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Educação Matemática Debate

Problem Solving as a creative means: a study on the lake area of Ramiro Ruediger Park in Blumenau/SC

Abstract: This article aims to analyze the implications of the Teaching-Learning-Assessment methodology in Mathematics through Problem Solving to apply the trapezoidal method, developed by students based on their prior knowledge, in a context involving the calculation of the area of the lake in Ramiro Ruediger Park, in Blumenau/SC. To this end, the Problem Solving approach and its connection to creativity are discussed. This article presents an applied, qualitative, and action-research study, detailing the context, the structure of the practice, and the analysis of the investigated problem. The results indicate that this approach contributed to the development of students' autonomy, reasoning, argumentation, and creativity during the educational practice, advancing the understanding of mathematical concepts, with problem solving established as a pathway and guidance for learning.

Keywords: Creativity. Problem Solving. Trapezoidal Method. Higher Education.

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Article

La Resolución de Problemas como medio creativo: un estudio sobre el área del lago del Parque Ramiro Ruediger en Blumenau/SC

Resumen: Este artículo tiene como objetivo analizar las implicaciones de la metodología de Enseñanza-Aprendizaje-Evaluación en Matemáticas a través de la Resolución de Problemas para aplicar el método de los trapecios, desarrollado por los estudiantes a partir de sus conocimientos previos, en un contexto que involucra el cálculo del área del lago del Parque Ramiro Ruediger, en Blumenau/SC. Para ello, se discute el enfoque de la Resolución de Problemas y su conexión con la creatividad. Este artículo presenta una investigación aplicada, cualitativa y de investigación-acción, detallando el contexto, la estructura de la práctica y el análisis del problema investigado. Los resultados indican que este enfoque contribuyó al desarrollo de la autonomía, el razonamiento, la argumentación y la creatividad de los estudiantes durante la práctica educativa, avanzando en la comprensión de conceptos matemáticos, con la resolución de problemas constituida como un camino y orientación para el aprendizaje.

Palabras clave: Creatividad. Resolución de Problemas. Método Trapezoidal. Educación Superior.

A Resolução de Problemas como meio criativo: um estudo sobre o cálculo da medida da área do lago do Parque Ramiro Ruediger em Blumenau/SC

Resumo: Este artigo tem como objetivo analisar implicações da metodologia de Ensino-Aprendizagem-Avaliação em Matemática através da Resolução de Problemas para aplicar o método dos trapézios, desenvolvido pelos estudantes a partir de seus conhecimentos prévios, em um contexto que envolve o cálculo da medida da área do lago do Parque Ramiro Ruediger, em Blumenau/SC. Para tanto, discute-se a abordagem da Resolução de Problemas e sua conexão com a criatividade. Este artigo apresenta uma pesquisa aplicada, qualitativa e de investigação-ação, detalhando o contexto, a estrutura da prática, e a análise do problema investigado. Os resultados indicam que essa abordagem contribuiu para o desenvolvimento da autonomia, do raciocínio, da argumentação e da criatividade dos estudantes durante a prática

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educativa, avançando na compreensão de conceitos matemáticos sendo a resolução de problemas constituída como um caminho e orientação para a aprendizagem.

Palavras-chave: Criatividade. Resolução de Problemas. Método dos Trapézios. Ensino Superior.

1 Introduction

Mathematics Education seeks to contribute significantly to the education of citizens, promoting an approach that values the development of essential abilities, such as the capacity for abstraction, critical thinking and creativity. Basic Education curriculum guidelines, such as the Base Nacional Comum Curricular [National Common Curriculum Base — BNCC], highlight the importance of methodologies that prioritize the construction of strategies, justification and argumentation of results, in addition to stimulating creativity, teamwork, personal initiative and autonomy, which are the result of confidence in one's own abilities (Brasil, 2018).

In the context of Higher Education, the *Diretrizes Curriculares Nacionais do Curso de Graduação em Engenharia* [National Curricular Guidelines for the Undergraduate Course in Engineering], for example, have guided that, in the training of engineers, the student has an active role in the construction of their knowledge, emphasizing the value of inter and transdisciplinarity, as well as indicating the role of the teacher as an agent driving the necessary changes, inside and outside the classroom (Brasil, 2019). For the Royal Academy of Engineering (2007, p. 4), “it is this combination of understanding and abilities that underpins the role that engineers now play in the business world”.

However, in many classrooms, teachers are ability limited to transmitting content in a traditional way, while students remain in a passive role, struggling to assimilate what is taught to them. This dynamic often results in superficial, meaningless learning, harming not only the school and academic trajectory of students, but also their future careers (Possamai et al., 2021).

