EDUR • Educação em Revista. 2024; 40:e39763
DOI: http://dx.doi.org/10.1590/0102-469839763t
Preprint: https://doi.org/10.1590/SciELOPreprints.4033

ARTICLE

THEORY OF STUDY ACTIVITY AND REALISTIC MATHEMATICS EDUCATION: ARTICULATION AND POSSIBILITIES[1]

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ABSTRACT: This paper is the result of a bibliographic review, whose research is of a qualitative nature and aims to verify the limits and possibilities of a theoretical articulation between the Study Activity Theory (SAT) and the didactic current of Realistic Mathematics Education (RME). The Theory of Study Activity has its origins in Vygotsky's Cultural-Historical Theory, while Realistic Mathematics Education is a proposal by the mathematician educator Freudenthal. The theory about the activity of study relies on the foundation of researchers such as Davidov, Elkonin, Luria, Leontiev, and Repkin, among others, developed in Russian schools in the 1960s. In a short period, Freudenthal's approach appeared to be applied in Dutch schools. Both groups of researchers (Vygotsky and Freudenthal) were concerned with understanding the teaching and learning process: the Russians were focused on the development of the human psyche, specifically, of the child, while the Dutch educators were focused on the teaching of Mathematics in elementary schools. The research results indicate that there is a possibility of articulation between these approaches, especially in the teaching of Mathematics, since the SAT highlights human activity as a source for the development of humanity and that the teaching and learning process needs to consider the historical subject's culture; Freudenthal considers mathematics as a human activity and emphasizes the need to adopt practices that value the "rediscovery" of mathematical concepts through historical and cultural recovery.

Keywords: Theory of Study Activity, Realistic Mathematics Education, Teaching Mathematics.

[1] Article published with funding from the *Conselho Nacional de Desenvolvimento Científico e Tecnológico* - CNPq/Brazil for editing, layout and XML conversion services. The translation of this article into English was funded by Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - CAPES/Brasil.

TEORIA DA ATIVIDADE DE ESTUDO E EDUCAÇÃO MATEMÁTICA REALÍSTICA: ARTICULAÇÃO E POSSIBILIDADES

RESUMO: Este artigo resulta de uma revisão bibliográfica, cuja pesquisa é de natureza qualitativa e tem como objetivo verificar limites e possibilidades de uma articulação teórica entre a Teoria da Atividade de Estudo (TAE) e a corrente didática da Educação Matemática Realística (RME – sigla da expressão em inglês "Realistic Mathematics Education"). A Teoria da Atividade de Estudo tem origens na Teoria Histórico-Cultural de Vygotsky, enquanto a Educação Matemática Realística é uma proposta do educador matemático Freudenthal. A teoria sobre a atividade de estudo conta com a fundamentação de pesquisadores como Davydov, Elkonin, Luria, Leontiev, Repkin, entre outros, desenvolvida nas escolas russas nos anos de 1960. Em um período próximo, surgiu a abordagem de Freudenthal a ser aplicada nas escolas neerlandesas. Ambos os grupos de pesquisadores (os de Vygotsky e os de Freudenthal) preocupavam-se em compreender o processo de ensino e aprendizagem: os russos estavam focados no desenvolvimento do psiquismo humano, especificamente, da criança, ao passo que os educadores neerlandeses se voltavam para o ensino de Matemática nas escolas de níveis básicos. Os resultados da pesquisa indicam que há possibilidade de articulação entre essas abordagens, especialmente ao ensino da Matemática, uma vez que a TAE destaca a atividade humana como fonte para o desenvolvimento da humanidade e que o processo de ensino e aprendizagem precisa considerar a formação histórico-cultural do sujeito; Freudenthal considera a matemática como atividade humana e enfatiza a necessidade de se adotar práticas que valorizam a "redescoberta" de conceitos matemáticos por meio de regaste histórico e cultural.

Palavras-chave: Teoria da Atividade de Estudo, Educação Matemática Realística, Ensino de Matemática.

