not in the use of these materials, but in the way in which to use them."

It seems that there was a maxim in the imagination of students and, for us, in many thoughts of teachers who teach school mathematics in Early Childhood Education and in the early years of Elementary School, that, to combat the harsh and cold image of school mathematics, it would be essential to teach it using manipulative materials. Thus, it would be taught through play. However, those who have this view forget that each type of material only manifests through how human beings use it. It is not the material that will improve the teaching of that school subject. This was a culture of school mathematics classes that was often narrated by students, and it was costly for the teacher-researcher to try to deconstruct it.

We are not opposed to teaching school mathematics using manipulative materials. However, we emphasize, in agreement with the research of Nacarato (2005), that there is an urgency for understanding and apprehension by those who embark on teaching and will teach school mathematics in Early Childhood Education and in the early years of Elementary School, of the concepts mathematical, of the meanings that can be constructed and negotiated through the use of manipulable materials and the mathematical relationships to be represented in and with them. It is not possible to teach anyway, nor to use these resources just for the sake of using them.

There is a second highlight of Terezinha in her formation memorial: the relationship she established with the professor-researcher. She narrates another teacher's attitude that enabled her to get closer to Mathematics and expand the potential for teaching this area of knowledge. In her pedagogical narrative, she reveals what the meetings were like in the subject *Fundamentals and Methodology of Teaching Mathematics*:

> Counting on their understanding was important to partially overcome the problems experienced. With his welcoming, calm, and safe way of transmitting content, I was able to realize that mathematics is not a big deal and that we can, with effort and dedication, develop satisfactory work, even though understanding mathematics without knowing its foundations and His methodology makes clear the need for us to learn how to learn the content. (TEREZINHA, NP, 2016)

We understand that this relationship favored - not only in her case but also in other students - the construction of bonds and trust, resulting from the sensitivity of seeing, feeling, and perceiving the need to be together with the students, helping them to remember and to think about the facts and what marked them, why this brand is impactful and how to help them transform.

The elements mentioned above are part of the quality of the relationship. For Contreras (2018), the essence of a relationship is based on respect, moving away from the idea of hierarchy, and requiring authority conquered and built by both parties, rejecting authoritarianism. Quality is seen in the sense of the incarnate (NAJMANOVICH, 2001). Therefore, this aspect becomes the flagship of the movement to record student narratives, even though the institutional aspect of the production of the formation memorial and pedagogical narratives is clear.

This quality has helped us respond to our concerns regarding our life path and education. It provided us with devices, offered by students, for us to repeatedly ask ourselves: In pedagogy courses, what has been done by trainers to deconstruct the various bitter feelings that students narrate concerning teaching and learning mathematics school? Under what conditions do these same teachers find

themselves with their school marks to problematize or provide these students with moments and/or situations for individual or collective reflection?

Contreras (2018) has raised some warnings about the quality of the relationship. For him, we usually need to ask ourselves what keeps a relationship alive. What makes it keep pulsating, and be continuous? Are we capable of, by seeing certain qualities of the relationship, transforming ourselves into learners and good listeners? If this happens, a new possibility of being in the world appears, allowing growth, and opening other possibilities. But, if this doesn't happen, what can we do, who can we turn to, and what strategies can we use to (re)exist? To this end, Contreras (2013) suggests attentive listening, as it has the purpose of continuing the relationship and expanding the conversation, so as not to know what the other person is like, what their characteristics or attitudes are, just listening to them.

This is because the relationship, from the perspective of Contreras (2018), is always personal and, therefore, abstract. It is not given; it is necessary to enter a relationship. It is necessary to give in and not enter a relationship beforehand, as it is unpredictable. It is circular. It may be the subjective exchange of subjectivities, that is, it involves receiving and giving. It is always open to events. It is mysterious because we have the mystery of the other, the mysteries of the between.

These mysteries can also be found, indirectly, in teacher practices and/or recreated practices. They can be teaching cases. The teacher-researcher used them and still uses them as a education device in his classes in the subject *Fundamentals and Teaching Methodology of Mathematics*. In the understanding of Nacarato, Mengali, and Passos (2009, p. 137), "cases can be used as education practices (initial or continued) because they enable the mobilization and production of theoretical and practical knowledge, as well as values and conceptions – by both those who analyze and those who build the case."

However, for a teaching case to be considered as such, it is necessary to understand it as inserted in contextualized knowledge and connected to teaching action. It differs from narratives because it provides "a classroom problematization and theoretical knowledge underlying it, leaving the reader with possible interpretations" (NACARATO; MENGALI; PASSOS, 2009, p. 134). This exercise is another of the trainer's challenges.

