

An analysis of Mathematical Modeling conceptions in didactic proposals produced from a training course

Abstract: This article is an excerpt from a research study that analyzed didactic proposals developed by teachers during a training course and presented in a notebook. Bibliographical in nature, the research aimed to analyze the didactic proposals from the Teaching Proposals Notebook for an Antiracist Mathematics, identifying connections with the conceptions of Mathematical Modeling. Of the 16 proposals analyzed, most reflected concepts from this approach, with emphasis on the perspectives of Ademir Donizeti Caldeira and Maria Salett Biembengut. The diversity of understandings about Mathematical Modeling in the literature justifies the different approaches observed. The proposals reveal the potential of Mathematical Modeling as a tool to make mathematics more meaningful while promoting the appreciation of diversity and culture.

Keywords: Teacher Training. Mathematics Education. Ethnic-Racial Theme. Mathematical Modeling.

Un análisis de concepciones de Modelado Matemático en propuestas didácticas producidas a partir de un curso de formación

Resumen: Este artículo es un recorte de una investigación que analizó propuestas didácticas elaboradas por docentes durante un curso de formación y presentadas en un cuaderno. De carácter bibliográfico, la investigación tuvo como objetivo analizar las propuestas didácticas del Cuaderno de Propuestas de Enseñanza para una Matemática Antirracista, identificando aproximaciones con las concepciones de Modelado Matemático. De las 16 propuestas analizadas, la mayoría reflejó conceptos de este enfoque, destacando las perspectivas de Ademir Donizeti Caldeira y María Salett Biembengut. La diversidad de comprensiones sobre el Modelado Matemático en la literatura justifica los diferentes enfoques observados. Las propuestas revelan el potencial del Modelado Matemático como herramienta para hacer que las matemáticas sean más significativas, al mismo tiempo que promueven la valoración de la diversidad y la cultura.

Palabras clave: Formación Docente. Educación Matemática. Tema Étnico-Racial. Modelado Matemático.

Uma análise de concepções de Modelagem Matemática em propostas didáticas produzidas a partir de um curso de formação

Resumo: Este artigo é recorte de uma pesquisa que analisou propostas didáticas elaboradas por professores durante um curso de formação e apresentadas em um caderno. De caráter bibliográfico, a pesquisa teve como objetivo analisar as propostas didáticas do *Caderno de propostas de ensino para uma matemática antirracista*, identificando aproximações com as concepções de Modelagem Matemática. Das 16 propostas analisadas, a maioria refletiu conceitos dessa abordagem, com destaque para as perspectivas de Ademir Donizeti Caldeira e Maria Salett Biembengut. A diversidade de entendimentos sobre Modelagem Matemática na literatura justifica as diferentes abordagens observadas. As propostas revelam o potencial da Modelagem Matemática como ferramenta para tornar a matemática mais significativa, ao

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Palavras-chave: Formação de Professores. Educação Matemática. Temática Étnico-Racial. Modelagem Matemática.

1 Introduction

Mathematical Modeling serves to mathematically explain various everyday situations. However, its insertion into human life is not a current phenomenon. Biembengut (2009) states that, since the beginning of the 20th century, Modeling has been present in the literature of Engineering and Economic Sciences. Furthermore, its application can be found, albeit implicitly, in various situations, such as in the architecture of historical monuments.

In the mathematical field, its influence is noticeable in demonstrations of theorems, such as in the calculation of the height of the pyramids carried out by Thales of Miletus in Antiquity. In the educational context, Mathematical Modeling is generally seen as an application of mathematics in other areas of knowledge. Its relevance lies in stimulating the development of students' critical thinking, allowing them to understand the sociocultural role of Mathematics.

Barbosa (2001, 2004) and Caldeira (2009) believe that working with Mathematical Modeling provides an investigative and problematizing environment for students regarding the construction, application, and social meaning of Mathematics. This work provides an environment that is favorable to the development of students' autonomy and critical thinking about mathematics in their reality.

When taught in a contextualized way, involving real-life situations associated with the knowledge acquired in the classroom, mathematics becomes recognized as an important means of understanding the world and reducing inequality in mathematical learning.

With regard to inequality in mathematical learning, the call for proposals *Racial Equity in Basic Education: applied research and scientific articles* (CEERT, 2020) points out that, according to data from the Sistema de Avaliação da Educação Básica [Basic Education Assessment System — Saeb] from 2017, only 29.9% of black students had satisfactory learning in the 5th grade, while this percentage was 59.5% for white students. In High School, the discrepancy is even greater: 16% of white students achieved a sufficient level of learning, in contrast to only 4.1% of black students. Furthermore, the 2018 study by the Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira [National Institute of Studies and Educational Research Anísio Teixeira — Inep] reveals that the school trajectory of these students is marked by dropout and failure, being 39% for black students, 34% for mixed race students and 22% for white students. (CEERT, 2020).

The inequality in learning between black and white students throughout their school career is an obvious challenge. In this sense, it is necessary to question which paths can be adopted in the classroom to overcome this inequality. Mathematical Modeling can be one of the possible directions to make Mathematics teaching more meaningful, allowing students to recognize and understand Mathematics present in their sociocultural context.

With this purpose, professors from the Center for Research and Studies in Mathematics Education at the Federal University of Uberlândia (NUPEM/UFU) submitted the project *Ethnomathematics, Mathematical Modeling and Teacher Training: possibilities for implementing Law n. 10.639/03 in Mathematics Teaching*, in response to the call for proposals *Racial Equity in Basic Education: applied research and scientific articles*. The research project was developed within the scope of NUPEM/UFU in partnership with the Centro de Estudos das Relações de Trabalho e Desigualdades [Center for Studies on Labor Relations and Inequalities — CEERT] and resulted in the offering of the teacher training course called *Teacher education*

course: for an anti-racist Mathematics Education.

