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Educação Matemática Debate

Neurodidactic analysis of the difficulty in learning the notion of rational number in the 3rd grade of Elementary School

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Article

Abstract: The objective of this article was to analyze the monitoring and remediation of learning difficulties related to the ability to operate with the notion of rational numbers in the 3rd grade of Elementary School. The methodology used was documentary and bibliographical research, based on the Common National Curricular Base, followed by references from Cognitive Neuroscience and Mathematics Didactics to justify the neurodidactic analysis. In the practical explanations and pedagogical considerations to the teacher of the documents analyzed, weaknesses were found that could be mitigated by neurocognitive principles related to attentional functioning added to principles of the Theories of Didactic Situations and Didactic Engineering.

Keywords: Neurodidactic Analysis. Didactics of Mathematics. Cognitive Neuroscience. Learning Difficulties. Rational Numbers.

Análisis neurodidáctico de la dificultad en el aprendizaje de la noción de número racional en 3º de Educación Primaria

Resumen: El objetivo de este artículo fue analizar el seguimiento y la remediación de las dificultades de aprendizaje relacionadas con la capacidad de operar con la noción de números racionales en el 3er año de Educación Primaria. Como metodología se utilizó la investigación documental y bibliográfica, partiendo de la Base Curricular Nacional Común, seguido de referencias de la Neurociencia Cognitiva y de la Didáctica de la Matemática para justificar el análisis neurodidáctico. En las explicaciones prácticas y consideraciones pedagógicas brindadas al docente en los documentos analizados, se encontraron debilidades que podrían ser mitigadas con principios neurocognitivos relacionados con el funcionamiento atencional combinados con principios de las Teorías de Situaciones Didácticas y la Ingeniería Didáctica.

Palabras clave: Análisis Neurodidáctico. Didáctica de las Matemáticas. Neurociencia Cognitiva. Dificultad de Aprendizaje. Número Racional.

Análise neurodidática da dificuldade de aprendizagem da noção de número racional no 3º ano do Ensino Fundamental

Resumo: O objetivo deste estudo foi analisar o acompanhamento e a remediação de dificuldades de aprendizagem relacionadas à habilidade de operar com a noção de número racional no 3º ano do Ensino Fundamental. Como metodologia utilizou-se das pesquisas documental e bibliográfica, partindo-se da Base Nacional Curricular Comum, seguidas de referências da Neurociência Cognitiva e da Didática da Matemática para justificar a análise neurodidática. Nas explicações de caráter prático e considerações pedagógicas ao professor dos documentos analisados, constataram-se fragilidades que poderão ser atenuadas por princípios neurocognitivos relativos ao funcionamento atencional somados a princípios das Teorias das Situações Didáticas e da Engenharia Didática.

Palavras-chave: Análise Neurodidática. Didática da Matemática. Neurociência Cognitiva. Dificuldade de Aprendizagem. Número Racional.

1 Introduction

The aim of this study was to analyze the monitoring and remediation of learning difficulties related to the ability to operate with the notion of rational numbers in the 3rd grade of Elementary School. To this end, a didactic-methodological manual was selected, which is part of the *Coleção Desafio da Matemática* [Mathematics Challenge Collection], part of the *Programa Nacional do Livro e do Material Didático* [National Book and Teaching Material Program — PNLD], focusing on the ability EF03MA09, prescribed in the *Base Nacional Comum Curricular* [National Common Curricular Base — BNCC] (Brasil, 2017).

The motivation for this analysis arose from the disagreement regarding the low academic performance of students in the area of Mathematics, despite the current scientific advances in learning, mainly in light of evidence from Neuroscience.

A relevant indicator for this study are the results of the Programme for International Student Assessment (PISA), conducted by the Organisation for Economic Co-operation and Development (OECD), in which 81 countries participate. There is a significant gap between the academic performance of French and Brazilian students. PISA assesses the areas of Reading, Mathematics and Sciences and, although France has lost some positions, it currently occupies the 22th position in Mathematics and Sciences and the 24th in Reading. Brazil, on the other hand, despite improving its performance over the years, is in the 65th, 62th and 52th positions, respectively (Brasil, 2023). It is notable that Mathematics has the worst performance in this comparison between the two countries and, when compared to the 81 participants, it remains far from the best classifications.

Student performance in Mathematics is so low that it is causing concern. Factors such as teacher training, the quality of teaching resources used in schools, learning difficulties, didactic and methodological concepts, and other issues involving the teaching and learning process may be the causes of this scenario. However, the main question to be answered in this study was: how are they organized didactically and what neuroscientific references can support the learning situations proposed to students, aiming at effectiveness in this process? Are the definitions of the didactic principles used clear? Are they related to the field of Neuroscience? The path taken in the search for answers resulted from documentary and bibliographical research to understand which contributions would help improve specific aspects of a manual for monitoring and remediating learning difficulties in the area of Mathematics, considering the theories of Mathematics Didactics and Cognitive Neuroscience. Based on the general objective, the following specific objectives were proposed: (i) to analyze, based on Cognitive Neuroscience, how attention recruitment can contribute to effective learning in didactic sequences for monitoring and remediation of learning; (ii) to present the different typologies of the Theory of Didactical Situations (TSD) to align them with the functionalities of attention in this context; (iii) to suggest Didactic Engineering as a method to structure and organize didactical situations in the service of effective learning; and (iv) to develop a neurodidactic analysis, detecting impacts on the learning of the notion of rational number.