By associating the teaching cases with the students' autobiographical narratives, other cultures of school mathematics classes appear and become a constant, for example, learning the multiplication tables. In the subject of *Fundamentals and Methodology of Teaching Mathematics*, the teacher-researcher exercised the art of questioning with the students: *"are we going to think that the multiplication table never existed and that you will have to teach arithmetic operations to children? So, how to proceed? Which route to walk? What materials would you use?"* These questions led us to find other artifacts, for example, the number line, but required us to think about the process carried out to solve arithmetic operations.

This exercise was slow and difficult for the students, as a whole set of practices that allowed, and still allow, the teaching of multiplication tables in school mathematics classes created in them, and possibly has created in other students, almost untouchable behaviors. Even defending the end of memorizing the multiplication tables, do they question how to know it *by heart* without knowing the multiplication table? Thinking about the non-existence of multiplication tables in school mathematics classes is almost impossible for many students, including those who have suffered punishment for not having the answer *on the tip of their tongues*. It is the tradition mentioned by Hobsbawm (2020) and the school culture that defines practices for transmitting knowledge and incorporating it into minds and bodies, as Julia (2001) says.

This was the case, mainly, of Daniel, who taught school mathematics, as an intern, in the third-year elementary school class at a public school in the municipal education network. He narrates how he planned the class and how he worked with the children:

> Another important lesson was teaching the 6th multiplication table to the class. But we [he and his internship colleague] worked on a different multiplication table, involving games, and making things in practice. We used A4 paper and asked them to stay in pairs. The objective, in addition to knowing the place of 6, was the interaction between both genders since in this age group the distinction between what is a boy, or a girl is very strong. Next, we asked each student to pay attention to the multiplication table of 5 that their teacher had already taught because today we were going to teach the multiplication table of 6. On the blackboard, we placed a poster with all the multiplication tables written, but without the answers [on the memorial of formation there is an image that shows this attitude]. We asked them to write in their notebooks, just as it was

> on the poster, then we started asking how much 6x1, 6x2 is... But it wasn't just asking. We showed them the relationship between one answer and the next, for example: 6x1=6; but 6x2=12 because it is 6 twice, so just add 6+6 which gives 12, and so would be the others. Then we played multiplication tables bingo, reviewing from the 2nd place to the 6th place. I confess that it was a very funny and exciting situation, as everyone wanted to get it right, but when a couple said bingo and we checked the correct answers, one or another error would appear, and we would continue bingo. The students got involved in the activity and learned through playing. (DANIEL, MF, 2017)

During the biographical mediation, that is, during the construction of the formation memorial, the teacher-researcher tried to make Daniel think about his practice as an intern. Then, he raised the question: *"Even though you claim to have worked playfully with the children, do you think there are other aspects besides playfulness in the task?"* He also stated, with a reflective tone: *"I understand that you and your colleague are trying to look for alternatives to teaching mathematics to children but take a good look at the path you have followed."* These mediations aimed at making him realize the existence of the memorization nature of the proposed task.

In our understanding, Daniel created a class in the internship somewhat like the ones he had when he was a Basic Education student. We imagine that his way of seeing school mathematics and his desire to better understand its more complex concepts led him to teach the multiplication tables proposed by him, even incorporating *elements* that could characterize games. Therefore, his emphasis may be on the development of the algorithmic process, on the linear way of seeing this process, and on the (mis)use of Arithmetic, leaving aside all the potential that the number line offers.

Valente (2016) highlights Arithmetic in other themes in school mathematics. His historical analysis of the teaching of this content in the last decades of the 19th century shows that it was necessary to consider intuitive knowledge as the first form of knowledge, which is why active teaching comes into play.

> It is up to the teacher to actively teach arithmetic in the first years of school. The teacher's active teaching, considering intuitive processes, has the great challenge of overcoming the legacy of memorization. In the case of arithmetic, memorizing the multiplication tables. Knowing the multiplication tables by heart is an icon of the times to be overcome, there is a need to construct alternatives that break with memorization, without doing without the tables, the calculation tables. (VALENTE, 2016, p. 17)

The alternative found by Daniel was, according to him, to build with the students the square for the number 6 of the multiplication table, and the bingo game was a device to review the previous squares, an activity that he called playful. However, we question: what meanings could be constructed and negotiated from the use of this material with the mathematical relationships to be represented in and with the material? Daniel uses the manipulative material as illustration and/or support and his focus was on memorizing the multiplication tables. At this time, the teacher-researcher wonders whether his classes in the subject *Fundamentals and Teaching Methodology of Mathematics* did not contribute to thinking about school mathematics from elsewhere, to overcome the marks that his students had in the Basic Education in which they studied.