The education course provided an environment for discussions on Mathematics Education and ethnic-racial issues and the National Curriculum Guidelines for Education on Ethnic-Racial Relations, as well as Law n. 10.639/2003. Remote meetings and asynchronous activities were held throughout the course, addressing the theme. One of the activities developed was the development of didactic proposals that articulated Ethnomathematics, Mathematical Modeling, local and global cultures, and Law n. 10.639/2003 in the teaching of Mathematics. The didactic proposals, which are the objects of analysis in this study, were organized in a notebook entitled *Book of teaching proposals for an anti-racist Mathematics*, available free of charge in digital format.

Thus, this study seeks to identify the contributions resulting from the discussions held in the education course in the development of the conceptions of Mathematical Modeling, manifested directly or indirectly, manifested in the didactic proposals that make up the book. To this end, the study was based on the conceptions presented by Caldeira (2009), Barbosa (2004), Biembengut (2014, 2016) and Burak (2004, 2019).

Based on this perspective, the study sought to answer the following question: What conceptions and/or approximations of conceptions of Mathematical Modeling can be identified in the proposals present in the *Book of teaching proposals for anti-racist Mathematics*?

2 Theoretical assumptions

Mathematics can be found in various social and cultural activities and situations, presenting its particularities in each way of conceiving the knowledge inherent to this area. This perspective can be introduced in the classroom, seeking to contribute to the teaching of Mathematics and awakening students' interest in the subject, leading them to identify mathematical content in different situations that are part of their social or cultural environment.

This work strategy places the student as the protagonist of their learning, valuing the knowledge already acquired by them throughout their school career and their experiences.

From this perspective of working with Mathematics, it is understood that Mathematical Modeling is capable of bringing Mathematics closer to students' experiences. Modeling can be a way to work with and recognize Mathematics present in real situations, to know and identify Mathematics used by other peoples and in other cultures, and to obtain mathematical models.

Mathematical Modeling has been a topic of international discussion since the 20th century, but according to Biembengut (2009), the discussions took place specifically in the 1960s. In the context of Brazilian Mathematics Education, discussions on Mathematical Modeling began almost at the same time, being consolidated in Brazilian education between the 1970s and 1980s.

Although discussions on this perspective are not new, there is no single conception of what Mathematical Modeling is. Barbosa (2004) argues that this approach provides an environment in which students are invited to problematize real-life situations through mathematics. Furthermore, according to the author, the activities involved in the Mathematical Modeling process function as resources to investigate and question the role of Mathematics in society.

For Araújo (2009), Modeling consists of interpreting real-life situations using Mathematics, allowing for the understanding of different social phenomena with the support of this science.

When used in the classroom, Mathematical Modeling enables an environment for research into mathematics involved in different real-life situations, in which the student

assumes an active stance, connecting everyday situations to the acquired school knowledge. In this sense, its inclusion as a pedagogical strategy is essential to stimulate the active process among young people and the formation of critical thinking about the role of mathematics in the classroom.

Thus, the study was based on the concepts of Mathematical Modeling defended by Barbosa (2004), Biembengut (2014, 2016), Burak (2004, 2019) and Caldeira (2009).

Biembengut (2014) considers that “Mathematical Modeling is the process involved in developing a model in any area of knowledge. It is a research process” (p. 21). Furthermore, Modeling aims to investigate, understand, research, solve or improve something (Biembengut, 2014).

According to this concept, Modeling is a research method applied to Education, and the work with Mathematical Modeling is aimed at developing a model to support other applications, not just a particular solution.

According to Bonotto, Scheller and Biembengut (2015), the inclusion of students in activities involving Mathematical Modeling contributes to the learning process, since it allows them to learn from problem situations. In addition, this approach encourages the development of interpretation and understanding on various subjects, since the work with Modeling encompasses problems from different areas of knowledge.

Therefore, the Modeling process encourages students to understand a situation of interest to them using Mathematics, while encouraging creativity and valuing the skills and knowledge acquired by them.

Biembengut (2016) presents three phases to guide the Mathematical Modeling process with the aim of encouraging the understanding of various problems in society based on Mathematics, with the intention of obtaining a mathematical model.

Stage 1) *perception and apprehension*: this is the time to seek information about the situation that one wishes to research and formulate the research problem situation.

Stage 2) *understanding and explanation*: the problem is formulated, the model is formulated and the problem is solved.

Stage 3) *meaning and expression*: this is the time to interpret, evaluate and validate the results of the applied model. If the model achieves the expected objectives, its meaning is shown; if it has not achieved the objectives, the process returns to the first or second phase. Finally, the Mathematical Modeling process is presented.

This Modeling process allows the student to research, investigate, problematize and solve a real-life situation with mathematics, giving the student the opportunity to be critical and active in the process of constructing knowledge. Unlike Biembengut (2014, 2016), Mathematical Modeling, according to Burak (1992), is a set of procedures that seek, based on mathematics, to explain everyday phenomena, so that human beings are able to make decisions and predictions. In this Modeling process, the starting point is derived from a situation of interest to the group involved and data collection takes place in the environment of interest of the subjects involved in the process.

Burak (2004) presents five stages of the Modeling process to be followed. The first stage, *choosing the topic*, is the moment in which the group, formed by three or four people, will choose a topic of interest. In the second stage, *exploratory research*, a search for characteristics about the topic of research interest is carried out; it is a process that encourages the group to play a more critical and attentive role. In the third stage, *problem-solving*, the group members organize and interpret the data to conduct the discussion on this data. In the fourth

stage, problem-solving and the development of mathematics related to the topic, the mathematical content is addressed in a meaningful way in the Modeling process. And, in the last stage, *critical analysis of the solutions*, considerations are made about the research process and the evaluation of hypotheses in relation to Mathematics.

