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Educação Matemática Debate

Investigating the concept of area for teaching and Creativity in Mathematics: interactions based on a story

Abstract: This paper describes a virtual teacher education forum based on the perspective of Mathematics for Teaching and Problematized Mathematics. The data produced and presented derives from the story *Imagined experience: the concept of area straight from the classroom* and included multiple resolutions, ways of listening and shifts in the meanings of *error* and *non-understanding*. In methodological terms, considering the Development Research, there were indications of the potential and weaknesses of the Training Situation portrayed. In conclusion, the alignment with Creativity in Mathematics was found to be relevant, since it contributed to the mobilization of interactions, favoring the reorganization of Mathematics for teaching the concept of area to participants.

Keywords: Teacher Education. Concept of Area. Mathematics for Teaching. Problematized Mathematics. Creativity in Mathematics.

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Article

Investigación del concepto de área para la enseñanza y la Creatividad en Matemáticas: interacciones a partir de un cuento

Resumen: Este artículo retrata un foro virtual de formación docente, a partir de la perspectiva de la Matemática para la Enseñanza y la Matemática Problematizada. Los datos producidos, presentados aquí, se basaron en el cuento *Experiencia Imaginada: El concepto de área directamente desde el aula* y contemplaron las múltiples resoluciones, los modos de escucha y los desplazamientos en los sentidos de error y no comprensión. En términos metodológicos, considerando la Investigación para el Desarrollo, verificamos las potencialidades y debilidades de la Situación de Formación retratada. Como conclusión, se brinda la relevancia del alineamiento con la Creatividad en Matemáticas, ya que contribuyó a la movilización de interacciones, favoreciendo la reorganización de la Matemática para la enseñanza del concepto de área a participantes.

Palabras clave: Formación Docente. Concepto de Área. Matemática para la Enseñanza. Matemática Problematizada. Creatividad en Matemáticas.

Investigação do conceito de área para o ensino e Criatividade em Matemática: interações a partir de uma história

Resumo: Este artigo retrata um fórum virtual de formação docente, baseado na perspectiva da Matemática para o Ensino e da Matemática Problematizada. Os dados produzidos e apresentados decorreram da história *Experiência imaginada: O conceito de área direto da sala de aula* e contemplaram as múltiplas resoluções, os modos de escuta e os deslocamentos nos sentidos de *erro* e de *não entendimento*. Em termos metodológicos, considerando a Pesquisa de Desenvolvimento, verificaram-se indícios de potencialidades e fragilidades da Situação de Formação retratada. Como conclusão, constatou-se a relevância do alinhamento à Criatividade em Matemática, visto que este contribuiu para a mobilização de interações, favorecendo a reorganização da Matemática para o ensino do conceito de área a participantes.

Palavras-chave: Formação Docente. Conceito de Área. Matemática para o Ensino. Matemática Problematizada. Criatividade em Matemática.

1 Introduction

This paper presents an action from an educational course with teachers and undergraduates, based on the perspective of Mathematics for Teaching (Davis and Renert, 2014) and Problematized Mathematics (Giraldo and Roque, 2021). Using the Concept Study methodology, it focused on discussions, interactions and reflections on the concept of area for teaching.

This education aimed to make research, teaching and extension inseparable. In terms of research, it corresponds to the educational process/product of a thesis linked to the teacher education line of the Science and Mathematics Education Program at the Federal Institute of Espírito Santo — Educimat/Ifes.

As far as extension is concerned, the education, implemented by members of the Espírito Santo Mathematics Education Studies and Research Group (Gepem-ES), took the form of a course, through an agreement signed between Ifes campus Vitória-ES, the Education Secretariat of the Cariacica and Vila Velha Municipalities and the Brazilian Society of Mathematics Education — Espírito Santo Regional (SBEM-ES).

In terms of teaching, the education involved Mathematics undergraduates and postgraduates linked to Gepem-ES. In this sense, it is worth noting that researchers from this group have carried out several studies focusing on the concept of area, the results of which can be found in the research by Campos, Coutinho and Paiva (2022), Campos and Paiva (2020), Campos, Lorenzutti and Paiva (2024), and Campos, Paiva and Soares (2023).

The following sections set out the theoretical and methodological references; briefly describe the organization of teacher education; explain the factors that motivated the inclusion of Creativity in Mathematics in the teacher education proposal; and discuss a Training Situation (TS), implemented in the form of a virtual forum, which aimed to provide interactions and reflections through multiple resolutions.

2 Theoretical and methodological references

The question: *What knowledge does a teacher need to teach?* has been the subject of investigation by various researchers since the 1970s. The answers to this question reflect the way in which research conceives the specificity of the knowledge that underpins teaching practice and, consequently, implies different contributions to the field of teacher education.

The studies begun by Ball and Bass (2002) and continued by Ball, Thames and Phelps (2008) have made a significant contribution to the education of teachers who teach Mathematics, by highlighting the difference between teachers' knowledge of Mathematics and that demanded by other areas. In addition, these researchers warn that mathematical knowledge for teaching goes beyond mastery of advanced Mathematics, emphasizing that “there are aspects that go beyond pedagogical content knowledge that need to be discovered, mapped, organized and included in Mathematics courses for teachers” (Ball, Thames and Phelps, 2008, p. 398).

Davis and Simmt (2006) and Davis and Renert (2014) acknowledge the contributions of Ball and Bass (2002) and Ball, Thames and Phelps (2008) with regard to the knowledge mobilized in Mathematics teaching. However, they point out that these studies highlight what a teacher knows or doesn't know on an individual basis, while the domain of knowledge for teaching Mathematics “is much more than a readily catalogued or objectively tested set of concepts” (Davis and Renert, 2014, p. 14).