We asked ourselves about the space-time of the internship and the limitations of each intern in what the supervisor and the teacher want the student to teach and/or develop in the classroom. Generally, the student begins his internship at a time when the school's students have been developing a classroom culture led by the teacher for a long time. Therefore, incorporating alternatives that open fissures in the pedagogical practice of the teacher can be a complicating factor, not a facilitator. But we believe this is not a rule.

Let's see what Terezinha says about her performance in the *Teaching Internship for Early Years I and II of Elementary School*. She and her internship colleague developed their activities in a first-year elementary school class and were instructed by the head teacher to work only with Portuguese language and school mathematics.

Therefore, Terezinha and her colleague used playful activities to teach school mathematics to children. She remembered the discussions held in the *Fundamentals and Teaching Methodology of Mathematics*

subject on the topic and built her strategy to appropriate mathematical content and, thus, be able to teach it to children:

> Thanks to the mathematics and art teachers we had during the pedagogy course, who introduced us to play as an indispensable resource in the classroom, we decided to follow in the teachers' footsteps and apply them to the classes taught in the 1st grade during our teaching. I confess that it wasn't easy at all, as I remembered the golden material class and the need to know the content to be able to teach. I always turned to my math and Portuguese teacher friends when I had doubts about the content I was going to teach with the game, and of course, I asked my internship teacher how to relate material x with the math content. I realized that, for me, it was difficult, but for the students, it was easier to understand the content, with greater participation, and interaction among the entire class. (TEREZINHA, MF)

This was also Patrícia's feeling when taking her internship classes in the first year of Elementary School. When doing a magnitude and measurement activity with the children, she remembered our meetings in the *Fundamentals and Teaching Methodology of Mathematics* subject, in particular the discussions about the multiplication tables. She begins the narrative by saying:

> I will talk about the discipline that, for me, was always very difficult to deal with, but that, through the teachings of the teacher [name of trainer], I managed to do a better job. In the mathematics subject, when we started, the students were studying magnitudes and measurements, and one of our first classes was working with mass. To do this, we used the construction of a homemade scale. The textbook proposed a very simple activity for children to weigh some objects and observe which was the lightest and the heaviest. I soon remembered the mathematics classes at college that asked us to think about situations that were related to the mathematics content and, also, to the level of the children I was teaching. All that came to mind was our clashes in multiplication tables class... (PATRÍCIA, MF)

Patrícia also narrates a class she developed with her colleague from the *Pedagogical Disciplines Teaching Internship held in Early Childhood Education*, held in a daycare class in the municipal education network, showing her discoveries and the joy of 2- and 3-year-old children:

> We did another fitting activity with the children. My colleague and I took a large bunch of grapes made from cardboard and the children needed to fit together the purple balls of different sizes to form the bunch of grapes. Our goal was for them to notice the variation in the size of the balls and try to fit each ball in the correct space. They would put a ball in one space, see that it didn't work, take it out, and try it in another. Even when a child placed the ball in a space that wasn't correct, but left it, another child would come and watch, take it away, and look for another place to fit it.
>
> We noticed that children enjoy fitting activities. Even the children who had a little more difficulty managed to do it. This type of activity is like a challenge... I even they wouldn't be able to do it, but through the internship classes, I noticed that challenges are essential in Early Childhood Education and children like to demonstrate that they can achieve the goals we propose to them. (PATRICIA, MF)

In her pedagogical narrative, at the end of the *Teaching Internship for Early Years I and II of Elementary School*, Patrícia (NP, 2017, our emphasis) carries out a self-assessment and points out elements, highlighting her experiences with multiplication tables and classes in the internships she completed:

> Upon finishing my internship and writing this pedagogical narrative, I have to say about the experience when the professor [name of the teacher-researcher] tried to deconstruct our idea about the multiplication table. I always found the way my teachers taught me the multiplication tables boring, but today there are many alternatives for children to learn the multiplication tables, and I think that's very valid. Now, using the number line with the other things it presents, I've never seen it before, and I think it's very difficult for children to learn. But, after the classes I gave in the internships, my vision is changing, because what is difficult is not so much for the children, but for the teacher who learned one way and believes that it is the best way to teach. I experienced this firsthand when my colleague and I were planning. Working with the scales in the 1st-grade class and the fittings with the daycare group made me very excited to think about

> alternatives, even though I had to study things that I didn't learn at school, to explore the children's abilities and even make me study more because mathematics is not my thing.