TEORÍA DE LA ACTIVIDAD DE ESTUDIO Y EDUCACIÓN MATEMÁTICA REALISTA: ARTICULACIÓN Y POSIBILIDADES

RESUMEN: Este artículo resulta de una revisión bibliográfica de naturaleza cualitativa y tiene como objetivo verificar límites y posibilidades de una articulación teórica entre la Teoría de la Actividad de Estudio (TAE) y la corriente didáctica de la Educación Matemática Realista (RME - expresión en inglés "Realistic Mathematics Education"). La Teoría de la Actividad de Estudio tiene su origen en la Teoría Histórico Cultural de Vygotsky, mientras que la Educación Matemática Realista es propuesta del educador matemático Freudenthal. La actividad de estudio se apoya en los fundamentos de investigadores como Davydov, Elkonin, Luria, Leontiev, Repkin, entre otros, desarrollados en las escuelas rusas en la década de 1960. En tiempo cercano, surgió el enfoque de Freudenthal para ser aplicado en las escuelas neerlandesas. Ambos grupos de investigadores (Vygotsky y Freudenthal) se preocuparon por comprender el proceso de enseñanza y aprendizaje: los rusos se enfocaron en el desarrollo de la psique humana, específicamente, del niño, mientras que los educadores neerlandeses estudiaron la enseñanza de las Matemáticas en primaria. Los resultados de la investigación indican que existe una posibilidad de articulación entre estos enfoques, especialmente para la enseñanza de las Matemáticas, una vez que la TAE destaca la actividad humana como fuente para el desarrollo de la humanidad, el proceso de enseñanza y aprendizaje necesita considerar la perspectiva del sujeto histórico y cultural; Freudenthal considera las matemáticas como una actividad humana y enfatiza la necesidad de adoptar prácticas que valoren el "redescubrimiento" de los conceptos matemáticos a través de la recuperación histórica y cultural.

Palabras clave: Teoría de la Actividad de Estudio, Educación Matemática Realista, Enseñanza de las Matemáticas.

INTRODUCTION

Human activity is essential for the development of human beings since through activity, individuals develop their abilities and skills to adapt, relate to others, and transform the world. As the historical-cultural approach points out, the development of humanity is based on understanding its existence and the environment in which it lives. For this reason, several cultural artifacts have been created and analyzed in an attempt to transform man into a producer of himself and of nature itself, but also seen as a product of the environment (CEREZUELA; MORI, 2015). The relationship between man and the environment has triggered the creation of cultures, the evolution of the human psyche, the development of technologies, and complex life in society.

The human need to understand its existence and explain how nature works gave rise to different sciences, including Mathematics, as an attempt to explain and learn about the world. When human beings become familiar with the environment in which they live and the relationships they establish within it, new tools and ideas are created or improved. In this sense, researchers such as Vygotsky (1978) and Freudenthal (1994), each in their respective fields, sought to understand the relationship between human beings and the environment, specifically, how we learn and develop.

Although Vygotsky developed studies in the field of Psychology, while Freudenthal developed work in the field of Mathematics Education, both ventured into research on teaching and learning processes. Their contributions were fundamental to understanding how and what motivates human beings to learn, particularly in the teaching of Mathematics.

In the 1920s, Vygotsky and psychologists Luria and Leontiev worked together to establish the THC; in the 1930s, Leontiev and his collaborators developed the Activity Theory (AT) in Russia, focusing on analyzing the task of studying as a human activity during the school period. Davydov and Repkin, continuing Elkonin's studies, carried out experimental work in 1960 and developed the Study Activity Theory (SAT) to understand how children develop and learn (QUEROL; CASSANDRE; BULGACOV, 2014). This theoretical line is the result of research from historical-cultural psychology that discusses the development of humanity considering its formation and historical and cultural production.

Also around 1960, Freudenthal and his team were concerned about student education and proposed a change in the teaching of Mathematics through the WISKOBAS project, as a form of resistance to the Modern Mathematics Movement, one of whose objectives was to offer meaningful teaching, closer to the reality of students, considering Mathematics as a human activity (FERREIRA; BURIASCO, 2016). The project was implemented in a basic-level school in the Netherlands, equivalent to elementary and secondary education in Brazil. Although there was a change in the entire Dutch curriculum, much was produced about the teaching of Mathematics, and, as a consequence, the Realistic Mathematics Education approach was developed.

Although the research groups of Vygotsky and Freudenthal, as well as their followers, applied their ideas in different places, their concerns and historical moments were similar. These groups sought to understand the teaching and learning processes, integrate culture and reality into this process, and combat the deep-rooted traditionalism of the time that was already shown to be insufficient to meet the demands and needs of human development.