In considering these reflections by Davis and his collaborators, it is understood that Mathematics for teachers' teaching should not be understood as a static body, but as something that is constructed in practice, involving an explicit and tacit character. Explicit knowledge is codified and therefore more easily shared, while tacit knowledge, because it is unstructured and highly personal, can be difficult to communicate.

Therefore, in view of the diversity of notions that are adopted by teachers that are not yet conscious or systematized, and as opposed to a domain to be appropriated by teachers as an established structure, Davis and his collaborators have stressed that Mathematics for teacher education is neither static nor individual, but dynamic and collective. Thus, it is possible to observe a contrast between the way Ball and Bass (2002) and Ball, Thames and Phelps (2008) and Davis and Simmt (2006) and Davis and Renert (2014) conceive of knowledge and teacher education: the former focusing on the teacher and their knowledge for action, and the latter considering the collective and emerging possibilities, valuing knowledge in action.

This alignment in terms of knowledge in action — in other words, considering teaching practice as a starting point rather than an end point — has been explored in various studies, and is an approach that provides reflections and contributions to Mathematics teaching (Campos and Gualandi, 2020; Paiva, Sousa and Campos, 2023; Ponte and Serrazina, 2004).

In line with the arguments of Davis and Simmt (2006) and Davis and Renert (2014), the training presented in this text, when dealing with the specificity of the knowledge that underpins teaching, considers Mathematics for Teaching, defended by Davis and Renert (2014), characterized as

> an open disposition towards mathematical knowledge that enables the teacher to structure learning situations, interpret students' actions carefully, and respond flexibly, so as to allow students to broaden their understandings and expand the range of their interpretive possibilities through access to powerful connections and appropriate practices [...] Teachers must have a deep understanding of emergent Mathematics (p. 117).

For Davis and Simmt (2006) and Davis and Renert (2014), teachers' Mathematics for Teaching is constituted by the existence and inseparability between established Mathematics — objectified Mathematics and curriculum content — and produced Mathematics — collective interpretation and subjective understanding.

Based on this conception and with the aim of developing Mathematics for Teaching, the authors propose Concept Study, defined as “a participatory methodology through which teachers interrogate and elaborate their Mathematics” (Davis and Renert, 2014, p. 35). This methodology combines elements of two notions: concept analysis with a focus on the mathematical concept, and lesson study, which adopts a collaborative structure, based on the following assumptions:

> Individual knowledge and collective knowledge cannot be dichotomized; collective possibilities are involved and unfold in individual understandings; Mathematics for Teaching (M_4T) is too vast and too volatile to be considered in terms of mastery by any individual. On the contrary, it is both an individual and a collective phenomenon; at the individual level, understandings of mathematical concepts and conceptions of Mathematics are emerging; at the collective social level, teachers' knowledge of Mathematics is largely tacit, but

> critical elements of this knowledge can be questioned in groups. At the cultural level, teachers are vital participants in the creation of Mathematics, mainly through the selection and preferential emphasis given to particular interpretations (Davis and Renert, 2014, p. 33).

In addition to the training being based on the aspects and assumptions of Conceptual Research, the reflections of Davis and Renert (2014) were also taken into account. These modes were adopted both for structuring the proposal and for implementing the training. But what are these listening modes anyway?

> [...] evaluative listening focuses on the past with its concern for fidelity to established knowledge; interpretive listening adds a concern for the present, as it attends to students' interpretations; and hermeneutic listening includes and transcends the others by superimposing an interest in the future, in emerging possibilities (Davis and Renert, 2014, p. 87).

These modes of listening relate to the way in which teachers who teach Mathematics listen to and deal with the Mathematics produced by students. In the teacher education proposal presented, these modes correspond to the way in which the meanings and knowledge of teachers and trainees are considered as part of the educational content.

Giraldo and Roque (2021, p. 19), in their reflection about Mathematics-for-Teaching, assert that for this perspectiva “teachers are considered vital participants in the production of mathematical possibilities, giving form and substance to cultural Mathematics, that is, not only formal Mathematics, but also a diversity of culturally situated practices, perspectives and applications”. In addition, the authors elucidate a rapprochement between this perspective and Problematized Mathematics, since both seek to enable and relativize understandings, understanding mathematical constructions as something that goes beyond a simple means of achieving formal Mathematics.

When referring to the exposition of Mathematics, Giraldo and Roque (2021) contrast two forms: the order of structure and the orders of invention. Through Problematized Mathematics, they defend the orders of (re)invention. For these authors, “ the order of structure, corresponding to the organization according to logical implications and legitimation criteria accepted today; and the orders of invention, referenced in various ways of knowledge production mobilized in practices today recognized as Mathematics” (Giraldo and Roque, 2021, p. 1).

In line with the propositions of Problematized Mathematics, the research was developed with problems as the only *a priori* of Mathematics, so that “in the spaces and times of the classroom, the various solutions they engender are no longer reproduced and can be (re)invented, as they can gain other understandings, other meanings with the experiences of the subjects” (Giraldo and Roque, 2021, p. 21). Along these lines, problems are understood as having a nature that transcends solutions.

In addition, in this training, we considered the perspective of Problematized Mathematics, which proposes conceiving *error* as a power of creation and *non-understanding* as a possibility for new meanings to emerge (Giraldo and Roque, 2021). From this perspective, *errors* and *non-understanding* are expressed as elements that contribute to the reorganization of Mathematics for the teaching of teachers and trainees, promoting the re-signification and/or expansion of the meanings attributed to the concept of area.

By observing the assumptions and ways of listening related to Mathematics for

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Teaching (Davis and Renert, 2014) and the notes on the relevance of shifts in the meanings of *error* and *non-understanding* as gradations of knowledge of Problematized Mathematics (Giraldo and Roque, 2021), aspects related to Creativity in Mathematics were identified. Based on this interest, some considerations and reflections on this approach are presented below.