Based on these objectives, the development of this study involves the description of the aforementioned manual, addressing its structure, the proposed didactic sequence, presentation EF03MA09, the proposed exercises, and the way in which mathematical knowledge is elucidated. Next, knowledge from theories of Mathematics Didactics of French origin was covered, such as Brousseau's Theory of Didactical Situations (TSD) (1986) and Artigue's Classical Didactical Engineering (1988), as well as theories of Cognitive Neuroscience, specifically those related to attention, presented by Gazzaniga, Ivry, and Mangun (2006). The choice of this content and teaching level is justified because it represents the fundamental moment in the students' school journey, marked by the insertion of new mathematical content.

Finally, the neurodidactic analysis of the mathematical learning situations presented in the manual revealed weaknesses, such as not fully covering the EF03MA09 ability of the BNCC, the need to restructure the statements of the questions in the didactic sequences to mobilize the cognitive resource of attention, as well as the importance of basing it on the typologies of the TSD. This analysis was guided by the principles of Didactic Engineering, considering the student as an actor in the knowledge construction process and the teacher as a mediator for effective learning.

2 A documentary analysis of the Manual of Learning Practices and Monitoring

The *Coleção Desafio da Matemática*, in addition to the textbooks in the student and teacher versions, is composed of the *Livro de Práticas e Acompanhamento da Aprendizagem* e o *Manual de Práticas e Acompanhamento da Aprendizagem* [Practice and Learning Accompaniment Workbook and the Manual of Learning Practices and Accompaniment]. From the 2th to the 5th grade, the Book consists of a review of knowledge, with the objective of monitoring and evaluating students' learning, as well as proposing the remediation of learning difficulties.

The *Practice and Learning Accompaniment Workbook* is fully contained in the *Manual of Learning Practices and Accompaniment*, however, it is not available in digital format. For this reason, the *Manual* is the focus of this analysis. Its purpose is to support teaching in the use of the book as a teaching resource, in addition to assisting in planning classes and remediating learning difficulties in relation to the proposed content. The didactic and methodological organization of the *Manual* consists of a presentation, the annual development plan, the proposal for teaching sequences, the distribution of BNCC abilities throughout the work, practical explanations and pedagogical considerations, annotated bibliographic references and exercise lists. The *Annual Development Plan* presents a planning suggestion for the school year, subdivided into two-month periods, with a structural sequence of content and itinerary to be followed by the teacher. In addition, it provides lesson plans for carrying out the practices proposed in the *Practice and Learning Accompaniment Workbook*. Although organized by two-month periods, these practices can also be adapted for quarterly planning. The *Manual* also indicates, in detail, the specific pages of the book to be used in each class.

As described in the BNCC document (Brasil, 2017, p. 286), the notion of rational number is characterized in the thematic unit referring to numbers in general and corresponds to the "meanings of half, third part, fourth part, fifth part and tenth part", which is expected to be developed in the 3rd grade of Elementary School. This thematic unit corresponds to a specific ability described as "(EF03MA09) Associate the quotient of a division with zero remainder of a natural number by 2, 3, 4, 5 and 10 with the ideas of half, third, fourth, fifth and tenth parts" (Brasil, 2017, p. 287).

The analysis of the *Manual of Learning Practices and Accompaniment* reveals that the contents related to the notion of rational number, as presented therein, seem incomplete in relation to what is described in the BNCC. It is observed that the approach prioritizes the description and construction of the meaning of half and third part, followed by the notion of exact division, but the meanings of fourth, fifth and tenth part are excluded, as provided for in the normative document. These contents are distributed between pages 27 and 30 of the *Practice and Learning Accompaniment Workbook*, which correspond to pages 67 to 69 of the *Manual* analyzed.

2.1 Description of the proposed teaching sequence

The *Manual* is organized into two teaching sequences, proposed for the application of the *Practice and Learning Accompaniment Workbook*, one for each semester. Each sequence is

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composed of the following items: theme, resources, thematic unit, guidelines, objectives, BNCC abilities and classes. Each class, in turn, contains the following items: specific content, teaching resources and guidelines. The proposal for the teaching sequences foresees a total of seven classes to be implemented.

Therefore, a hypothetical example follows (Table 1), however, analogous to the organization of the teaching sequence of the *Practice and Learning Accompaniment Workbook*, contained in the *Manual*, since the publisher prohibits the reproduction of the digital version.

Table 1: Hypothetical example 1

Theme: Quantities and Measurements
Objectives
▪ Assess students' prior knowledge of units of measurement.
▪ Recall units of measurement for length and measure length in centimeters.
BNCC abilities favored
▪ (EF03MA17) Recognize that the result of a measurement depends on the unit of measurement used.
▪ (EF03MA19) Estimate, measure and compare lengths, using non-standardized and most common standardized units of measurement (meter, centimeter and millimeter) and various measuring instruments.
Lesson 1
Specific content: length measurements
Teaching resources:
▪ Practice and Learning Accompaniment Workbook
▪ sulfite paper
▪ ruler
Forwardings
▪ Have a discussion with students to inform them that they are going to study units of measurement.
▪ Ask them to observe the classroom and ask: does everything have the same measurement? Is there the same way to measure objects? Among other questions.
▪ Suggest that they measure the objects of interest.
▪ Complete exercises 3 and 4 from list 2 in the *Book*.