Based on this, we developed a qualitative bibliographical research, to establish a theoretical articulation between the Study Activity Theory and the Realistic Mathematics Education (RME) approach, based on the works of authors who discuss the ideas of the Study Activity Theory (SAT) and RME. At the end of this article, we describe our final considerations and reaffirm the importance of considering mathematics as a human activity in the teaching process.

METHODOLOGY

This article is the result of theoretical research in which we conducted a qualitative bibliographic review. We searched for academic texts that discussed the main aspects of SAT and RME to articulate concepts from these didactic currents that can be applied together through pedagogical tasks focused on teaching Mathematics. In the search engines, the main indications were: “Educação+Matemática+Realística”, “Realistic+Mathematics+Education”, “RME+Mathematics”, "RME+Freudenthal", "Teoria+Histórico+Cultural", "Cultural+Historic+Activity+Theory", "THC+Vygotsky" and "CHAT+Vygotsky". The search returned articles and publications with the highest number of citations, as well as reproductions of the original texts of the authors involved. With this search, it was possible to find other authors related to Vygotsky and Freudenthal, including Brazilian authors. To use the articles and books, we chose a selection of authors with the largest number of publications in the area: Vygotsky, Davydov, Leontiev, Luria, Repkin, Elkonin, Miller, Freudenthal, De Lange, Van Del Heuvel-Pahuizen, Gravemeijer, Buriasco, Silva, Ferreira and Lopez.

As defined by Marconi and Lakatos (2003)

> Bibliographic research, or secondary source research, encompasses all bibliography already public in the subject of study, from individual publications, bulletins, newspapers, journals, books, research, monographs, theses, cartographic material, etc., to oral media: radio, magnetic tape recordings, and audiovisual media: films and television. Its purpose is to put the researcher in direct contact with everything that has been written, said, or filmed on a given subject, including conferences followed by debates that have been transcribed in some way, whether published or recorded (MARCONI; LAKATOS, 2003, p. 183).

The authors emphasize that, despite the resumption of studies, bibliographic research is not limited to the reproduction of sources, since it can provide a new focus on a known topic and relate it to other areas, leading to innovative conclusions (MARCONI; LAKATOS, 2003). Qualitative research allows the study of different dimensions of reality that cannot be measured quantitatively and seeks answers to specific problems but can be generalized (MINAYO; DELANDES; GOMES, 2000).

In this article, we show the development of SAT, its main concepts, and its vision of human development, especially in the learning process. We then describe the motivations for creating the RME approach, its fundamental principles, and concepts. Subsequently, we articulate the ideas defended by Vygotsky and Freudenthal, relating them to mathematics teaching practices that can value the historical and cultural formation of students whose goal is human development.

We emphasize that we did not find documents that indicate possible partnerships between the precursors of SAT and those of RME; however, what motivated us to develop this article was the similarity between the ideas defended, the understanding that man develops through human activity, the consideration of mathematics as a human activity and the possibility of complementarity between the approaches to compose a set of tasks whose objective is to lead the student to appropriate historically constituted mathematical concepts.

THE THEORY OF STUDY ACTIVITY

According to Asbahr (2005), in historical-dialectical materialism, human activity is the central category. Through sensory practical activity, human beings develop historically and socially, but also individually (MARX, 1989). Davydov (1988) states that the first forms of human activity come from the historical-social practice of humanity to discover the universe of the human subject. Based on this proposal, Russian psychologists presented the concept of activity as one of the fundamental principles for the development of the human psyche (ASBAHR, 2005).

Querol, Cassandre, and Bulgacov (2014) indicate that there is a philosophical current, introduced by Hegel, called Hegelianism, which refers to human consciousness as a result of the knowledge accumulated and historically constituted by society and that this knowledge is objectified through the creation of artifacts. Influenced by Hegel's ideas, Marx (1976) understands man as a product of history and culture, as well as a transformer of nature. Since man is capable of actively creating and changing the world, humanity develops capabilities that go beyond simple adaptation to the system.