Burak (2019) states that working with Mathematical Modeling stimulates interest, motivation and contact between mathematics and other areas of knowledge. Thus, the Modeling process aims to contribute to the development of critical, creative and active thinking among students, in which there is motivation on the part of the group, as it creates a connection between the group's topic of interest and Mathematics.

Mathematical Modeling, according to Barbosa (2001, 2004), is a learning environment in which students are involved in the process of investigating real-world situations through mathematics, in which it is not necessary to use pre-defined concepts from the area or obtain a mathematical model. Furthermore, in this process, students have the freedom to choose the level of involvement according to their interests.

Barbosa (2004) argues that the situations in the Modeling process should originate from the social context, allowing students to use mathematical concepts and knowledge to problematize these situations. This process provides students with the opportunity to investigate and understand the mathematics present in their daily lives, making them protagonists in the process of knowledge construction.

As it is an investigative process, which involves students in an environment of problematization, investigation, data collection, organization and evaluation of results, the research situation can start from a non-mathematical problem and the mathematical concepts are constructed as the process progresses.

Working with Mathematical Modeling based on this concept promotes the development of research skills, critical thinking, interest and use of mathematics in other areas of knowledge. Thus, Modeling favors the understanding of the sociocultural role of mathematics, enabling students to act critically in society and recognize its presence in their reality.

Barbosa (2004) suggests three approaches to the process with Mathematical Modeling, which he calls cases: 1st case) the teacher presents the theme and collects the data, leaving the students responsible for the investigation and discussion; 2nd case) the teacher presents the problem and the students collect, investigate and discuss the data; and 3rd case) all the steps are carried out by the students, from the formulation to the resolution of the problem, starting from a theme that does not necessarily belong to the area of Mathematics.

Thus, Mathematical Modeling does not only aim to apply Mathematics in social contexts, but also to make students critical and participatory individuals in the classroom and in society, being able to reflect on the applications and meaning of mathematics in different contexts.

In a conception close to Barbosa's, Caldeira (2009) argues that Mathematical Modeling is an approach to Mathematics Education that is essential for inserting the relationships between mathematical knowledge and contributing to the construction of a democratic and participatory society. For the author, in this process, mathematical content should be worked on in relation to the context and reality of students and teachers, avoiding isolated and decontextualized approaches to Mathematics.

Caldeira (2009) also argues that Mathematics is a construction derived from social and cultural interactions and that, based on Modeling, it is possible to break with the idea that mathematics is ready-made and definitive knowledge. From this perspective, it is not enough to just consider the reality of students, but also to encourage them to understand it and transform

it through mathematics. Furthermore, it is essential to recognize that each culture has different ways of doing mathematics, thus promoting the formation of critical citizens.

In view of the above, Mathematical Modeling allows students to have a broad and critical view of mathematics, relating it to the culture and society in which they are inserted. This process encourages reflective thinking and values the different mathematical knowledge that exists, leading them to understand that there is no single Mathematics.

3 Methodology

In this article, a bibliographical research was developed with the objective of knowing, researching and exploring a problem, using as reference books, dissertations, theses, videos, magazines and articles, in digital or printed format (Cervo, Bervian and Silva, 2007; Marconi and Lakatos, 2003). In addition, bibliographical research allows the researcher to have direct contact with the research sources (Marconi and Lakatos, 2003).

This article presents the results of a research that had as one of its products the *Book of teaching proposals for an anti-racist Mathematics*, published in 2022 and object of analysis in this study.

The choice of the research topic and the book of proposals as the object of investigation resulted from the researcher, first author of this article's approach to the Mathematical Modeling trend during her undergraduate studies and her participation in the *Teacher education course: for an anti-racist Mathematics Education*.

Thus, it was decided to analyze the proposals developed by the students, in an attempt to reveal the contributions of the discussions and activities carried out during the course for the development of the conceptions about Mathematical Modeling manifested in the didactic proposals related to ethnic-racial issues. To this end, the study was based on the conceptions of Mathematical Modeling defended by Caldeira (2009), Barbosa (2004), Biembengut (2014, 2016) and Burak (2004, 2019).

The *Book of teaching proposals for anti-racist Mathematics*, published by Editora Siano, was organized by the project coordinators and is the result of the *Teacher education course: for an anti-racist Mathematics Education*.

The course was held between November 2020 and August 2021, with a workload of 80 hours, and was conducted remotely, with guest researchers, discussion forums, and assessment activities on the topic. Its objective was to provide opportunities for discussions on Mathematics Education focused on ethnic-racial issues, contributing to the training of Mathematics teachers by establishing relationships between teaching, the National Curricular Guidelines for Education on Ethnic-Racial Relations, and Law n. 10639/2003.

The teaching proposals were developed in groups of up to 6 participants, as a final assessment of the course. In total, 21 groups were formed, each named after a black celebrity, including: Lélia Gonzalez, Nilma Lino Gomes, Kabengele Munanga, Katherine Johnson, Dorothy Vaughn, Mary Jackson, Conceição Evaristo, Abdias do Nascimento, Aleijadinho, André Rebouças, Antonieta de Barros, Chiquinha Gonzaga, Jackson do Pandeiro, João do Vale, Xica da Silva, Zumbi dos Palmares, Pixinguinha, Cartola, Thereza Santos, Lima Barreto and Ana das Carrancas. However, only 19 groups completed and presented their teaching proposal at the end of the course.