Leikin (2009) points out that there are a variety of perspectives on creativity and states that they continue to evolve over time. However, even though there is no single meaning for the term, researchers such as Vale and Barbosa (2015) identify similarities between attempts to define Creativity in Mathematics, considering the following aspects: (1) it involves divergent thinking; (2) it is mostly associated with the dimensions fluency, flexibility and originality; and (3) it is related to problem solving and formulation.

In the educational field, several studies on creativity have been developed, covering: the context of teacher education (Barbosa, Vale and Ferreira, 2015; Vale and Pimentel, 2012); pedagogical practices (Andreatta and Allevato, 2020; Vale, 2015); and university students from other areas of knowledge related to Mathematics (Leikin and Guberman, 2023).

Therefore, with the aim of contributing to studies on Creativity in Mathematics and explaining the approximation identified between this area and the perspectives of Mathematics for Teaching and Problematized Mathematics, in the context of training with undergraduates and teachers, the characterizations of the aforementioned aspects related to Creativity in Mathematics are set out below.

We begin by distinguishing between convergent and divergent thinking, based on the contributions of Vale and Pimentel (2012):

> Convergent thinking is a way of thinking oriented towards obtaining a single answer to a situation [...]. It usually involves a thought process that follows a set of rules according to logic, while divergent thinking looks at the problem, analyzing all the possibilities [...]. It is in this type of thinking that the solver engages in a creative elaboration of ideas provoked by a given stimulus. It is the opposite of convergent thinking, being a creative process that involves imagining as many solutions as possible; it is generally more spontaneous and free-flowing (p. 351).

With regard to the dimensions fluency, flexibility and originality, it can be seen that:

> *Fluency* is the ability to generate a large number of ideas and refers to the continuity of these ideas, the flow of associations and the use of basic knowledge. *Flexibility* is the ability to produce different categories or perceptions in which there are a variety of different ideas about the same problem or situation. *Originality* is the ability to create new, unique or extremely different ideas or products (Vale and Barbosa, 2015, p. 4).

Considering the dimensions addressed, we identified the possibility of articulation between Mathematical Creativity, Mathematics for Teaching and Problematized Mathematics. Mathematics for Teaching proposes that teachers act in such a way as to "enable students to broaden their understandings and expand the range of their interpretive possibilities through access to powerful connections and appropriate practices" (Davis and Renert, 2014, p. 11). In turn, Problematized Mathematics, by addressing the destabilization of the meanings of error and non-understanding that underpin conventional conceptions of mathematical learning, seeks to ensure that this destabilization goes beyond the epistemological dimension of mathematical knowledge, also reaching “a dimension of teacher education and practices” (Giraldo and Roque, 2021, p. 18).

Given these perspectives, in the context of reformulating the educational process/product, it is proposed to formulate a specific question to mobilize participants to interact about the contributions of considering error as a power of creation.

In order to address the aspect of problem solving and formulation related to Creativity in Mathematics, we used the contributions of Leikin (2009). This researcher proposes the notion of solution spaces, which correspond to ways of investigating different aspects of problem solving through multiple-solution tasks. These solution spaces can be organized into three categories: expert, individual and collective.

In general terms, expert solution spaces refer to the most complete set of known solutions to a problem. They can be conventional, recommended by the school curriculum, or unconventional, based on alternative strategies. Individual solution spaces correspond to solutions generated by a single individual and can be classified as available solution spaces — solutions that the individual can come up with independently - and potential solution spaces — solutions that are possible with external help. Collective solution spaces combine the solutions of a group of individuals and are generally broader than individual solutions and are even relevant to the development of other individual solutions. Along these lines, individual and collective solution spaces are subsets of expert solution spaces and seek to explore the multidimensional structure of problem solving (Leikin, 2009).

Therefore, multiple resolution tasks, implemented through the solution spaces mentioned, involve solving a problem in different ways and with different solutions based on different representations. This aligns with the perspective of Mathematics for Teaching, which considers that “collective possibilities are involved and unfold in individual understandings [...] teachers' knowledge of Mathematics is largely tacit, but critical elements of this knowledge can be questioned in groups” (Davis and Renert, 2014, p. 33).

It is also worth highlighting Leikin's (2009) contributions regarding the notion of relative and absolute creativity. When dealing with Creativity in Mathematics, through the lens of multiple resolution tasks, the author presents some notes based on previous research:

> Naturally, Creativity in school Mathematics differs from that of professional mathematicians [...] any definition of creativity is relativistic. Seeing personal creativity as a characteristic that can be developed in schoolchildren requires a distinction between relative and absolute creativity [...]. Absolute creativity is associated with "great historical works" [...], with discoveries on a global level [...] Relative creativity refers to the discoveries of a specific person within a specific reference group (Leikin, 2009, p. 131).

These contributions, in line with Mathematics for Teaching, provide reflections on adopting a relativistic stance towards established Mathematics (objectified Mathematics and curricular content) and produced Mathematics (collective interpretation and subjective understanding), based on relative rather than absolute points of view.

In addition to the references relating to teacher education and knowledge, there are also those that support the treatment of the concept of area. By considering area as a geometric quantity, we draw on the contributions of Douady and Perrin-Glorian (1989), Baltar (1996) and Bellemain and Lima (2002).

In relation to the concept of area, Douady and Perrin-Glorian (1989) carried out research with French students and observed that they solved tasks relating to the concept of area based on two types of conceptions: *geometric*, when students considered area as a surface, or *numerical*, with an exclusive focus on calculation. Some students even approached both conceptions independently.

According to Douady and Perrin-Glorian (1989), these approaches can result in incoherent understandings. This is because, when students only consider the geometric conception, they may understand that changing the shape of a figure implies changing its area. On the other hand, when they adopt only the numerical conception, they may make incorrect extensions of formulas and may also have difficulties understanding the units of measurement and even omit them.

Given this scenario, the researchers proposed an approach that considers area as a quantity, highlighting the need to differentiate the concepts of area and surface and area and number, structuring frameworks that articulate and distinguish these notions.