Based on Marx's ideas (1976), Vygotsky (1978) used the concept of activity in his early studies and indicated that human activity, constituted in a collective and meaningful way, explains the development of consciousness that can be culturally mediated (VYGOTSKY, 1978; ASBAHR, 2005). "Cultural mediation means that the relationship between the subject and the object is mediated by cultural artifacts. An artifact refers to an aspect of the material (and conceptual) world that has been modified throughout the history of its constitution through actions" (QUEROL; CASSANDRE; BULGACOV, 2014, p. 408).

Asbahr (2005) and Miller (2019) explain that consciousness and activity are fundamental elements of historical-cultural psychology, understood as a dialectical unity, and it was Leontiev (1983) who conceptualized activity, founding the general psychology of activity. Leontiev (1983) states that consciousness is shared knowledge and social achievement, but that it is only possible to establish an individual consciousness from a social consciousness. This process of internalizing knowledge occurs from the "outside-in" to perform external sensory and practical activities, fundamentally social, transforming them into individualized internal activities. "Activity, mediated by the psychic reflection of reality, is the unity of life that guides the subject in the world of objects. Its main constitutive characteristic is the objectal character [...]." (ASBAHR, 2005, p. 109).

Human activities differ from the activities of other living beings and each other for many reasons, but the main one is the motive. Motive is understood as the trigger of the activity, which links an object to its need (LEONTIEV, 1978). Isolated objects and needs cannot produce activities (ASBAHR, 2014).

> The first condition of all activity is necessity. However, necessity cannot determine the concrete orientation of an activity, for it is only in the object of the activity that it finds its determination: it must, so to speak, find itself in it. Since necessity finds its determination in the object (is "objectified" in it), the said object becomes the motive of the activity, that which stimulates it (LEONTIEV, 1978, p. 107-108).

In the search to meet its needs, the human psyche evolved and the psychic reflection of reality began to be done consciously. Amidst the abstract and the concrete, man was able to differentiate the real object from the subjective and we call this differentiation consciousness. "Consciousness refers, therefore, to the human possibility of understanding the social world and the world of objects as capable of analysis" (ASBAHR, 2014, p. 111). Man appropriates social knowledge and meanings, gives them his meaning, and links them to his motives, objects, and needs.

> Meaning is the generalization of reality that is crystallized and fixed in a sensible vector, usually the word or the locution. It is the ideal, spiritual form of the crystallization of the social experience and social practice of humanity. Its sphere of representations of a society, its science, and its languages exist as systems of corresponding meanings. Meaning therefore belongs, first and foremost, to the world of objectively historical phenomena (ASBAHR, 2014, p. 111).

Based on these foundations, Russian psychologists from the Laboratory of Primary School Psychology of the Institute of Psychology of the Russian Soviet Republic, under the leadership of Elkonin and Davydov, proposed a radical organization in the teaching methodology of Russian schools in the 1960s (PUENTES, 2020). The innovative proposal triggered the Study Activity Theory, which defines a study as a human activity that ensures the completion of the so-called study task, which can be understood as one or more activities that aim to modify the subject of the action (who performs the study task) and is related to the content (object of the study task), defined as the mastery of certain modes of action (PUENTES, 2020).

Research and experiments indicated that this type of activity could be schematized considering the interconnection of the following components: the study task, the study actions, the control action, and the action of evaluating the degree of transformation of the subject.

> 1) the study task, which by its content consists of the modes of action to be assimilated; 2) the study actions, the result of which is the formation of the mode of action to be assimilated and the primary execution of the didactic model; 3) the control action, which consists of comparing the action executed with the model and; 4) the action of evaluating the degree of cognition of the changes that occurred in the subject (PUENTES, 2020, p. 3-4).

At the beginning of the development of SAT, pioneering researchers were not clear about what to consider as the content of the study task and adhered to a traditional definition that, years later, was reformulated by Repkin and Dorokhina (PUENTES, 2020).

The understanding gradually became established that the development of the Study Activity is revealed in the plans of interactions and the development of the human psyche, through the set of components that define its processes:

> The theoretical relationship with reality and with the orientation models corresponding to this relationship constitute the specific need and motive of the child's Study Activity. The interrelated modes of theoretical consciousness (scientific, artistic, moral, and legal) act as the content of this activity. Mastering generalized modes of action from the point of view of the (theoretical) content of solving hundreds of concrete practical tasks is the substantial characteristic of the study task. Formulating a study task for the student means placing him in a situation that requires an orientation toward a generalized mode of action from the point of view of the content of its solution in all possible particular and concrete variants of conditions (DAVYDOV, 2019, p. 172).