Source: Own elaboration

It is important to consider that the document analyzed presents explanations and considerations regarding possible difficulties that students may have in solving the activities and exercises, as well as suggestions for resolving them. There are two distinct sections that organize the pedagogical work: one corresponds to practices and knowledge review and the other to monitoring learning.

In the *Manual of Learning Practices and Accompaniment*, the interval between pages 27 and 30 is provided for exercises that address ability EF03MA09. Pages 27 and 28 contain exercises that encompass ideas of division, reinforcing the importance of the student recognizing two main ideas: (1) dividing equally and (2) identifying how many times one quantity fits into another.

On page 29, the notion of half is addressed, which, in essence, should be understood as the exact division of a quantity distributed into two parts. Next, on page 30, we find the notion of a third, which is equivalent to the distribution of a certain quantity into three equal parts, also resulting in an exact division.

2.2 Structuring, organizing and solving proposed exercises

In the *Manual of Learning Practices and Accompaniment*, page 29 of the *Book of Practices and Monitoring of Learning* deals with the meaning of half, while page 30 addresses the meaning of third part. Both follow the analogous structure in organizing the content for presenting mathematical content and solving exercises.

To illustrate the statements used, here is an example: To determine half of a number, we divide it by 2. Joana bought 14 apples at the market and gave half to her friend. How many apples did each one have left?

Beside the problem situation, there is an image representing a girl, holding a box of apples in each hand, each of the boxes containing seven apples. This visual resource, combined with the written explanation, provides a simplified contextualization to aid the student's understanding. In addition, there is objective instruction, explaining the definition of half.

Following this image, there is a set of three exercises. Table 2 presents an example analogous to the exercises proposed in the material.

Table 2: Hypothetical example 2

1. Complete the sentence with the correct number: a) Half of 10 is ________. b) Half of 12 mangoes is ____ mangoes. c) Half of 8 liters of orange juice is _____ liters. 2. A lemonade recipe uses half a dozen lemons and 2 cups of milk. Roberta wants to make half of this recipe. a) How many lemons will she use? ______ b) How many glasses of milk will she use? _____ 3. Complete the sentences with the corresponding numbers: 9 6 12 a) Half of 18 eggs is _____ b) Half of two dozen is _____ c) Half of a dozen is _____

Source: Own elaboration

A similar teaching structure is used to present the meaning of third part. The content is presented through a simple problem situation, accompanied by an illustrative image. In addition, a set of three exercises is provided. The first exercise is the same as the one shown above, however, the number of items to be answered is increased. The second and third consist of simpler mathematical problems, each with a single answer. These questions involve situations of exact division, in which the quotient is equal to 3 (three).

3 Theoretical framework: elements for neurodidactic analysis

Neurodidactic analysis integrates knowledge from Mathematics Didactics — specifically the Theory of Didactical Situations (TSD) and Classical Didactic Engineering — and Cognitive Neuroscience, with an emphasis on the attentional processes studied by Cognitive Psychology. Based on this perspective, criteria were defined for the neurodidactic analysis of learning difficulties regarding the notion of rational numbers, which will be applied in the *Manual of Learning Practices and Accompaniment*, as presented in Table 3.

Table 3: Criteria for neurodidactic analysis

Order	Referential	Criteria	Characterization
1sh	Cognitive Neuroscience	Attention recruitment identification	Signals of elements of detection, selectivity and maintenance of attention (Gazzaniga, Ivry and Mangun, 2006; Sternberg, 2008)
2nh	Theory of Didactical Situations	Verification of typologies of Didactic Situations	Elements of the action, formulation, devolution, validation, institutionalization phases (Brousseau, 2008)
3rd	Typologies of teaching situations	Existence of Didactic Sequence	Information about situations structured over a number of classes with interconnections. Its objective is to allow the acquisition of knowledge, without exhausting the content covered, considering the needs and difficulties of students in this process. (Brousseau, 2008).
4th	Didactic Engineering	Identification of communicable and reproducible devices	Elements of the phases preliminary analysis, conceptions and *a priori* analysis, experimentation, *a posteriori* analysis and validation (Artigue, 1988)

Source: Own elaboration

3.1 Principles of Mathematics Didactics

According to Almouloud (2007), Mathematics Didactics developed in France in the 1970s, in the context of the reform of Modern Mathematics and the advent of Piaget's ideas on the development of intelligence and the acquisition of fundamental concepts. It encompassed the concern of studying the problems of teaching mathematical content, arising from the demands of mathematical knowledge. According to Serra et al. (2024, p. 26), "mathematics didactics enables the understanding of the different forms and practices of interaction between teachers and students in the context of the circumstances of teaching and learning mathematics, enabling the organization of its teaching". It deals with different theories, including Brousseau's Theory of Didactical Situations (TSD) (2008), based on Jean Piaget's psychology; Classical or first-generation Didactic Engineering, developed in the 1980s by Chevallard (2009), Brousseau (1982) and later by Artigue (1998); and Robert's Theory of Levels of Knowledge Functioning (1998), which underpins his research within the scope of Vygotsky's psychology. In this study, the focus will be on SDT and Classical Didactic Engineering.