After the course was completed, the teaching proposals were organized and formed the *Book of teaching proposals for anti-racist Mathematics*, organized by the project coordinators.

4 Presentation and discussion of the results

This section presents analyses of the teaching proposals that make up the *Book of teaching proposals for anti-racist Mathematics*. The material is composed of 16 chapters, each of which corresponds to a teaching proposal developed and presented during the *Teacher education course: for an anti-racist Mathematics Education*.

The proposals cover a diverse target audience, from Basic Education to Higher Education, and relate ethnic-racial issues to Mathematics Teaching with a view to implementing Law n. 10.639/2003 in Mathematics classes.

Five categories were listed, in which the categorized works are included, providing a comprehensive view of the various approaches and contributions present in the analysis. The categories correspond to a Mathematical Modeling perspective.

The first of these is Caldeira's socio-critical view, which seeks to work with mathematical content related to social and cultural situations. The second category is the process of developing Mathematical Models as discussed by Biembengut. Next, Barbosa's social vision stands out, emphasizing the importance of considering the social and cultural context when applying Mathematics in different situations. The fourth category is Burak's conception of Modeling, understood as a process of research and investigation. Finally, there is the category of works that do not approach any of the visions of Mathematical Modeling presented.

4.1 Sociocritical view of Caldeira

The second proposal, entitled *Ethnomathematics in the daily life of a quilombola community*, was written by Adriana Paula Corrêa de Souza, Amanda Cristina Martins, Angélica de Fátima Vieira Costa, Heloísa Oliveira Marques e Souza, Regiane Aparecida Dias Martins and Vicentina Sebastiana de Magalhães. The proposal is aimed at students in the initial and final years of elementary school and seeks to recover the history of the Quilombola Community of the city of Conselheiro Lafaiete (MG) through workshops that include Ethnomathematics.

The didactic proposal aims to bring students closer to the activities and knowledge of the Quilombola Community in Conselheiro Lafaiete, integrating mathematical content in a contextualized way, as advocated by Caldeira (2009), in which mathematical content is worked on by making connections with the context and society. Despite this, there is no approach to the main aspect of the Mathematical Modeling process, which is the existence of an opportunity for reflection and analysis of the results found.

In the proposal in question, there are no moments for discussion and reflection of the mathematical content, nor exploration of the possible existence of a mathematical approach different from the Western one in the Quilombola Community.

The third proposal was entitled *Applicability of Law n. 10.639/03: experiences and possibilities through African games in Mathematics classes* and was developed by Wellington Gonzaga Brandão and Taís de Sousa Ferreira. Its target audience is students in Elementary School and aims to use the African game called Ntxuva in Mathematics classes to work on logical-mathematical reasoning, mathematical concepts already known to students, and to promote reflections on African social and cultural aspects.

The didactic proposal aims to develop logical-mathematical reasoning, using concepts familiar to students. During the activities, reflections on African social, cultural, political and historical issues related to the games are proposed. The proposal is in line with what is advocated by Caldeira (2009) and Silveira and Caldeira (2012), since it incorporates the process of Mathematical Modeling, allowing students to understand and attribute meaning to

mathematical knowledge from different cultures. The mathematical contents are linked to African cultural objects, highlighting the presence of Mathematics in diverse cultural situations, according to the perspective of Caldeira (2009). Thus, the proposal reflects an approximation with the Mathematical Modeling process defended by Caldeira (2009) and Silveira and Caldeira (2012).

The fifth proposal, called Education of ethnic-racial relations: an affective perspective, was developed by Ailda Damasceno Ayrosa, Alessandra Guimarães dos Santos Medina and Camila Santos da Silva. Aimed at teachers who work in the Youth and Adult Education (EJA) modality — 1st segment, its objective is to address, recognize and value Brazilian culture and its ethnic-racial relations, through problematizations and reflections on these themes and on the mathematical content presented in the classroom.

The proposal values Brazilian culture and ethnic-racial relations, promoting reflection and problematization of these themes, including mathematical content. However, it does not specify how this problematization will occur or whether it will consider the applicability of the content in social contexts or in other mathematical knowledge. The approach suggests an approximation with Caldeira's Mathematical Modeling perspective (2009), which highlights the importance of relating mathematical knowledge to society. Although the form of this reflection is not made explicit, the proposal seeks to contextualize the content according to the competencies and skills presented in the Base Nacional Comum Curricular [National Common Curricular Base — BNCC], aiming to value the student and their culture.

Entitled *The Mancala game: a guiding activity from the perspective of Law 10.639/03*, the sixth proposal was developed by Noelly Susana Goedert de Souza and Roberto Perides Moisés, and is aimed at students in the 6th grade of Middle School. The objective of this proposal is to hold discussions to combat structural racism, promoting the self-identification of Afro-descendants among students, the appreciation of African culture and the exploration of mathematical relationships present in the African game Mancala.

The appreciation of African culture through the game Mancala is one of the main intentions of the proposal, promoting the recognition of this culture and strengthening the self-identification of Afro-descendants and the self-esteem of students. Despite being an asynchronous activity, the proposal addresses mathematical content related to the construction of the game and the reasoning involved in the process of playing. It is aligned with the perspective of Caldeira (2009), seeking to make students recognize the presence of Mathematics in a game from another culture, in accordance with the implementation of Law n. 10.639/2003 in the classroom.

The seventh chapter describes the activity entitled *Art in South Africa: a journey to the Geometry of the Ndebele people*, developed by Adriana Ferreira Rebouças Campelo and Marlei Budny dos Santos Souza. Intended for Elementary School students, its objective is to provide discussions on mathematical knowledge from the perspective of African culture, understand the cultural diversity of the African continent and carry out reflections that value the cultural aspects of Africa.