> Geometric frame: made up of flat surfaces. Numerical framework: consisting of the measurements of the area of the surfaces, which belong to the set of non-negative real numbers. Framework of magnitudes: a context specific to the notion of area, which integrates the first two and is formally characterized as equivalence classes of surfaces of the same area [...] functional algebraic framework to which formulas expressing area as a function of lengths relative to geometric figures belong (Bellemain and Lima, 2002, p. 21-28).

In addition to the frameworks described, to structure the teacher education proposal, we considered the situations that give meaning to the concept of area: comparison, measurement, production (Baltar, 1996) and unit conversion (Ferreira, 2010, 2018), as we understand that these, as well as the frameworks (geometric, numerical, of quantities and functional algebraic), allow for the contemplation of various aspects related to the concept of area, both in the context of training and in the classroom.

3 Methodology

This paper presents an action developed as part of a continuing education course, based on a proposal for teacher education which corresponds to an essential part of ongoing doctoral research. As this study is linked to a professional postgraduate program, it is expected to produce a thesis and an educational process/product, which in this case is the teacher education proposal.

In short, this educational process/product — the teacher education proposal — aims to provide participants with the reorganization of Mathematics for teaching the concept of area. To this end, the proposal is made up of Educational Situations (ES), understood as strategies that mobilize discussions, interactions and reflections on the concept of area for teaching, according to the theoretical-methodological frameworks adopted. The ES are implemented on the basis of demands identified in the training, so that the participants' knowledge and meanings relating to the concept of area for teaching are incorporated, relativized and interpellated as educational content.

For the methodological processes, aspects of Design Research were adopted which, as Plomp (2009) puts it, is understood as:

> the systematic study of the design, development and evaluation of educational interventions — such as programs, teaching and learning strategies and

> materials, products and systems — as solutions to identified problems, which aim to advance our knowledge of the characteristics of these interventions and processes for the design and development of solutions (p. 9).

Development research consists of identifying a demand, followed by carrying out an investigation that is structured in periodic cycles. These cycles cover the design, development and evaluation of the resulting product. This process, based on theory and its own development, is constantly refined with each cycle. In line with these theoretical understandings, this methodological alignment was considered during the design, development and evaluation of the Teacher Education Proposal, which is the subject of this thesis.

Therefore, as Barbosa and Oliveira (2015) describe, the educational process/product "goes through the process of analysis and refinement so that, at the end of the investigation, it can be used by other people in other contexts" (Barbosa and Oliveira, 2015, p. 530). Through this research, we intend to contribute to and support the practices of other teachers, trainers and researchers interested in learning about and implementing teacher training actions and pedagogical practices based on the perspectives of Mathematics for Teaching and Problematized Mathematics and/or relating to the concept of area.

Therefore, Development Research aims, from the identification of a problem — in this case, the demand for training focused on investigating the concept of area for teaching — to generate an intervention to be materialized through an educational process/product. In this research, it corresponds to the proposal for teacher education, based on the theoretical and methodological foundations presented.

When discussing the adoption of Developmental Research, Plomp (2009) points out that, although there may be differences in the details of its presentation, there is consensus on the composition of its phases, namely:

> a) preliminary research: needs and context analysis, literature review, development of a conceptual or theoretical framework for the study; b) development or prototypical phase: iterative design phase consisting of iterations, each a research micro-cycle, with formative evaluation as the most important research activity focused on improving and refining the intervention; c) improvement phase: (semi)summative evaluation to determine whether the solution or intervention complies with predetermined specifications. As this phase also often results in recommendations for improving the intervention, we can call it the semi-summative phase (Plomp, 2009, p. 34).

With these guidelines, the *initial phase* of the research began with the design of the educational process/product. In 2021, the first author devised a proposal for initial teacher education in the form of a research project and submitted it to the selection process of the Ifes Educimat Program. After taking part in the selection and entry process, as well as verifying the viability of the diversity of the participating public and the relevance of continuing studies aimed at investigating the concept of area for teaching — based on a review of theses, dissertations and published articles — the first version was reformulated, beginning the prototypical phase.

The *prototypical phase* included the examination and implementation stages. The instances of examination corresponded to the boards to which the research was submitted and argued — selection process boards, simulated Gepem-ES boards, internal seminar boards, qualification boards and the defense board scheduled for the second half of 2025. The implementation instances covered the education contexts, divided into preliminary, pilot and main. The preliminaries, aimed at teacher education actions, took place through two invitations and one submission to a scientific event. The pilot, consisting of three teacher education workshops, and the main education context, structured as a teacher education course, were extension activities submitted to and approved by the Human Research Ethics Committee under No. 70981423.0.0000.5072, and institutionalized by the IFES Extension Coordination, Vitória campus, to which the research is linked.

The notes of the members of these panels were obtained through transcriptions of the audiovisual material and written records sent via e-mail. The data from the implementation was produced through: (1) *questionnaires* — Google forms, academic and professional profile of the participating group and feedback on the evaluation of the educational process/product; (2) *observations* — audio and video recordings; (3) *education situations* — records of the participating teachers and trainees; and (4) *observations* — notes from the members of the research and education team.

The evaluation phase is being undertaken by reviewing the notes obtained from the examining boards, which made it possible to rectify and ratify the educational process/product. In addition, the analysis of the data produced through the preliminary, pilot and main studies makes it possible to verify potentialities and weaknesses, supporting the necessary refinements for the validation and materialization of the final version of the educational process/product.

In this way, the validation process has been carried out throughout these phases, with revised versions being produced with a view to ascertaining design principles, i.e. guidelines that appear to be promising for favoring the reorganization of Mathematics for teaching the concept of area to teachers and trainees in education.

Thus, the forum portrayed in this production results from the implementation of the fourth version of the educational process/product, considered to organize the teacher education course (main study). This scenario allowed for the emergence and incorporation of Creativity in Mathematics, as will be discussed below.