After the 1970s, Guy Brousseau, through his article Fundamentals and Methods of Mathematics Didactics (1986), incorporated into Mathematics Didactics notions of didactic variations, theory of situations and didactic contract. In addition to expanding the student-teacher-knowledge pedagogical triangle, he emphasized that "one should not limit oneself solely to the classroom to study teaching and learning; one must consider the organization of the educational system (programs, curriculum, teaching materials, textbooks, timetables, etc.)" (Almouloud, 2007, p. 26)

In this sense, Mathematics Didactics supports the need to analyze teaching documents, their organization and the structure of exercises related to mathematical content taught in school. Regarding TSD and its influence in the classroom, it is important to highlight that

> This theory brings reflections on how we can design and expose mathematical content to students, in order to obtain an education that has meaning and

> context for the student. A didactic situation is established when pedagogical relationships occur between the triad teacher, student and mathematical knowledge in a learning situation, taking into account the environment. To understand the interaction between the larger space of life and the school environment, that is, the student's daily life and academic life, reference is made to adidactic situations that consist of the student's search for solutions, autonomously, in a situation that is beyond the teacher's control (Barbosa, 2016, p. 1).

Below, we see a conceptual map that presents some of the principles of TSD, created by Guy Brousseau (2008), illustrated in Figure 1.

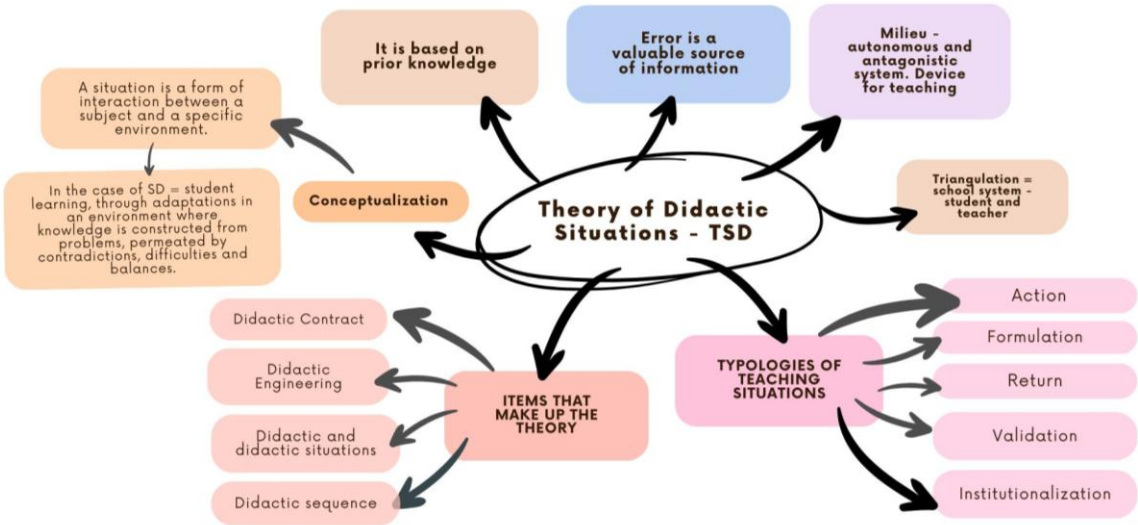

Figure 1: Principles of the Theory of Didactic Situations (Own elaboration)

As a way of associating TSD with the proposal of didactic sequences presented in the analyzed textbook, it is important to understand that a didactic sequence is defined as

> a series of situations that are structured over a pre-determined number of classes. Properly structured, these situations aim to make it possible to acquire quite clear knowledge, if the subject matter is exhausted. Thus, a didactic sequence cannot, a priori, have a stipulated duration according to the schedule, as its fulfillment takes into account the needs and difficulties of the students during the process. (Teixeira and Passos, 2014, p. 162).

Brousseau (1986) developed a theoretical model to assist mathematics teachers in presenting content in elementary school classes, guiding students to develop activities that provide opportunities for the appropriation of new knowledge. Artigue (1988) uses TSD to support the methodology of Didactic Engineering. In turn, this is concerned with constructing a theory based on didactic situations, configuring a methodology that, specifically, recruits a relationship between action and investigation in didactic situations in the classroom. The term *Didactic Engineering*

> was "coined" for didactic work as being that which is comparable to the work of the engineer who, in order to carry out a precise project, relies on scientific knowledge in his field, accepts to submit to a set of scientific types, but, at the same time, finds himself obliged to work on objects much more complex than the refined objects of science and, therefore, to face practically, with all the means at his disposal, problems that science does not want or cannot take into

account (Artigue, 1998, p. 283).

With Artigue (1988), Almouloud (2007) and Almouloud and Silva (2021) as the foundations, Figure 2 presents the principles of Didactic Engineering.