The teaching proposal seeks to value African culture, especially that of the Ndebele people, allowing students to learn about and reinterpret this culture through their paintings. It establishes connections with mathematical content and stimulates students' creativity. Although it does not focus on the students' social context, the authors explicitly acknowledge the African ancestry present in Brazilian culture, suggesting that valuing African culture can contribute significantly to the learning process, in addition to stimulating student engagement.

The approach presented shares similarities with Mathematical Modeling, as advocated by Caldeira (2009), by relating mathematical content to the African cultural context. This

perspective demystifies Mathematics as an isolated science, presenting it in a contextualized and meaningful way. This approach can promote greater student participation by recognizing the presence of Mathematics in different social and cultural environments and situations.

Entitled *The history of the church of Our Lady of the Rosary in Cuiabá and the teaching of Mathematics: implementing Law 10.639/03*, the ninth proposal was prepared by Maria do Socorro Lucinio da Cruz Silva, Ana Laura Thezolin and Iara Maria Soares de Assis Frade, aimed at Middle School students. The proposal aims to: i) present the history of the construction of the church of Our Lady of the Rosary, which is located in the city of Cuiabá (MT, Brazil), highlighting the participation of enslaved blacks in the construction of this temple; and ii) analyze the geometric content and characteristics present in the church's structure and façade, using a ruler and compass to reproduce the geometric figures identified during the process.

The proposed activity establishes a connection between Mathematics and a historical point related to enslaved black people, aiming to lead students to reflect on and value the contribution of black people in the construction of the church. This approach reflects the perspective of Caldeira (2009), who highlights the importance of finding a link between school mathematical culture and its relations with society, making learning more meaningful.

The proposed activity establishes a connection between Mathematics and a historical point related to the enslaved black people, aiming to lead students to reflect and value the contribution of black people in the construction of the church. This approach reflects the perspective of Caldeira (2009) who highlights the importance of finding a link between the school's mathematical culture and its relations with society, making learning more meaningful.

The tenth proposal was entitled *The use of Jongo as a cultural practice and the school environment* and was prepared by Cristiano Gomes de Oliveira for students over the age of 12 and for teachers at the school unit where the author worked. The proposal seeks to present Jongo as a cultural manifestation for the school community, in an attempt to promote the appreciation of knowledge and practices present in the local culture and to relate mathematical knowledge with the activities involved in Jongo.

The proposal encourages students to conduct investigations through interviews and the creation of scripts, stimulating the development of critical thinking throughout the process, through discussions and reflections. The results of the investigations lead students to perceive and relate knowledge acquired in Jongo and community practices, reflecting Caldeira's (2009) perspective on Modeling, in which Mathematics is associated with the context analyzed, promoting the formation of critical, reflective and participatory students. The proposal also follows the approach of defining mathematical content throughout the process, as presented by Caldeira (2009), reinforcing the presence of elements related to Mathematical Modeling in its execution.

The proposal entitled *Black women in Science: a didactic proposal for the implementation of Law 10.639/03* was prepared by Paulo Ricardo de Andrade Oliveira and Talita Lopes Moreno. With the target audience focused on students from the 6th to 9th grade of Middle School, the objective is to conduct a study of the trajectory of two black women who participated in the First World Meeting of Women in Mathematics, the Ndebele women, and the Geometry present in the arts of these women. Furthermore, it seeks to value the arts of the women of the Ndebele people, establishing a relationship between paintings and the teaching of Geometry.

The Mathematical Modeling approach, according to Caldeira (2009), is similar to the proposal discussed, by relating Geometry concepts to the culture of the Ndebele people. Even though this culture is not directly inserted in the students' context, the proposal is to break down prejudices about the African continent and encourage the participation of women, especially

black women, in the field of Mathematics. In addition, it intended to recognize and explore the mathematical concepts present in African culture through the arts of Ndebele women.

This approach contributes to the education of students, allowing them to recognize mathematical concepts in different situations, including African culture. In this way, Mathematics gains meaning when presented in a contextualized way, different from the isolated view adopted in the school environment.

The thirteenth proposal was entitled *Ndebele paintings: culture, identity and difference colored by women*, prepared by Eric Machado Paulucci and Élide de Sousa Peres and is aimed at undergraduate students and continuing teacher training. Its objective is to discuss Law n. 10.639/2003 in anti-racist Mathematics Education and to investigate the paintings of Ndebele women, relating them to the mathematical knowledge involved in these works, and the participation of African women in the production of knowledge. This approach goes beyond the Western Mathematics traditionally taught in schools, as it encourages students to explore their imagination and reflections.

According to Caldeira (2009), Mathematics is intrinsically linked to culture, and the work with Mathematical Modeling seeks to address mathematical content related to society, in this case, to Ndebele culture. The proposal aimed to value African arts and the role of women in this context, associating Mathematics with a cultural context.

According to Caldeira (2009), working with Mathematical Modeling is not limited to relating mathematical content to the context, but shows students that there is not just one form of Mathematics. In the proposal presented, students are encouraged to explore the presence of mathematical concepts in Ndebele arts, providing a new perspective on Mathematics.

Although not explicitly mentioned, the proposal presented is close to the concepts of Mathematical Modeling defended by Caldeira (2009) and Silveira and Caldeira (2012). This is evidenced by the approach that seeks to associate Mathematics with the African cultural context and stimulate discussion about this relationship.

Proposal 14 was entitled *African representation in games: applicability in EJA classes* and was written by Marici Anne C. e Silva and Thathyany Freitas Miranda. It is aimed at students and teachers of the EJA modality, with the objective of combating discrimination and providing an environment for discussion about racism and social inequality, while promoting anti-racist learning, using the games Fanorona, Tsoro Yematatu and Vector Mancala in the classroom.