The teacher education course, the main study, was organized into 65 hours of face-to-face actions — five meetings and three directed studies — and 15 hours of non-face-to-face actions — three web conferences and three asynchronous interactions, carried out in the Virtual Learning Environment (VLE). Throughout the course, 16 participants took part, including Mathematics undergraduates, Pedagogy undergraduates and teachers who teach Mathematics in Basic Education.

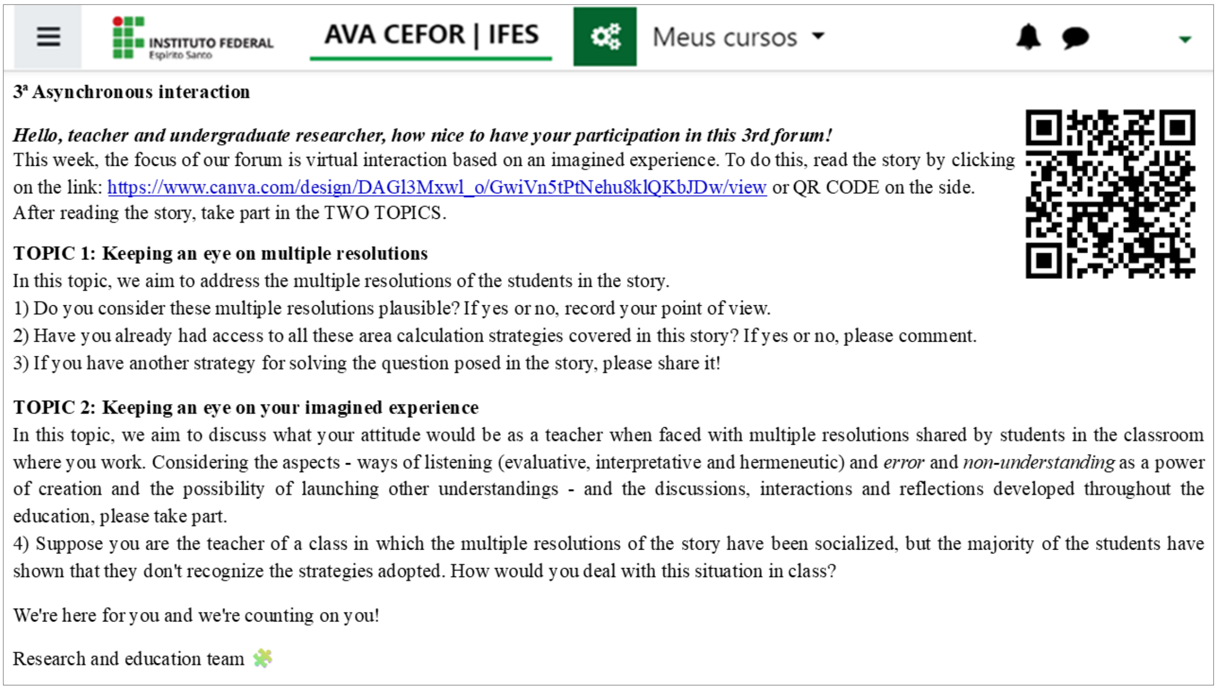
The course began on April 6, 2024 and, at that time, two of the authors attended the *live event The LEM of IFES campus Cachoeiro de Itapemirim invites you to a conversation about: some ideas about Creativity in Mathematics class*, whose speakers were Ana Barbosa and Isabel Vale, with mediation by Jorge Henrique Gualandi. Through this moment of self-education, there was an opportunity to expand knowledge on the subject, identify references for study and also the proposal to consider Creativity in Mathematics in the educational process/product.

By reading the articles, we observed similarities between Creativity in Mathematics and the perspectives of Mathematics for Teaching and Problematized Mathematics, presented in the section on theoretical and methodological contributions. In addition, in dialogue with members of the research and education team, some SFs were formulated and others reformulated to be in line with the dimensions associated with Creativity in Mathematics and to contribute to the implementation of the course and the training of the participants.

More specifically, the data units presented in this article come from posts made by participants in the virtual forum corresponding to the 3rd asynchronous interaction of the teacher education course. This interaction took place through the SF virtual forum, which was based on the story *Imagined experience: the concept of area straight from the classroom*, the aim of which was to provide interactions on the themes of multiple resolutions, Mathematics for Teaching and Problematized Mathematics.

4 Keeping an eye on interactions in the virtual forum

This section presents and analyzes posts made by teachers and trainees, which include their points of view in relation to the multiple resolutions of the story, the production of other resolutions and the description of how the participants and teachers would deal with the multiple resolutions in their teaching practices. Figure 1 shows the direction of the virtual forum and Figure 2 shows the story *Imagined experience: the concept of area straight from the classroom*.



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3ª Asynchronous interaction

Hello, teacher and undergraduate researcher, how nice to have your participation in this 3rd forum!
This week, the focus of our forum is virtual interaction based on an imagined experience. To do this, read the story by clicking on the link: https://www.canva.com/design/DAGl3Mxwl_o/GwiVn5tPtNehu8klQKbJDw/view or QR CODE on the side.
After reading the story, take part in the TWO TOPICS.

TOPIC 1: Keeping an eye on multiple resolutions
In this topic, we aim to address the multiple resolutions of the students in the story.
1) Do you consider these multiple resolutions plausible? If yes or no, record your point of view.
2) Have you already had access to all these area calculation strategies covered in this story? If yes or no, please comment.
3) If you have another strategy for solving the question posed in the story, please share it!