In line with Daniel's report, the two highlights we made in Patrícia's narrative give us the conditions to think about the transformations that the multiplication table underwent until it reached the present day, as highlighted in the previous subsection. They also reveal signs of their awareness of the transfer we make, from ourselves to the children, when we say that this or that type of activity is difficult for them.

The second idea reported by Patrícia, transferring the idea of the difficulty of the task from us to the children, is quite common, but should not be taken for granted. We are not blaming Patrícia for making this transfer. We had already done this when we taught Basic Education for a few years, and our awareness only came with the development of our research, with the guidance provided, in discussions at conferences and scientific events and in the research group in which we are part of — History of Formation for Teachers who Teach Mathematics (Hifopem)[5]. We are happy that Patrícia realized, even as an undergraduate, this peculiarity that many fellow teachers take time to realize before becoming aware of the need to change this pattern.

The highlight made by Patrícia is also in the narrative of Hilsaneide and Luciana C. However, these two set out to search for greater possibilities than what they learned at school and/or knew from hearing about it. The two, who were interns in the first year of Elementary School, prepared two classes on estimation but used different paths. Hilsaneide (MF, 2017) says:

> During the course, there was a need to develop a lesson plan that addressed estimation, but the textbook did not contain sufficient information and instructions. So, we carried out research and looked for a more appropriate and easy-to-understand way for 1st year students. This worried me a little, as I didn't know how to approach this topic; So, to obtain a satisfactory result, my colleague and I had to study and look for explanations from those who knew and even understood how to work with estimates with 6-year-old children. We even remember mathematics classes with Professor J. The objective of the class was for the student to make estimates in everyday situations, having a notion of approximate values of objects and foods, and analyzing different ways of making estimates. We conceptualize for the students what an estimate would be, with examples, and we move on to some situations, such as estimating the value of some objects, such as clothes, and school materials, among others. We distributed fake money to the students: R$2.00 bills; R$ 5.00; R$ 10.00; R$ 20.00; R$50.00; and R$ 100.00. With the banknotes, the children imagined how much each object drawn on the board cost and pasted the approximate value onto the picture.

From what appears in Hilsaneide's narrative, they got together, studied the topic, asked for help from other people, and prepared tasks for the children, as the information they had available in the textbook was not enough to move forward with the children and, perhaps, caused them even more doubts. We highlight the collective work, the sharing of ideas, and the joint commitment they had, which could also be carried out by practicing teachers, as, in the municipal network where the internships were carried out, teachers have the right to seven hours of activities outside the classroom, guaranteed by federal law.

The importance that Hilsaneide gives to this exercise of sharing ideas and doubts, group study, and respect between the two, for us, reveals how much she and Luciana C. have grown as people, as students, and as professionals who will one day practice teaching. Below is another excerpt from Hilsaneide's narrative (MF, 2017):

> I can say, from my own experience in this internship, carried out in pairs, in which my colleague and I met several times to plan the classes. In these exchanges of knowledge, we learn to listen

[5] The group was created in 2010 and is certified in the Directory of Research Groups of the National Council for Scientific and Technological Development (CNPq), bringing together Postgraduate and Postdoctoral students and former researchers who still maintain ties with the group. It focuses on the (auto)biographical method, dedicating to studying and analyzing research that takes narratives from their multiple perspectives: pedagogical narratives, life and pedagogical narratives and narrative research.

> to and respect each other's ideas and opinions, not to mention sharing the experiences and practices that would be most appropriate for each class.

For us, this is an experience that we need to live constantly, as there are still few moments of exchange that we have with our work colleagues. We live on our islands, surrounded by doubts and concerns. In many cases, we don't realize that we need others for our progress, to realize how fruitful it is to share experiences.

We ended up doing what Patrícia did: transferring our difficulties to our students and, thus, failing to experience something more daring and/or creative. We forget to look at ourselves, see our limitations, and ask for help from others, who are also one of us. We create models to guide our actions, in many cases, based on theories of self that camouflage our weaknesses and silence many students.

We need to say that problematizing school mathematics class cultures was not and has not been something simple for us. The culture of play for play's sake, using manipulable materials, but without due theoretical-methodological care to teach school mathematics, has been one of the issues that stand out most in the student narratives we receive when we teach the subject *Fundamentals and Teaching Methodology of Mathematics*. Another culture of school mathematics classes is the teaching of multiplication tables, as already mentioned. We still see a few difficulties in students' defenses against using the multiplication table, even if they agree with the idea that memorization should be discarded.