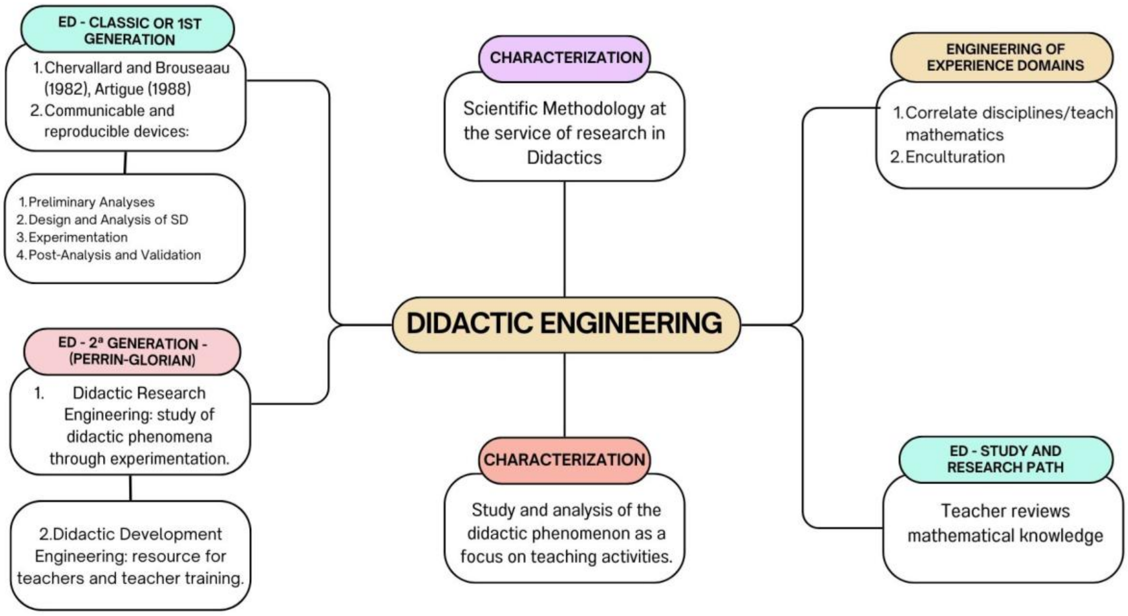

Figure 2: Principles of Didactic Engineering (Own elaboration)

In addition to the types of teaching situations described by Brousseau (2008), as well as the definition of teaching sequence, this analysis will incorporate the idea of Almoloud and Silva (2012), which addresses the principles of Classical Didactic Engineering in greater depth. These principles permeate the phases of preliminary analysis of the general theoretical teaching framework, the conception and *a priori* analysis of existing teaching situations, considering macro and micro-didactic variables, experimentation, consisting of the application of the teaching sequence, and, finally, *a posteriori* analysis and validation.

With a clear understanding of the types of teaching sequences and how they can benefit from the principles of Classical Didactic Engineering, it will be possible to better understand how the teaching sequences in the textbook could be better structured to promote the mobilization of the notion of rational number.

3.2 Principles of Cognitive Neuroscience

The term Cognitive Neuroscience, according to Gazzaniga, Ivry and Mangun (2006), also emerged in New York in the 1970s and, on second analysis, it is possible to understand that it occurred concomitantly with the emergence of Mathematics Didactics in France. Until then, Cognitive Psychology already had assumptions about how we perceive, learn, think and act, as reported by Sternberg (2008). However, this new term proposed studies on the functioning of the human mind, based on the paradigm of information acquisition and processing, which occur from internal transformations.

Kandel et al. (2014), in their book *Principles of Neuroscience*, reported that the human nervous system is organized into (1) central and (2) peripheral. The periphery is a region where the sense organs are located, responsible for capturing information, which is sent to the brain. This process is fundamental for the processing, decoding, understanding, storage and recall of the extracted information.

From an organic point of view, it is explained that the simultaneous activity of different sets of neurons, through an integrative action of the brain, generates cognition. Therefore, the study of the brain must understand this part of the nervous system as an information processing organ. The sets of cells that give rise to cognition are structured in a precise and orderly manner, and can undergo changes according to activity and learning.

At this internal and organic level, Cognitive Psychology focuses on studying and receives Cognitive Neuroscience as a technical and scientific improvement. Both study the functioning of human cognition and its structures. Thus, they seek to understand how processes occur at a mental level involving attention, language, memory, perception and behavior. "The term 'cognition' refers to all processes by which sensory input signals are transformed, reduced, elaborated, stored, retrieved and used [...]". (Neisser, 1967 apud Kandel et al., 2014, p. 238).

However, human cognition depends directly on two processes for the acquisition and active processing of information — attention and consciousness. According to Sternberg (2008), conscious attention serves three purposes: it helps to monitor the interaction of individuals with the environment, to establish a relationship between the past and the present, building a meaning for experiences, and, finally, to control and plan future actions, based on information from monitoring and connections between the past and the present. In this way, it can be understood that cognition, as a process involved in the processing of information, is directly connected with learning.