The proposal aims to relate mathematical content to the African cultural context, which is closer to the perspective of Mathematical Modeling defended by Caldeira (2009). This author emphasizes that Mathematics is linked to culture, and the proposal seeks to give meaning to Mathematics by presenting students with mathematical content present in African games. This can lead them to realize that Mathematics is present in various everyday situations.

The proposal entitled *Elaboration of affective maps in a context of Popular Education*, developed by Giselle Corrêa de Souza and Flávia de Fátima Santos Silva, is aimed at students in the second stage of EJA. The objective of the proposal is to define relationships between students' memories and the context of Popular Education, create affective maps working with Mathematics and Geometry and develop portfolios with the maps.

Caldeira (2009) highlights the importance of Mathematical Modeling in relating mathematical content to the individual context of students, providing meaning to the subject. In the proposal in question, Mathematics is applied to the construction of maps, using calculations of distances between locations chosen by the students and in defining the scale. This process not only addresses mathematical concepts but also values the history and trajectory

of the students by recalling significant places in their journeys, highlighting the intention to value the students' culture.

The sixteenth proposal, entitled *Capulanas: possibilities for teaching Geometry*, was developed by Bárbara de Paula Motta Mirson and Luciellen Shitini Rosa de Souza. Aimed at Middle School students and adult education, the proposal aims to promote new possibilities for teaching and learning Geometry and other areas, implementing Law n. 10.639/2003 in Mathematics classes.

This perspective connects with the ideas of Caldeira (2009), who emphasizes the importance of relating Mathematics to society or to the context of the students. Although Capulanas are not part of the students' immediate environment, the proposal seeks to integrate mathematical concepts with the African cultural context, aligning itself with the implementation of law no. 10.639/2003, which provides for the teaching of Afro-Brazilian culture in schools.

Furthermore, Caldeira (2009) argues that Mathematics is related to culture, therefore, when approaching Mathematics in a meaningful way, students are encouraged to perceive the presence of mathematical content in different situations, including in cultural aspects of other continents, as is the case of Capulanas.

4.2 Process of creating Mathematical Models (Biembengut's conception)

The proposal presented in the first chapter is entitled *An anti-racist approach to teaching graphs and tables* and was developed by Tamires Torres da Purificação and Washington Santos dos Reis. The proposal is aimed at 9th grade Middle School students and proposes that they seek data involving racial issues on the themes of race and the labor market, race and the map of violence in Brazil, social inequality, gender and race (focusing on black women), quota policy, Higher Education and race, race and income/economy, and entrepreneurship and race, with the aim of creating graphs and tables with the data collected.

The proposal presented is similar to Biembengut's (2014) concept of Mathematical Modeling, when it is proposed that students build a mathematical model, graphs and tables, allowing them to understand and reflect on real-life situations.

The eighth proposal was entitled *Art of Ndebele women: exploring Geometry and African Culture* and was produced by Flávia Odenheimer Trevisan and René Aparecido Santos. Its target audience is EJA students and its objective is to hold discussions and reflections on cultural aspects of Africa, with a focus on the work of Ndebele women, and to work on concepts of Geometry in the construction of a hut inspired by the work of these women.

The proposal encourages students to research and collect data on African culture, especially that of the Ndebele people, while encouraging them to reflect on beliefs and stereotypes associated with Africa.

The proposal incorporates Biembengut's (2016) Modeling perspective by following the steps of the Modeling process, from initial research on Ndebele culture to the creation of a model of the hut and its application in drawings inspired by the culture.

Cited in section 4.1 of this article, the proposal *The history of the church of Our Lady of the Rosary in Cuiabá and the teaching of Mathematics: implementing Law 10.639/03* seeks to work with the history of the church of Our Lady of the Rosary, located in Cuiabá (MT, Brazil), discussing the participation of enslaved blacks in its construction. It also seeks to work with

geometric concepts present in the façade and structure of the church. Thus, the proposal shares similarities with the approach of Biembengut (2014), encouraging students to investigate and identify geometric figures in the structure of the church, recognizing mathematical elements in daily life and in the local culture. In addition, the stages of the Mathematical Modeling process of Biembengut (2014), which result in obtaining a mathematical model from the problem situation, are similar to the stages of the proposal, in which students reproduce the drawing of the church, highlighting geometric figures and performing calculations of area, perimeter and volume.

The eleventh proposal, called *Transdisciplinarity in the braiding of ribbons in the Congada culture*, developed by Beatriz Akiria de Assis Quaresma and Lúcia Helena dos Santos Oliveira, involves 7th grade Middle School students. Its objective is to present Congada in an interdisciplinary manner and to work on mathematical and historical concepts present in the cultural manifestation.

The proposal is based on the Mathematical Modeling approach from the perspective of Biembengut and Hein (2003), who advocate modeling as a means of relating mathematics to reality, aiming to obtain a mathematical model. The phases presented in the Modeling process, which include perception and apprehension, understanding and explanation, and meaning and expression, present similarities with the steps proposed in the execution of the activity. However, it is worth noting that the third phase, which involves the interpretation, evaluation and validation of the results of the application of the model, does not occur fully in the proposal, since the students do not apply the mathematical model formulated in the situation, beginning with the relationships of the mathematical content of Geometry with the problem addressed.

Proposal 14, entitled *African representation in games: applicability in EJA classes*, cited in sections 4.1 and 4.3, seeks to combat racism and to hold discussions on racial and social issues using the games Fanorona, Tsoro Yematatu and Vector Mancala.