TOPIC 2: Keeping an eye on your imagined experience
In this topic, we aim to discuss what your attitude would be as a teacher when faced with multiple resolutions shared by students in the classroom where you work. Considering the aspects - ways of listening (evaluative, interpretative and hermeneutic) and *error* and *non-understanding* as a power of creation and the possibility of launching other understandings - and the discussions, interactions and reflections developed throughout the education, please take part.
4) Suppose you are the teacher of a class in which the multiple resolutions of the story have been socialized, but the majority of the students have shown that they don't recognize the strategies adopted. How would you deal with this situation in class?

We're here for you and we're counting on you!

Research and education team

Figure 1: Direction of the virtual forum (Research data, 2024)

As explained above, the direction of the 3rd *Asynchronous Interaction* was organized into two topics and four guiding questions, to which 41 posts were submitted.

For question *1) Do you consider these multiple resolutions to be plausible? If yes or no, register your point of view*, It was found that 11 of the 13 participants who posted said they considered the resolutions to be plausible. The two participants who said they didn't, said that because they didn't know some of the strategies presented, it wouldn't be possible to make such a statement.

> Participant BC: *Now, the solutions of counting dots, the Cartesian plane and solving using the integral, I had no idea that I could use these strategies to solve area questions, so I don't know if all the resolutions are plausible.*

In relation to this first question, regarding the potential and fragility of this SF of the educational process/product, it was possible to identify that there was a double meaning attributed to the term *plausible*. When analyzing the posts, it was observed that some participants presented their answers considering *plausible* as a synonym for a resolution that was solved coherently in the sense of *being correct*, as was the case with the two participants mentioned above. However, it was also possible to see that the term was interpreted by other

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participants as an opportunity to position themselves as to whether or not the resolution was *coherent* in the sense of being considered for Basic Education. Therefore, for the materialization of the educational process/product, we consider reformulating this question in order to explicitly contemplate the two meanings mobilized by the training participants.

Figure 2: Story (Research data, 2024)

For the second question 2) *Have you already accessed all these area calculation strategies covered in this story? If yes or no, please comment*, there were responses that can be organized into three situations: there was knowledge of all the resolutions; not all the resolutions were known to the participants; and/or they were resolutions that had not previously been thought of as being part of the story. Below is a unit of data from each of these situations:

> Participant EC: *All resolutions are plausible and, in a way, arrive at the same number (some with different units of measurement). I was able to see all these strategies beforehand in my experiments.*
>
> Participant AD: *They're all plausible, but I didn't know about the 2nd resolution using the dots and the 4th question in which the student uses the Cartesian plane.*
>
> Participant SD: *I think these resolutions are quite different , I confess that not all of them I had thought or could solve those forms.*

In the story, the aim was to align Creativity in Mathematics and, in this second question, it was understood that the *flexibility* dimension was evidenced by mobilizing interaction on the "variety of different ideas about the same problem or situation" (Vale and Barbosa, 2015, p. 4), thus meeting the intentionality.

Then, for the third question *3) If you have another strategy for solving the question proposed in the story, share it!*, there were posts involving three new solutions. One possibility presented was covering and counting the area units. When showing this strategy, one of the entries was as follows:

> Participant AD: *When I started the content on the area of flat figures, I only used formulas, because I learned that way and passed it on to my students, but after participating in the first meetings on the concept of area, in the year 2023 [...], I was already interested in working with manipulable materials which, in addition to making knowledge more interesting, makes it enjoyable and we reach the largest number of students. I could then use the form we learned here on the course, using superposition.*

In this post, it can be seen that Participant AD reports that she experienced the formation of the pilot study, which took place in 2023. Later, in 2024, when the course was offered, she and two other participants decided to join the main study. This post, as well as the participants' interest in experiencing this other training, indicates the contributions of education contexts developed from the proposal of teacher education based on the perspective of Mathematics for Teaching and Problematized Mathematics.

Another resolution was sent in by Participant LR, a strategy known as the *Brahmagupta Formula*. In presenting her contribution, this participant continued the story. To do this, she produced an imagined experience between the teacher and a student via WhatsApp, as illustrated in Figure 3.

Sending this other strategy, which was not present in the story, gave the participants access to a new resolution. It also mobilized the beginning of the interaction about the importance of multiple resolutions.

> Participant KR: *Another 1000-score student [...]. That's an interesting formula, I don't remember if I've ever seen it myself.*
>
> Participant RB: *I didn't know many of these resolutions and I think it's important to present the student with different possibilities.*