> The word learning, therefore, involves an individual with his or her brain capturing information from the environment, holding it for some time, and eventually retrieving it and using it to guide subsequent behavior. The concept of learning largely overlaps with that of memory, although the two should be distinguished, considering memory as the complete process, and learning, only as the stage of acquisition. (Lent and Buchweitzm, 2018, p. 56)

Therefore, understanding what attention is and how it works is an essential condition for selecting stimuli aligned with efficient didactic and methodological proposals in promoting effective learning. Therefore, it is understood that:

> Attention is the means by which we select and process a limited amount of information from among all the information captured by our senses, our stored memories, and other cognitive processes [...]. Attention includes both conscious and unconscious processes [...], because if a person paid conscious attention to everything around him, he would soon feel overwhelmed. (Sternberg, 2008, p. 107).

Attention performs four basic functions: signal detection and surveillance, search, selective attention, and divided attention. As it is an important cognitive function for acquiring any information from the environment, understanding the processes involved in it helps to better structure environmental stimulation, favoring new learning. Table 4, based on Sternberg (2008), describes each of the functions of attention, focusing on gathering information from the environment.

By presenting Table 4, it is possible to understand that defining appropriate stimuli, aligned with the curricular contents and with significant and efficient learning experiences, is an essential action for successfully extracting information from the environment. Knowledge about the types of attention also influences the decision regarding the choice of these stimuli, since it affects the connection between the information extracted from the environment and the students' existing repertoire. By understanding how the nervous system selects the stimulus based on attention, or can share it during the performance of tasks, an interesting path emerges for organizing more efficient didactic sequences, regarding the capture and processing of information.

Table 4: Functions of Attention

Function	Description	Examples
Signal detection and surveillance	Tries to detect the emergence of stimuli	▪ In a research submarine, listen for intermittent and unusual sounds ▪ In a dark street, listen for unwanted signals or sounds ▪ After an earthquake, be cautious about the smell of gas or smoke
Search	Active searches are made for certain stimuli	▪ When smoke is detected (through surveillance), an active search is made for the source of the smoke ▪ Look for keys, glasses and other items that you missed
Selective attention	Choose to pay attention to some stimuli and ignore others. Focusing attention helps you perform other cognitive processes, such as verbal comprehension or problem-solving.	▪ Paying attention to reading a book or attending a class while ignoring stimuli, such as a nearby radio or parallel conversations in the classroom
Divided attention	Can perform more than one task at the same time and redirects attentional resources, distributing them according to needs	▪ In many situations, drivers talk while driving, but if something emerges as a potential risk, they stop talking and focus their attention on the road

Source: Own elaboration

Based on the constructs of Psychology and Cognitive Neuroscience, the relevance of attention and its processes in promoting effective learning in human life is evident, as well as in the construction of concepts and abilities in regular education. Considering attention as a cognitive function and a fundamental neuroscientific principle for learning, the need arises to question: Given the abilities provided for in the BNCC, the didactic sequences structured in the analyzed document and the understanding of the principles of Mathematics Didactics and the functions of attention, based on Psychology and Cognitive Neuroscience, what ways are possible to improve the sequences and enhance the effective learning of the notion of rational number in the 3td year of Elementary School?

4 Neurodidactic analysis on the difficulty of learning the concept of rational numbers in the 3rd grade of Elementary School

In the first analysis, it is clear that the *Manual of Learning Practices and Accompaniment* of the *Coleção Desafio da Matemática* [Mathematics Challenge Collection] presents weaknesses in its relationship with the BNCC. When exploring the ability EF03MA09, which includes the notion of rational numbers, it is observed that there are no essential meanings, such as fourth, fifth and tenth parts, being restricted to only half and third part in contexts of exact division. Considering that the *Manual* intends to work with monitoring learning and remediation of difficulties, the lack of this knowledge in the 3rd grade of Elementary School, a stage in which the curriculum introduces this knowledge in the school day, will possibly cause a gap in the students' learning process.

The absence of elements of Mathematics Didactics as a reference in the *Manual* is significant, since this area underpins the didactic and instructional procedures appropriate for teaching mathematical knowledge. Although there is a concern with activities, pedagogical resources and innovative methodologies, there is no indication of bibliography or articles that offer specific didactic guidelines for teaching mathematics. The objective of the *Manual* is to assist teachers in monitoring, assessing and remediating learning difficulties regarding rational numbers, in addition to guiding teaching planning. However, these aspects are partially met due to the lack of didactic foundation. Both the forms of remediation and the teacher's planning itself require didactic foundations for instructional practices so that they can effectively contribute to the learning of mathematical knowledge. Regarding the first criterion, it is interesting to consider that connections between the students' previous repertoire and the new information to be acquired contribute to the understanding of the meaning of the content presented. In the *Manual*, the statements of the questions present weaknesses in this sense, as they lack elements that promote this connection. Thus, an alignment of the statements of the proposed exercises with the situations experienced by students in their daily lives is recommended, making the content more meaningful to them.

A study carried out on Mathematics Education justifies the relevance of the relationship between students' repertoire and the content, using excerpts from the BNCC itself:

> The thematic unit Numbers aims for students to develop numerical thinking "which implies knowledge of ways to quantify attributes of objects and to judge and interpret arguments based on quantities" (p. 268). In this educational perspective, students need to develop, among others, "the ideas of approximation, proportionality, equivalence and order, fundamental notions of Mathematics" based on "significant situations, successive expansions of numerical fields", in which "records, uses, meanings and operations must be emphasized" (Brasil, 2017, p. 268 apud Almouloud and Silva, 2021, p. 135).