There is an approximation with the Mathematical Modeling advocated by Biembengut (2016), as it results in the construction of a model. The proposal leads students to build African games to work with African culture related to mathematical concepts, considering the games as models that represent something to be accomplished, understood or explained.

The proposal entitled *Capulanas: possibilities for the teaching of Geometry*, cited in section 4.1, seeks to promote new possibilities for the teaching and learning of Geometry and other areas, implementing Law n. 10.639/2003 in Mathematics classes.

Throughout the stages of the proposal, students will make Capulanas, aimed at exploring concepts of Geometry. The creation of a Capulana aligns with Biembengut's (2016) conception of Mathematical Modeling, as it results in the creation of a model, allowing students to carry out investigations that relate Geometry to the arts present in Capulanas.

4.3 Barbosa's social vision

The proposal entitled *Art of Ndebele women: exploring Geometry and African culture*, mentioned in section 4.2, seeks to conduct research, discussions and reflections on cultural aspects of Africa, focusing on the work of Ndebele women, and to work on concepts of Geometry in the construction of a hut inspired by the work of these women.

The use of mathematical knowledge and everyday practices throughout the process makes the application of this knowledge significant for student learning, going beyond the school environment. The approach resembles the Mathematical Modeling perspective defended

by Barbosa (2004), which emphasizes the use of mathematics to understand situations in the social world, highlighting the importance not only of African culture, but also of the student's knowledge construction process. Furthermore, the proposal adapts to level (case) 2 of the work with Mathematical Modeling, in which students conduct research and collect data based on a theme presented by the teacher.

The tenth proposal, entitled *The use of Jongo as a cultural practice and the school environment*, cited in section 4.1 of this work, encourages research and reflections on the game Jongo, as a cultural manifestation, and its relationship with mathematical knowledge.

As there is an intention to associate school content with community knowledge and the cultural practice of Jongo, this approach is aligned with the Mathematical Modeling process, as stated by Barbosa (2004), highlighting the importance of research, questioning and formulating questions, which are present in the proposal.

Cited in sections 4.1 and 4.2, the proposal *African representation in games: applicability in EJA classes* aims to promote discussions on racial and social issues using the games Fanorona, Tsoro Yematatu and Vector Mancala.

The proposal presented provides students with an environment focused on conducting investigations and problematizing the topic addressed, in which students, with the teacher, will conduct bibliographic research on the topics in the areas of knowledge of Physics and Mathematics, approaching the perspective defended by Barbosa (2004). Despite this, the proposal distances itself from the idea of Barbosa (2001, 2004) by not leading the student to problematize social situations through mathematics, but rather to make connections between mathematical and physical content present in games of African culture.

4.4 Mathematical Modeling as a Research Process (Burak's conception)

The proposal entitled *An antiracist approach to teaching graphs and tables* aims to work with data involving racial issues in Brazil, mentioned in section 4.2. This approach is similar to the perspective advocated by Burak (2004) because it is structured in stages, including the choice of the theme, exploratory research, problem-solving, mathematical resolution and development, in addition to critical analysis of the solutions.

4.5 Absence of approaches to Mathematical Modeling

Called *The importance of Law 10.639/03 to train teachers who will teach Mathematics*, developed by Michela Caroline Macêdo and Tuane Pacheco, it was the fourth proposal analyzed. Aimed at undergraduate students of the Mathematics and Pedagogy courses, it seeks to train teachers with knowledge about ethnic-racial issues and African culture, exploring the relationship between Law No. 10.639/2003, African culture and mathematical learning.

The proposal presented by the authors Michela and Tuane does not specify the topics covered, nor does it discuss aspects of Mathematics Teaching, making it difficult to identify a relationship with Mathematical Modeling processes. Although there is an intention to encourage the understanding of African history and culture, there is no evidence that the reflections seek to integrate mathematics with these contexts, indicating the possible absence of Mathematical Modeling.

5 Final considerations

The proposed course, entitled *Ethnomathematics, Mathematical Modeling and Teacher Education: possibilities for implementing Law 10.639/03 in Mathematics Teaching*, played a crucial role in the consolidation and effectiveness of the project, as it theoretically supported

the actions carried out by the students and researchers and also provided moments of reflection and discussion on the topic.

Through training meetings, even though they were held remotely, it was possible to create a welcoming environment that stimulated in-depth discussions on ethnic-racial issues in mathematics teaching. These discussions were fundamental for the development of teachers and future teachers, especially with regard to the implementation of Law n. 10.639/2003, which establishes the mandatory teaching of Afro-Brazilian culture in schools. The importance of initiatives like this lies in the need to include mathematical content that relates to African and Afro-Brazilian culture in the school curriculum. In a society where customs and beliefs of African origin are often forgotten or marginalized, it is crucial to value this culture and combat prejudice through educational actions.

Approaching Mathematics in a way that is connected to the beliefs and customs of different peoples not only demystifies mathematical knowledge, but also values contexts that are often marginalized, promoting more meaningful learning. This process allows students to perceive that elements of their cultures, customs and beliefs, and those of their ancestors, can be studied in schools. In this context, the use of Mathematical Modeling appears to be a promising approach, since by constructing mathematical models that explore the application of Mathematics in real-life, culturally and socially relevant situations, the development of critical and reflective thinking is stimulated. The didactic proposals developed aim to integrate mathematics with ethnic-racial issues, using African cultural games and artifacts from the Ndebele culture, among other elements.

To facilitate the analysis of the material and thus organize the results found, five categories of analysis were developed. The first four are related to the Modeling concepts presented by Caldeira (2009), Barbosa (2004), Biembengut (2014, 2016) and Burak (2004, 2019). The fifth is related to the absence of any of these concepts. Among the didactic proposals analyzed, only one of them, entitled *The importance of Law 10.639/03 to train teachers who will teach Mathematics*, did not demonstrate a connection with any of the four Mathematical Modeling concepts discussed.