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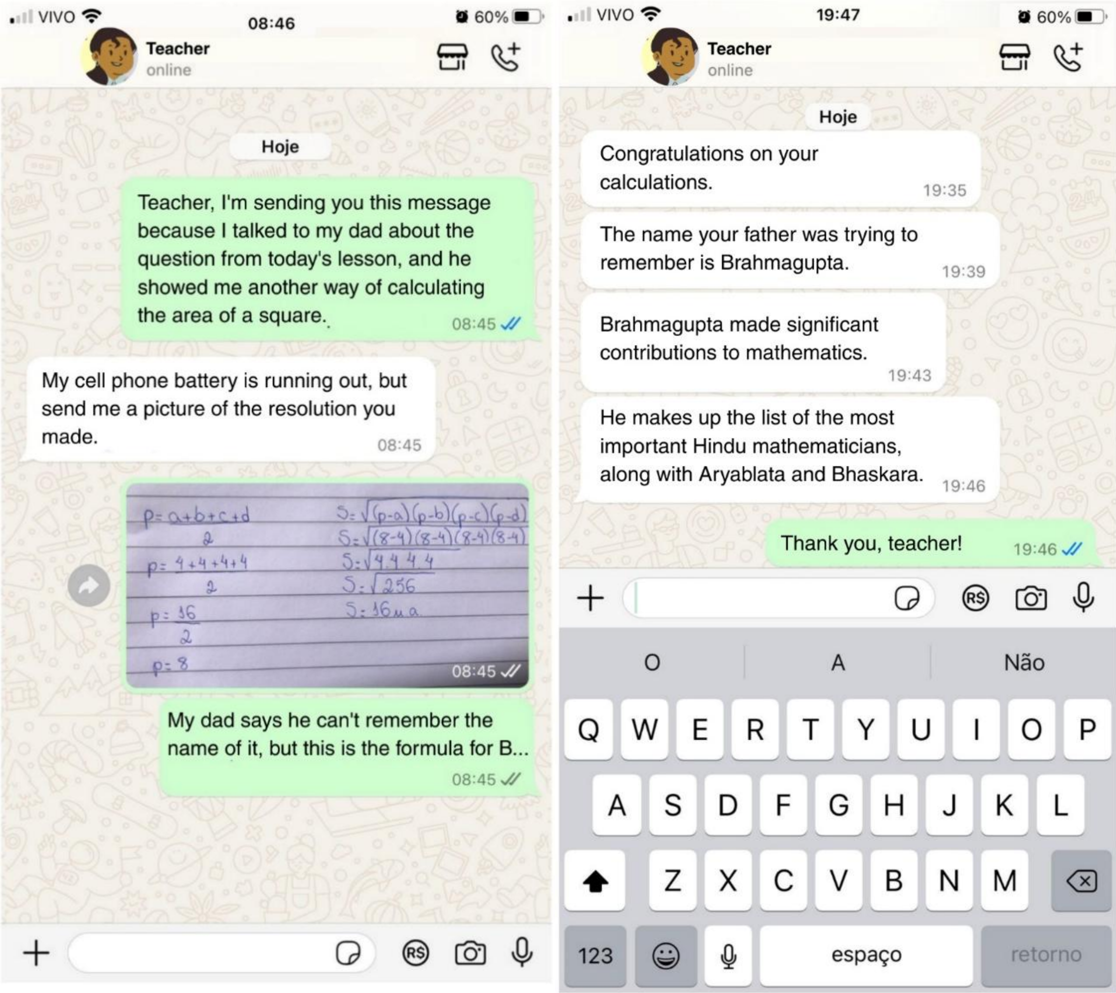

Figure 3: Participant's LR post (Research data, 2024)

In relation to the dimensions associated with Creativity in Mathematics, it was considered that the strategy known as *Brahmagupta's Formula*, presented by Participant LR, showed *originality*, since, by proposing a new resolution in relation to the story, she demonstrated the ability to "create new ideas or products" (Vale and Barbosa, 2015, p. 4). Through this contribution, it was possible to expand the set of strategies shared by the group, as well as validating the SF proposal to mobilize other resolutions.

Next, Participant RC described a practice in which it would be possible to treat the area of a triangle and a square in an articulated way.

> Participant RC: *By transforming the square into two equal triangles, we obtain the area of each triangle and then add the two areas together to find the area of the square. I believe this strategy can also help explain to students why the formula for the area of the triangle is divided by 2.*

In the context of Creativity in Mathematics, Participant RC's contribution can be related to the *fluency* dimension, in relation to the continuity of ideas and the "flow of associations" (Vale and Barbosa, 2015, p. 4), given that the participant shared the possibility of connecting the study of the area of a square with the area of a triangle. It should be noted that this movement of establishing associations was dealt with more specifically in the 1st Asynchronous Interaction of the course, when there was guidance, via the virtual forum, regarding the articulation of the concept of area with other mathematical topics.

This third question acted as an invitation to look for other solutions. Considering relative creativity, which "refers to the discoveries of a specific person within a specific reference group, to the human imagination that creates something new" (Leikin, 2009, p. 131), these participations showed the manifestation of this creative dimension, evidenced in the proposal of a new solution to the problem, bringing new perspectives to the group.

The last question *4) Suppose you were the teacher of a class in which the multiple resolutions of the story had been socialized, but most of the students in the class didn't recognize the strategies adopted. How would you deal with this situation in the classroom?*, generated posts that can be organized into two approaches. In the first, the teacher adopts a position aligned with hermeneutic listening, leading the discussion of each resolution collectively. In the second, students are directed to form groups, with each group investigating a specific resolution and then sharing their conclusions with the whole class.

Below are posts that mention the potential and challenges of pedagogical practice when working with multiple resolutions, especially in contexts in which not all the resolutions produced were recognized by the students.

> Participant AD: *If this situation happened in my class, I would try to split up groups [...] I know I would get a lot of resistance from some students, but it would be a way of demonstrating that mathematics has several ways of solving a given problem.*
>
> Participant MS: *[...] Nowadays, students want the quickest solution and one that doesn't require a lot of reasoning, but I believe that showing ways is very important for the growth of these students' knowledge.*

In terms of challenges, the posts by participants AD and MS, when dealing with how students have dealt with the tasks — more specifically in the passages *"I would have a lot of resistance"* and *"the student wants the quickest solution and the one that doesn't require much reasoning over the question"* — show students' points of view that run counter to the perspective of Problematized Mathematics.

However, when they present their points of view on multiple solutions, participants AD and MS say: *"It would be a way of demonstrating that Mathematics has several ways of solving a given problem and I believe that showing ways is super important for the growth of these students' knowledge"*. In these excerpts, it can be seen that, unlike the students, the teachers recognize the potential of working with multiple solutions and show alignment with the principles of problematized Mathematics.

Currently, in the Metropolitan Region of Greater Vitória, in the state of Espírito Santo, where the schools where the training participants are teaching are located, students generally have between three and five hours a week dedicated to math classes. However, it is common for other activities and events to take place during class time, including math classes. This situation leads to a challenge related to the workload of the Mathematics subject and presented by the participants.

> Participant BC: *I don't think we'd have time to work on all the strategies presented by the students in our day-to-day classes, but I would try to reflect on each strategy presented in the next few classes, working on the fundamentals of each one until we reached the concept of area. I would also try to use a simpler language that is more common to the students. Just as I did here on the course and as I did with my students in a class at Meet.*

Still on the fourth question, it was pointed out that the following aspects should be considered in the interaction about the participants' stance on the multiple resolutions: listening modes (evaluative, interpretative and hermeneutic) and *error* and *non-understanding* as a power of creation and the possibility of launching other understandings.