Strictly speaking, the appropriation of mathematical knowledge involves the creation of a system of coordinated actions, inserting objects into it, that is, establishing conditions of origin of some system of objects through real transformation or of a psychic nature.

As Miller (2019) presents, the proposal of the Theory of Study Activity allows us to understand one of the vital activities of man throughout the development of his psyche: many activities are developed at different stages of his life and during school age, the study activity is the main one. Under the influence of the environment in which he lives and the social relationships he establishes, the child tends to change his social situation as he grows and develops (LEONTIEV, 1983; VIYGOTSKY, 1978).

For each stage of a child's growth, there is an activity called the main activity, that is, from this activity, others can be performed, such as communication and perception. These activities that are based on the main activity are called guiding activities and will contribute to the child's development. The main activity is essential for the child to be able to develop other skills that allow him/her to be integrated into the community of which he/she is a part, be it family, school, or society. Examples of main activities developed in childhood, from infancy to school age, are manipulative activity, emotional communication, and role-playing (ELKONIN, 1987; MILLER, 2019). When the child enters school, his/her main activity is study, which will allow him/her to assimilate knowledge and expand his/her cognitive and intellectual capacities. The child's actions receive a new treatment and there is a greater demand for the fulfillment of his/her responsibilities. The relationships with knowledge and the people around him/her become more complex and prepare the child for the adolescent phase, in which the guiding activity is personal communication. Even in adolescence, study activity is still fundamental but personal communication will allow this young person to discover their role in society (MILLER, 2019).

According to Davydov and Márkova (1987), in this trajectory, it is up to the learner to try to develop his/her skills and abilities to the maximum, geared towards his/her education, according to his/her interests, life history and social relationships. Social differences can be justified by the correlation of the levels of education: the first level allows the student, through the guidance of the teacher, to carry out the study activity and correlate it to objectives and conditions; the second level concerns the personality in which each student carries out his/her task; finally, it is at the last level that the student becomes the subject of his/her activity, as he/she achieves autonomy and becomes capable of establishing other ways to carry out the study activity, transforming it into a creative activity.

Miller (2019, p. 81) indicates the study activity "[...] as a privileged path to insert the student in a learning process that generates development, that is, that transforms him/her qualitatively, making him/her a subject capable of acting autonomously and creatively in his/her environment." With this, we can conclude that the study activity highlights the theoretical relationship between student and knowledge, allowing the subject of the action to develop capacities to analyze, reflect, and understand reality through mental actions. This process is called "developmental teaching" proposed by Davydov (MILLER, 2019).

Back in 1960, while Russian psychologists were developing and experimenting with SAT in schools, Dutch researchers and educators were working to transform the Dutch curriculum, noting that the traditional method did not value the historical and cultural aspects of students and the school community and that teaching was boring, rigid and decontextualized. Freudenthal and his team developed an approach specifically aimed at teaching mathematics that integrates history, culture, and context into the process of teaching and learning mathematical concepts.

THE REALISTIC MATHEMATICS EDUCATION APPROACH

The German mathematics researcher and educator, Hans Freudenthal, developed a teaching approach that he named, together with his colleagues and followers, Realistic Mathematics Education. This approach emerged between the 1960s and 1970s, in the Netherlands, where mathematics educators, influenced by Freudenthal's ideas, sought to modify the teaching proposed by the Modern Mathematics Movement to offer a teaching that was opposite to the structuralist format predominant at the time (FERREIRA; BURIASCO, 2016). Freudenthal carried out studies in Mathematics, Sciences, and Literature and made many contributions to Mathematics Education, although discussions about educational practices were already taking place in the Netherlands and other countries (GRAVEMEIJER; TERWEL, 2000). In 1923, he entered the University of Berlin, where he studied Mathematics and Physics. His first works were on Algebra and Topology, but he also contributed to the areas of Geometry, Philosophy, History of Mathematics, and Mathematical Education (FERREIRA; BURIASCO, 2016).