However, significant similarities were identified with the concept of Mathematical Modeling proposed by Caldeira, whose category was entitled *Caldeira's Socio-Critical Vision*. Twelve of the proposals analyzed were classified in this category, since they seek to integrate mathematics with African and Afro-Brazilian cultural activities, as well as with situations from students' daily lives. This approach allows mathematical concepts to be presented in a contextualized manner, making them more meaningful to students and stimulating discussions and reflections on the role of mathematics in society and in different cultures.

Regarding Barbosa's perspective, whose corresponding category was called *Barbosa's Social Vision*, similarities were observed in only three of the proposals. In these cases, the proposals encourage students to investigate, research and reflect on the presence of Mathematics in different contexts, relating situations or activities with mathematical concepts. This process promotes the development of research skills, critical thinking and the application of Mathematics in different scenarios.

Biembengut's concept of Mathematical Modeling, *Processes of elaboration of mathematical models*, was identified in five of the proposals. In these situations, the approach occurs mainly through the construction of mathematical models as a result of the work carried out throughout the activities. Furthermore, these proposals follow the steps suggested by Biembengut in his conception.

Finally, Burak's perspective, called *Mathematical Modeling as a research process*, was identified in only one of the proposals, entitled *An anti-racist approach to teaching graphs and*

tables. This proposal is close to Burak's conception, since it follows the five steps he presented for carrying out the activity.

Therefore, although there are no explicit discussions about the Mathematical Modeling process, there are approximations with the perspectives defended by Caldeira, Biembengut, Barbosa and Burak. In this way, the training course and the resulting didactic proposals demonstrate the potential of Mathematical Modeling as an effective tool to make Mathematics more meaningful for students, while promoting the appreciation of cultural diversity. This approach not only enriches the teaching of Mathematics, but also contributes to the construction of a more inclusive and respectful society in relation to the different cultures that comprise it.

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References

ARAÚJO, Jussara de Loiola. Uma abordagem sócio-crítica da modelagem matemática: a perspectiva da Educação Matemática Crítica. *Alexandria*, v. 2, n. 2, p. 55-68, jul. 2009.

ASSUNÇÃO, Helena Santos; AIÚBA, Aiúba Ali. Capulanas e macuti – camadas de tecidos, folhas e histórias. *Cadernos de Campo*, n. 23, p. 101-124, jul./dez. 2017.

BARBOSA, Jonei Cerqueira. Modelagem matemática: O que é? Por quê? Como? *Veritati*, n. 4, p. 73-80, 2004.

BARBOSA, Jonei Cerqueira. Modelagem na Educação Matemática: contribuições para o debate teórico. In: *Anais da 24ª Reunião Anual da ANPEd*. Caxambu, 2001, p. 1-30.

BIEMBENGUT, Maria Salett. 30 anos de modelagem matemática na educação brasileira: das propostas primeiras às propostas atuais. *Alexandria*, v. 2, n. 2, p. 7-32, jul. 2009.

BIEMBENGUT, Maria Salett. *Modelagem matemática no Ensino Fundamental*. Blumenau: Editora da FURB, 2014.

BIEMBENGUT, Maria Salett. *Modelagem na Educação Matemática e na Ciência*. São Paulo: Livraria da Física, 2016.

BIEMBENGUT, Maria Salett; HEIN, Nelson. *Modelagem matemática no ensino*. 3. ed. São Paulo: Contexto, 2003.

BONOTTO, Danusa de Lara; SCHELLER, Morgana; BIEMBENGUT, Maria Salett. Modelagem nas Ciências e Matemática: percepção de professores em formação continuada. In: *Anais da 9ª Conferência Nacional sobre Modelagem na Educação Matemática*. São Carlos, 2015, p. 1-15.

BURAK, Dionísio. A modelagem matemática na perspectiva da Educação Matemática: olhares múltiplos e complexos. *Educação Matemática sem Fronteiras*, v. 1, n. 1, p. 96-111, jan./jun. 2019. <https://doi.org/10.36661/2596-318X.2019v1i1.10740>

BURAK, Dionísio. Modelagem matemática e a sala de aula. In: *Anais do I Encontro Paranaense de Modelagem em Educação Matemática*. Londrina, 2004, p. 1-10.

BURAK, Dionísio. *Modelagem matemática: ações e interações no processo de ensino-aprendizagem*. 1992. 460f. Tese (Doutorado em Educação). Universidade Estadual de Campinas. Campinas. <https://doi.org/10.47749/T/UNICAMP.1992.46030>

CALDEIRA, Ademir Donizeti. Modelagem matemática: um outro olhar. *Alexandria*, v. 2, n. 2, p. 33-54, jul. 2009.

CEERT — Centro de Estudos das Relações de Trabalho e Desigualdades. *Edital de equidade racial na Educação Básica: pesquisas aplicadas e artigos*. São Paulo: CEERT, 2020.

CERVO, Amado Luiz; BERVIAN, Pedro Alcino; SILVA, Roberto. *Metodologia científica*. 6. ed. São Paulo: Pearson Prentice Hall, 2007.

MARCONI, Marina de Andrade; LAKATOS, Eva Maria. *Fundamentos de metodologia científica*. 5. ed. São Paulo: Atlas, 2003.

SILVEIRA, Everaldo; CALDEIRA, Ademir Donizeti. Modelagem na sala de aula: resistências e obstáculos. *Bolema*, v. 26, n. 43, p. 1021-1047, 2012. <https://doi.org/10.1590/S0103-636X2012000300012>