These connections contribute to increasing the relevance of new information and maintaining attentional focus for longer to capture it more accurately. Thus, cognitive schemes will be evoked with previous information from the student's life context, which will be able to associate them with newly acquired knowledge. Considering that the *Manual of Learning Practices and Accompaniment* aims to monitor learning and contribute to overcoming difficulties, it is relevant to base teaching proposals on neuroscientific knowledge about attention.

According to Silva, Fonseca and Correia (2020, p. 251), "attention is one of the pillars of learning, because without it, the facilitation of the passage of information along the synapses with the purpose of forming lasting memories would not be possible". And, finally, without the consolidation of new information or knowledge in long-term memory, there will be no effective learning, which will hinder new learning.

To sustain the student's attentional focus for capturing information, it is possible to use the principles of SDT, according to criteria 2 and 3, cited in Table 3. These propose the provision of good problems to trigger in the student the search for new knowledge that will only be understood as learned when applied in situations outside the teaching context (Brousseau, 2008). Thus, the mathematical story of a problem that addresses a mathematical concept needs to be immersed in everyday situations and be functional in daily life, keeping the student aware of what needs to be assimilated.

Differentiating mathematical problems that serve effective learning from exercises is an important item to be considered in this analysis. The study *O ensino de números naturais e suas operações via Resolução de Problemas: uma análise em livros didáticos* [Teaching natural numbers and their operations through Problem Solving: an analysis in textbooks] can contribute to this understanding by indicating that

> differentiates problem and exercise by highlighting that a mathematical situation becomes a problem when it requires the mobilization of concepts, principles and prior mathematical knowledge, which do not directly belong to the content being learned. On the other hand, if the Mathematics situation requires only the direct use of formulas, this situation becomes an exercise (Lazarini, Mendes and Proença, 2024, p. 2).

In this context, the formulation of good mathematical problems must take into account the student’s mental state of vigilance, since it directly influences the level of alertness required to capture and select stimuli and information consciously. Alertness behavior is intrinsically related to the processes of detecting, searching for, and selecting information, as well as to sustaining attention (Gazzaniga, Ivry, and Mangun, 2006; Sternberg, 2008). If the didactic instruction does not have elements that allow the student to maintain attention for the time necessary to search for, detect, and select information, there will be losses in their appropriation, creating a gap in the understanding of a concept or knowledge. By combining SDT with the principles of Cognitive Neuroscience, well-structured mathematical problems should efficiently recruit the student's attention.

In this sense, a study by Fonseca (2019) demonstrates choices to understand the learning process. Considering High School and Higher Education in Brazil and France through the prioritization of the notion of Trigonometric Functions, the study contributes to the understanding of what is necessary to “redimension reflections on the cause of the phenomena of school difficulties, mobilizing efforts to mobilize their evolution and contagion in subsequent levels of Mathematics teaching, primarily” (Fonseca, 2019, p. 127). For this reason, combining Mathematics Didactics and Cognitive Neuroscience is a promising path for educational advances that lead to higher levels of academic performance.

Therefore, in the mobilization of the notion of rational number foreseen for the 3rd grade of Elementary School, it is possible, for example, to structure an ordered set of situations that make sense among themselves. A methodological proposal appropriate to this structuring foresees sequences structured in an orderly manner, considering the construction of meaning and duration, the didactic contract that favors learning through the teacher-student relationship to assist in the internalization of content and, finally, Didactic Engineering that has, in its methodological principles, investigation and action, enabling various meanings to pedagogical planning (Brousseau, 2008).

This structure contributes to the notion of rational number being properly consolidated, with no need for new remediation procedures, as can be seen in a study involving natural numbers:

> It is important to note that the prevalence of the meaning of partition in textbooks, as well as the reduction of calculation procedures over the years, can harm students' learning, since the teaching of the division operation of natural numbers ends up being centered on a limited vision (Cruz and Teles, 2020, p. 16).

In a study in Argentina, which also analyzes a teaching manual based on TSD, there is an example of how an appropriate teaching situation can be structured:

> [...] for the situation to be systematized, it is essential to go through the four moments, or situations, relevant to this process. The first of these, called action (Brousseau, 2008), is when students learn to create strategies, starting from something random that they themselves created, and not from what came from the educator. In the context of mental calculation within situations with rational numbers, for example, they realize that "answering randomly is not the best strategy" (Brousseau, 2008, p. 23). After realizing that they need strategies, it is essential that they move on to the second situation, called formulation, where "[...] they discover the importance of discussing and defining strategies" (p. 24). In this situation, they realize the value contained in dialogue and discussion about what they can develop to overcome challenges. This is called the formulation situation (Brousseau, 2008). The third situation to solidify a situation is based on the demonstration of truth in a given circumstance, called the validation situation (Brousseau, 2008). The fourth situation is institutionalization, the moment in which the teacher verifies everything that was systematized in the three previous phases, classifying what can be reused, reviewing what was done (Brousseau, 2008) (Cosme and Berticelli, 2024, p. 11).