Based on the studies developed, Freudenthal established RME based on "three pillars": the way mathematics is viewed, the way students learn it, and how it should be taught (VAN DEN HEUVEL-PANHUIZEN, 1996). In Freudenthal's view, mathematics should not be imposed on students without going through the process of experimentation and experience of mathematical concepts: the mathematization of reality must be developed (FREUDENTHAL, 1994). In short, the mathematization process can be interpreted as a way of organizing reality (and dealing with it) through mathematical concepts (DE LANGE, 1999; SILVA, 2015).

To understand mathematical concepts, it would be necessary to understand mathematics as a human activity, which means understanding it in a broader way, not reduced to definitions, algorithms, and equations, but as a process of organizing reality and, at the same time, treating this reality as mathematical ideas and objects.

The activities in which signs of mathematization are recognized are such as:

> • identifying the specific mathematics in a general context;
> • schematizing;
> • formulating and visualizing the problem;
> • discovering relations and regularities;
> • recognizing similarities in different problems;
> • representing a relation in a formula;
> • proving regularities;
> • refining and adjusting models;
> • combining and integrating models;
> • generalizing. (DE LANGE, 1999, p. 18, our translation).

Freudenthal considers that the set of knowledge historically accumulated and validated by humanity, the result of mathematizations and interactions is called "mathematical knowledge" (SILVA, 2015).

To break with traditional teaching, in which definitions and formulas are presented as ready-made and finished, and then their applications are shown, Freudenthal (1994) proposes that "guided reinvention" should be practiced as a teaching method. From this perspective, students are given the opportunity to explore and (re)invent these concepts (LOPEZ; FERREIRA; BURIASCO, 2014). In "guided reinvention", the main focus is on how students deal with tasks through mathematization, not on objects and products, with the teacher playing the role of "guide" in this process, offering tools and guiding students through questioning, investigations and assimilation with reality (FREUDENTHAL, 1994). Furthermore, Gravemeijer and Doorman (1999) state that guided reinvention proposes that students try to solve problems in different contexts and levels of understanding, sharing solutions to progress to more advanced levels.

In defense of guided reinvention, Freudenthal (1994) argues that, with this type of approach, the student understands mathematical ideas satisfactorily and effectively, because he or she begins to experience them. This experience allows the student to appropriate knowledge, which may not be possible when opting for a teaching method of repetition or memorization (FREUDENTHAL, 1994).

To carry out the mathematization process, the "phenomena" must be considered in the elaboration of the tasks and this consideration is the main objective of the didactic phenomenology proposed by Freudenthal (1983).

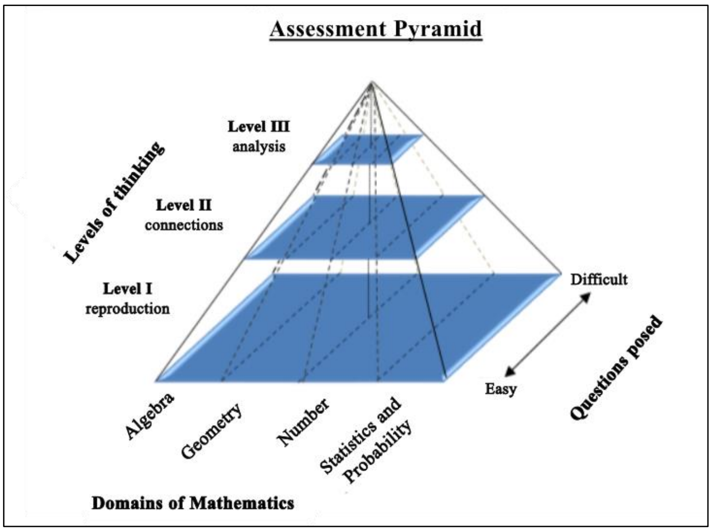
> From this perspective, the problems to be worked on during the teaching and learning process should suggest contexts that can be mathematized, that elicit the mathematics that is intended to be taught, and that offer opportunities for students to construct their responses and strategies. The contexts do not necessarily need to refer to everyday life situations, but they need to be "imaginable", and "achievable" by them, capable of receiving mathematical treatment, and be accessible to students (LOPEZ; FERREIRA; BURIASCO, 2014, p. 252).

Freudenthal developed some principles that organize teaching work and help in the elaboration of tasks (as pedagogical activities are called) that contribute to triggering a process of mathematization and guided reinvention: Principle of Activity, Principle of Reality, Principle of Levels, Principle of Interlacing, Principle of Interactivity and Principle of Orientation (FERREIRA; BURIASCO, 2016; SILVA, 2015).