With regard to the participants' position on the students' mathematical productions, two data units are presented below to discuss the aspects of error and *non-understanding*. Interactions about ways of listening will follow.

According to Giraldo and Roque (2021, p. 17), in the teaching of Mathematics, both in Basic Education and in Higher Education, *"error and non-understanding are commonly seen as undesirable deficiencies in students, which should be corrected or penalized by teaching"*. This scenario can be seen in the posts highlighted below. Due to their content, we have chosen not to identify the participants, avoiding any association of these records with previous contributions.

> Participant 1: *But if a different resolution comes up and if it's right, I'd think it's great and I'd be super supportive, I'd ask you where that resolution came from, how you learned it.*
>
> Participant 2: *In my practice, I often probe the students on their solutions. I check the results and analyze the processes [...] When I get the students themselves to explain their idea, and I confirm it as a valid resolution for that episode, they perceive themselves as "students" [...] and feel included [...].*
>
> Participant 3: *I think I could show the application of the square formula $A=l^2$, but even so, I know it would still be abstract for some students. By removing one part of the doubt, I would move on to a more tangible idea for the select group who still don't understand. The drawing on the grid is a good example, so I would bring it up in a way that the student could understand.*

The excerpts *"and if it was right, I'd think it was great and I'd be super supportive..."*, *"I check the results and analyze the processes [...] and I confirm it as a valid resolution for that episode"* and *"I know it would still be abstract for some students [...] I would move on to a more tangible idea for the select group who still don't understand"* are indications of points of view relating to the order of the structure, in which mathematical knowledge is characterized by *"the perfection of the structure and the correctness of the results"* (Giraldo and Roque, 2021, p. 2).

Throughout the education, we tried to provoke tension in relation to points of view that alluded to the order of structure, in other words, non-problematized Mathematics. However, considering that a diversity of perspectives is essential for the reorganization of Mathematics for the teaching of teachers and trainees in education, it was inferred that it was not feasible for members of the research and education team to directly question these points of view in the AVA.

When faced with situations such as those highlighted above, which required the need to challenge points of view with more specific interventions and/or challenges, these dialogues were carried out using directed study scripts and a guidance WhatsApp group. These contexts involved the participation of a teacher or graduate student, together with two members of the research and education team. Subsequently, situations considered relevant to be addressed with the other participants were taken up again, without identification, in face-to-face meetings and web conferences.

Dn view of these occurrences, a reformulation of the educational process/product was proposed, including a specific question that would encourage participants to interact and reflect on the contributions of considering error as a power for creation and *non-understanding* as a possibility for new meanings.

With regard to listening modes and how they could be considered when exploring multiple resolutions, two points of view were identified: one emphasizing hermeneutic listening and the other adopting complementarity between listening modes. Two posts are highlighted below to analyze these points of view.

> Participant SB: *Dealing with the situation where most students don't recognize the strategies adopted in the different resolutions of the story can be challenging, but it can be approached in a constructive and educational way [...] I would use more hermeneutic listening, putting these different resolutions to be explained and presented to all students, where you can include a guided discussion, where you go through each strategy, provide additional and practical examples and highlight the key points that differentiate each approach.*
>
> Participant LR: *Faced with the different strategies adopted by the students, I could use evaluative listening, providing immediate feedback for those answers [...] as in the resolutions using the formula* $A = l^2$*. I could use interpretive listening for those solutions where I didn't know the strategy, such as Pick's Theorem, enabling the student to explain how they came up with their answer. And to use hermeneutic listening, encouraging a variety of solutions to the question, even agreeing to receive resolutions after the end of the lesson. I believe that by practicing these ways of listening, the school environment would become more inclusive, encouraging students to express their ideas and doubts.*

Based on the description of how the students' mathematical productions would be considered, it was inferred that, for the participants, multiple resolutions are being taken as an integral part of knowledge and not just as a means to allow students to appropriate formal Mathematics. These attitudes are in line with the perspective of Mathematics for Teaching and Problematized Mathematics.

The last question therefore led to interactions related to how listening modes can guide teaching practices. It also made it possible, in research terms, to identify how the participants have dealt with the Mathematics produced by the students.

5 Final considerations

This text portrays interactions between teachers and undergraduate students that took place in a virtual forum, mobilized from the story *Imagined experience: the concept of area straight from the classroom*. These interactions aimed to address multiple resolutions (Leikin, 2009) and the aspects of listening modes (Davis and Renert, 2014) and shifts in the meanings of *error* and *non-understanding* (Giraldo and Roque, 2021).

Analyzing the results of these interactions revealed both potential and weaknesses in the proposal for teacher education focused on investigating the concept of area for teaching. This proposal, based on the perspective of Mathematics for Teaching and Problematized Mathematics, was an educational process/product of ongoing doctoral research.

In the prototypical phase, inherent to Development Research (Plomp, 2009), it was identified that Creativity in Mathematics would be the relevant articulation to be considered in the formulation and reformulation of SF, with a view to favoring the reorganization of Mathematics for teaching the concept of area for undergraduate students and teachers in training.

The aim of this text is to illustrate that the reformulation of the SF virtual forum: 3*rd Asynchronous Interaction aligned with Creativity in Mathematics* contributed to mobilizing interactions and reflections on how to deal with students' mathematical productions and enabled teachers and trainees to learn about other area calculation strategies to explore when teaching the concept of area.

In this way, the doctoral research proposal, to which the interaction portrayed is associated, seeks to contribute to studies in the field of teacher education. This objective is achieved through a proposal for teacher education that enables the reorganization of Mathematics for teaching the concept of area to teachers and trainees in education.

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