These 2 school mathematics class cultures were the ones that marked this research with the 11 students from a Pedagogy course at a federal university who agreed to participate in it. Our solution has been to establish a relationship of trust, and proximity, creating qualities for the relationship. We believe that the quality of the relationship has a direct link with teacher education, because, for Contreras (2010), deciding to train as a teacher is to do something with yourself, it is to think about what you did with yourself during your education journey as well as what the person does to themselves. The path we took in this research to encourage students to discuss the cultures of school mathematics classes and produce writings about themselves in the formation memorial can enhance thinking about this movement, to raise awareness.

Evidence of awareness can also be found in Cíntia's narrative. In the dialogue of the discussion-reflection group, the student treats teaching school mathematics in daycare as a challenge, which we think is not hers alone. Difficulties can be resolved and/or reduced with the appropriation of mathematical concepts and the relationship created between them and the manipulable materials, as we have said previously. Cíntia (NP, 2016) talks about this in her pedagogical narrative:

> I want to emphasize that, regardless of the classes, what struck me most was the "reality shock" when we were constantly asked if we wanted to be teachers if we were capable of really teaching, and mainly, that to be able to teach we must seek to learn what we are going to teach. And like I said, I have a lot of difficulty with mathematics, because I learned little at school, and now that I must teach it, I find myself questioning it, but as if I don't know the content.

Knowing the mathematical concepts that will be taught, that is, knowing the mathematics that will be taught, is one of the concerns that, every semester that the teacher-researcher receives students in the subject of *Fundamentals and Methodology of Teaching Mathematics*, haunts him. How can a future teacher who will soon take up teaching teach this or that mathematical content if he/she does not yet know that content? If the graduate already knows, he needs to go further, as Freire (2008, p. 95) says: "I can't teach what I don't know. But this, I repeat, is not knowing that I just must speak with words that the wind carries. It is knowing, on the contrary, that I must live concretely with the students."

To concretely experience this knowledge that Freire (2008) talks about, we studied the syllabus (shown above) of the subject *Fundamentals and Teaching Methodology of Mathematics,* as it is gigantic and caused us discomfort, strangeness, and restlessness, as we wanted to bring together many themes in a very small space-time. Furthermore, there is the psychologizing approach that guides the discipline. The search for paths and alternatives that could help us in classroom discussions about the topics on the subject's syllabus was and still is my premise when we stopped to plan the subject before the academic semester began.

One of the paths we have followed is inscribed in biographical studies. The use we make of the autobiographical narratives of students in the Pedagogy course has enabled a fruitful education practice, since their narratives show, in the words of Nacarato, Mengali, and Passos (2009), beliefs concerning teaching and learning Mathematics, contain marks from historical moments in Mathematics Education and also allow us to identify pedagogical trends from a given time and place.

FINAL CONSIDERATIONS

This article addressed school mathematics and the academic education of future teachers at a Brazilian federal university, to analyze how future teachers appropriated mathematical concepts throughout their school and academic career, and how their related representations of school mathematics and its teaching can influence the development of new representations. The students' autobiographical writings became objects of problematization and reflection for both them and us. We get to know their trajectories and their ideas about school mathematics, the representations constructed throughout their student itineraries for this subject. Therefore, these writings were taken as a starting point for our work as trainers, to help us in the (de)construction of representations and practices relating to Mathematics. This is because we believe that narratives make it possible to organize experience.

Believing in the above possibility, we see that the 11 students who participated in this research used their narratives to organize their experiences as elementary school, high school, and academic students. They realized how much a teacher who taught them school mathematics could leave a positive or negative impression on them, awakening feelings of hatred and contempt towards school mathematics or proximity and liking for the school subject, revealing another mathematics to them.

The research showed evidence that students appropriated mathematical concepts, in many cases, in a painful way. Punishment and/or the feeling of impotence are the most visible, creating representations of a cold, difficult-to-access, punitive school mathematics that few could learn. It contributed to the perpetuation of a culture of school mathematics classes in which punishment and suffering were fundamental to students' learning and their social advancement.

The clues left by the students in their narratives signaled the size of the challenge that the trainers need to face making the students aware of the obstacles they bring from their school mathematics education so that the difficulties can be the subject of reflection, overcoming, and (re) significance during their academic education, to produce meaningful school mathematics teaching for their future students. Therefore, the trainer needs to be constantly reflecting and redefining their practice, helping students to problematize and understand school content throughout their academic education.

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