According to Van Den Heuvel-Panhuizen (1996), and Ferreira and Buriasco (2016), the activity principle is related to the view that mathematics is a human activity and learning is constructive, in which case students can create mathematical concepts through their productions; the reality principle is based on contexts that are "rich" for the exploration of mathematical concepts and the appropriation of knowledge. These are situations that are close to the students, factual or figurative so that they can mathematize and learn from this process. In this principle, the teacher has the role of choosing tasks whose contexts are close to the students or contexts that the students can approach; the level principle foresees that students go through several levels of understanding, starting the learning process through informal procedures until they can formalize them, reusing primary models to reach more advanced levels of the same subject; the interweaving principle reinforces that mathematical contents are not isolated chapters, but strongly connected. By following this principle, students can acquire an integrated view of mathematics, which will allow them to use mathematical tools and knowledge appropriately; the principle of interactivity emphasizes that the learning process is not only individual but also social. Therefore, students should have the opportunity to share their knowledge with other colleagues and the teacher, moments that are important for reflection on the work; for the principle of guidance, the teacher must guide the "reinvention" of mathematics, through a set of coherent tasks that allow students to have space to explore mathematical tools and, thus, (re)invent the mathematical subjects involved.

Since the principles guide teaching actions, assessment must be following these principles. Recognizing the types of tasks can help teachers obtain the greatest possible amount of evidence about student learning. For this to be possible, tasks must raise different cognitive demands (reproduction, connection, analysis/reflection), involving different topics of mathematical knowledge (related to Algebra, Geometry, Numbers and Statistics and Probability, for example) at different levels of complexity (from complex to simple), as illustrated in Figure 1.

Figure 1: Evaluation pyramid proposed by De Lange (1999).



Source: Prepared by the authors based on De Lange (1999).

In this arrangement, the set of tasks that make up the assessment must:

> [...] describe a student's growth in all domains of mathematics and at all three levels of thinking, questions in a complete assessment program should fill the pyramid. There should be questions at all levels of thinking, of varying degrees of difficulty, and in all content domains. (DE LANGE, 1999, p.17).

According to De Lange (1999), the definition of levels does not have a clear distinction and therefore it is not possible to state when a level begins or ends. The "simple" or "complex" character depends on how the student relates to the tasks and can use informal or formal language in the resolutions (FERREIRA; BURIASCO, 2016), but the teacher needs to recognize the difficulties faced by the student, as well as the objectives of each stage of the proposed task.

POSSIBLE APPROACHES

Regarding the reasons that led the teams of Davydov, Elkonin, Repkin, and Freudenthal, together with their collaborators to rethink the curriculum of their respective countries, we realize that their concern is focused on transforming current education into a process of historical, cultural, and social formation, as they proved that the understanding of the time about human formation did not meet the needs of society, nor did it contribute to the critical development of individuals.

The SAT indicates that the main activity developed by school-age children is studying and that a reason is needed to trigger the child's interest in performing the task. This reason may or may not be associated with the objective of the task, but the teacher needs to promote favorable conditions so that the subject's reasons coincide with the learning objectives and this can be done using resources and contexts that can help the child create meanings and interest in learning.

Similarly, specifically focused on teaching and learning mathematical subjects, RME emphasizes that mathematics is a human activity developed to explain and transform the environment in which we live. When considering mathematics as a human activity, students need contexts and conditions that favor the exploration and "reinvention" of mathematical subjects. This process can occur through the historical recovery of these subjects, as well as incorporating elements of the student's culture in the presentation of the problems and tasks to be performed. As Miguel (2008) argues, the student,

> when interpreting data and information, they do so within a framework whose most fundamental aspect is the history of their previous experiences. The dissociation between the form and content of Mathematics teaching does not allow students to grasp the structure of a subject; grasping such a structure means learning how things relate to each other. Thinking

> about meaningful learning implies assuming the fact that learning presupposes an action of a dynamic nature, which requires teaching actions directed so that students deepen and broaden the meanings they develop through their involvement in learning activities (MIGUEL, 2008, p. 7-8).