Thus, although the author of the *Manual* mentions the expression didactic sequence in the didactic-pedagogical proposal, he does not necessarily use it in accordance with the TSD. The document contains practical explanations and pedagogical considerations for the teacher that suggest exploring concrete materials, in addition to the role-play, to help the student understand, if he does not do so through the tasks presented. This action seems to favor the remedy of the difficulty in capturing information. However, if the first task were structured with the TSD typologies, the notion of double and third part, the only knowledge presented in the Manual regarding rational numbers, would have the possibility of being learned effectively.

These typologies offer resources that sustain the attentional focus and improve abilities around mathematical knowledge by the student, without necessarily needing remediation. The pedagogical sense of the teacher's planning, then, is constructed through action and investigation, and becomes fundamental for maintaining the students' attention when implementing a didactic situation. In this way, Didactic Engineering becomes a way of equipping the teacher based on the analysis of the teaching and learning context. According to Almoloud and Coutinho (2008) and Kasahara and Sá (2023), through actions such as preliminary analyses, conceptions and analysis of the didactic situations themselves, experimentation, a posteriori analyses and validation, there will be an impact on the effective learning of mathematical content by the student.

An article on the integration between theory and practice in teaching mathematics through Didactic Engineering explains that it

> allows teachers and researchers to plan, execute, observe, and analyze the teaching and learning of specific mathematical concepts in the classroom. They can then use this information to continually improve their teaching practices, making mathematics more accessible and attractive to students. Machado (2012) states that this methodology was created with the aim of analyzing the didactic situations that are the object of study of Mathematics Didactics, thus inserting itself into this theoretical framework (Lopes, Costa and Costa, 2024, p. 5).

Based on the principles of Didactic Engineering, it can be seen that the proposed tasks are strongly linked to the teaching context, prioritizing the mechanical application of a single calculation procedure to obtain the answer, to the detriment of the meaningful construction of the concept of rational number. Furthermore, the lack of connection between the proposed questions compromises didactic coherence, making it difficult to sustain attention and mobilize cognitive schemes that favor effective learning.

Thus, in the preliminary analyses of the concepts and didactic sequences of the *Manual*, there are relevant weaknesses, even without advancing to the phases of experimentation, *a posteriori* analysis and validation foreseen in Didactic Engineering. Therefore, this approach presents itself as an interesting suggestion to be applied in a process of restructuring and updating the didactic sequences, as well as the way they approach the notion of rational number.

5 Final considerations

The objective of this study was to analyze the monitoring and remediation of learning difficulties related to the ability to operate with the notion of rational numbers in the 3rd grade of Elementary School. To this end, the document *Manual of Learning Practices and Accompaniment* was chosen, which contains the *Practice and Learning Monitoring Workbook*.

Based on the neurodidactic analysis carried out, some weaknesses were found, such as the impairment of the understanding of the meaning of the notion of rational numbers by students due to mathematical stories with impoverished contexts, with no connection to their reality. This factor does not motivate students to remain alert long enough to evoke their prior knowledge in search of a connection to the new information captured. Adding the acquired repertoire to new knowledge is an important action for the construction of lasting memories, which will result in effective learning.

Another weakness was the lack of meaning in the structuring of the didactic sequence that included simple exercises, with no connection between them. As suggested by TSD, this is a valuable characteristic for teaching situations. Furthermore, students' level of vigilance is increased in situations like this, thus sustaining their attention.

These two issues arise from the failure to consider, in a manual for teachers' use, a basic reference for those who teach mathematics: Mathematics Didactics. Furthermore, it is suggested that knowledge of Cognitive Neuroscience be provided, since the relationship between neuroscience and education is recognized as a path that contributes to effective learning.

Furthermore, the Manual that proposes monitoring learning and remediating difficulties does not address all the meanings provided for in the EF03MA09 skills. The notions of the fourth, fifth and tenth parts are not included, therefore, gaps in access to knowledge provided for in the BNCC may occur.

Didactic Engineering is also an interesting methodological and scientific suggestion for the author of the *Manual*, in order to improve the practical explanations and the pedagogical considerations directed to the teacher. The knowledge addressed in the theoretical basis, discussion and analysis of the criteria proposed in this study are based on knowledge, experiences and studies of researchers who have great potential to contribute to this improvement. Thus, both for the purpose of monitoring learning and for remediating difficulties, it is possible to increase the effectiveness of new teaching situations.

The practical explanations and pedagogical considerations may have aggregating elements arising from the knowledge of Cognitive Neuroscience:

> Although the issue of prior knowledge has been treated as the main factor to consider when developing pedagogical strategies, understanding how the

> brain works with cognitive functions such as "attention" can open doors to developing more effective strategies, since the student's learning production potential depends on their attention and cognitive engagement (Silva, Fonseca and Correia, 2020, p. 248).

In this way, the *Manual* can be a differential to meet the demands foreseen by the BNCC, aiming at monitoring learning and remediating difficulties. To this end, including Mathematics Didactics and knowledge of Cognitive Neuroscience as a reference, according to the Neurodidactic analysis carried out, will enable significant advances in the effective learning of the notion of rational number.

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