Scholars of SAT and RME emphasize that learning is not a process that occurs only individually and that social interaction is a human need. Knowledge is a historical and social construction; therefore, we can consider that teaching also occurs through the interaction between teacher, student(s), and knowledge. In both SAT and RME, social interaction is valued, so the teacher needs to promote tasks that allow for discussions and group work. In this way, students can validate their ideas with their peers before validating them with the teacher, awakening autonomy.

As discussed by researchers from SAT and RME, an individual learns when he or she assigns his or her meanings and becomes capable of applying the concepts learned in different situations. The perception of the degree of complexity of tasks and the ability to solve different problems are associated with the levels of understanding of the concepts involved. A set of tasks needs to consider problems at different levels so that the student can develop his or her problem-solving skills based on his or her experiences. In addition, the connection between acquired knowledge and subsequent knowledge should be emphasized, especially in the case of teaching Mathematics.

In SAT, secondary activities are fundamental to the child's development, and previous activities are not forgotten so that the child can perform others, but rather incorporated and improved in new activities. Regarding this interaction, RME proposes that we consider the principle of intertwining between the subjects of the domains of mathematical knowledge since every concept to be learned is related to some previous one and will be necessary for new learning. Therefore, it is clear that both theoretical schools understand that it is learning that guides the development of human beings.

In the approaches studied, the teacher plays a fundamental role as a guide. For SAT, the teacher must guide his/her students and is responsible for creating favorable conditions for the development of motives that coincide with the learning objective, in addition to knowing all the stages of task execution and recognizing the different solutions presented by the students. A similar process occurs for RME: in guided reinvention, the teacher promotes conditions similar to those that allow the reinvention of the subject being studied. It is also the teacher's role to be a “guide” of the process, allowing students to reach their conclusions with the help of guidelines that indicate how the problem can be solved based on their knowledge. In both approaches, the teacher plays a fundamental role in ensuring that learning takes place.

Below, we present our final considerations and indications of possible articulations between the Study Activity Theory and Realistic Mathematics Education in a pedagogical proposal for teaching Mathematics.

FINAL CONSIDERATIONS

We describe, in general terms, the theoretical approaches between SAT and RME. Despite their different origins, both didactic currents consider human activity as the main factor for their development and that teaching needs to consider the historical and cultural formation of society.

Both SAT and RME present steps and guiding principles for the teaching process, but the first theory is supported by the considerations of THC and TA, focused on the study of child development and the presentation of an alternative for basic education at the time, while Freudenthal's approach reveals a specific alternative for teaching Mathematics. Although they discuss learning in different contexts, the researchers involved agree that learning occurs as meaning is attributed to what is learned. This can be achieved through a didactic proposal that values the historical-cultural formation of humanity, that allows social interaction, and that the teacher plays the role of guide, being able to use artifacts that mediate learning.

The learning objectives and proposed tasks must be coherent and it is the teacher's responsibility to know the stages of development of the tasks, in addition to considering the different ways of resolving the situations presented to the students. It is necessary to place the student in conditions similar to those of the invention of the mathematical subject, because, as SAT and RME scholars present, the student can understand the importance of that knowledge for the development of humanity and, possibly, transform himself/herself and his/her relationship with the world.

We have presented only a small excerpt from the Study Activity Theory and Realistic Mathematical Education, in addition to a brief theoretical comparison, and many questions have arisen throughout the development of this article, such as: how could a pedagogical proposal for teaching mathematical concepts articulate the SAT and the principles of RME? Is there a connection between the results obtained after the interventions proposed by the groups of Davydov, Repkin, and Elkonin and the projects developed by Freudenthal's collaborators? What are the possible contradictions between the study activity theory and the approach of Realistic Mathematical Education? How can we theoretically define and structure a task if we choose to approximate the SAT and RME? We hope that these questions can guide future research that highlights the importance of didactic approaches that value historical and cultural processes in the development of humanity.

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Submitted: 05/16/2022
Preprint: 04/28/2022
Approved: 01/25/2023

AUTHORS´ CONTRIBUTIONS

Author 1 - Data collection, active participation in data analysis, and writing of the text.
Author 2 - Data analysis and review of the final text.
Author 3 - Data analysis and review of the final text.

DECLARATION OF CONFLICT OF INTEREST

The authors declare that there is no conflict of interest with this article.