Vygotsky (1987) already warned that direct teaching of concepts and procedures can be ineffective for the construction of knowledge, resulting in an empty repetition of words by students, similar to the behavior of a parrot that imitates without really understanding.

In view of this, recent literature that addresses the teaching of Mathematics in different stages and educational contexts has been concerned with rethinking the processes of teaching, learning and assessment, presenting new possibilities that encourage educators to innovate their practices. The proposal is that students become protagonists in the construction of their own knowledge, developing the ability to think of multiple alternatives for solving problems, which fosters creativity (Van de Walle, 2009; Allevato and Onuchic, 2021; Vieira, Possamai and Allevato, 2023).

The *Base Nacional Comum Curricular* (Brasil, 2018) reinforces the need for the ideas presented in classes to be fundamental to the development of mathematical thinking. The document emphasizes that mathematical learning must be intrinsically related to understanding, that is, to the apprehension of the meanings of mathematical concepts. In this sense, Problem Solving is presented as an essential ability to be developed throughout the teaching of various mathematical contents, in all stages of Basic Education.

This guideline is strongly aligned with the Methodology of Teaching-Learning-Assessment of Mathematics through Problem Solving, proposed by Allevato and Onuchic (2021). According to the authors, this methodology offers a way to teach, learn and assess mathematical contents, using the problem as a starting point and guide for the construction of new concepts and procedures, always based on the students' prior knowledge.

Within the scope of Problem Solving, it is worth noting that,

> the field of mathematics education has focused on problem solving for over 50 years. During this time, much research has been conducted and much has been written on the topic, resulting in a shared belief that problem solving is, and should be, an important part of teaching and learning mathematics. Indeed, during this time, problem solving has become incorporated into curricula around the world, both as a skill to be taught and as a means by which mathematics is learned. However, problem solving still poses great difficulty for students of all ages. Thus, the work continues (Liljedahl and Cai, 2021, p. 1).

In this context, a set of problems was developed with higher education students, specifically from the Food, Civil, Electrical, Mechanical, Production and Chemical Engineering courses at a Community University in Blumenau (SC, Brazil), who were taking the Numerical Calculus course. The objective of problem 1 was to recover the students' prior knowledge about the concept and calculation of the area measurement of regular figures, in addition to their relationships for the generalization of the formula for measuring the area of trapezoids. Problem 2 sought, based on questions involving irregular regions, to provoke the generalization of the trapezoid method, which was later applied in problem 3, the focus of analysis of this article.

In this context, this work aims to analyze the implications of the Teaching-Learning-Assessment Methodology in Mathematics through Problem Solving, when using the trapezoid method, developed by the students based on their prior knowledge, in a context that involves calculating the area measurement of the lake in Ramiro Ruediger Park, in Blumenau. It is worth noting that the educational practice was carried out remotely due to the Covid-19 pandemic.

To this end, the relationship between Creativity and Problem Solving is explored below, highlighting how these approaches can complement each other to promote more meaningful learning.

2 Problem Solving and Creativity: a path to active learning

Problem Solving and Mathematical Creativity are interconnected pillars that strengthen Mathematics Education. Researchers such as Allevato and Onuchic (2021) emphasize that Problem Solving should be the starting point for the construction of new concepts, promoting mathematical understanding. The Mathematics Teaching-Learning-Assessment Methodology through Problem Solving integrates teaching, learning and assessment, transforming problem solving into a meaningful means of learning.

The steps of this methodology begin with the posing of a problem, called a generator, with the aim of developing new learning. In the individual reading phase, students reflect on what is asked and what strategies can be used. Then, during the group reading, they discuss their interpretations, allowing different perspectives to emerge. When solving the problem in groups, students apply their prior knowledge and explore new strategies, demonstrating creativity by developing different methods.

The teacher acts as a mediator, observing and intervening to guide the discussion, while students record their resolutions and share solutions with the class. The plenary session provides a space for reflection on the approaches presented, leading to the search for consensus. The formalization of the content occurs when the teacher introduces relevant concepts, connecting theory and practice. At the end, new challenges are proposed to consolidate learning (Allevato and Onuchic, 2021).

This approach not only promotes mathematical understanding, but also stimulates creativity, allowing students to explore original solutions. Gontijo (2007) defines creativity as the ability to present multiple possible solutions to a problem, a central aspect of Problem Solving. When working with open-ended problems, for example, which allow for multiple approaches, students are encouraged to think outside the box, valuing originality and flexibility.

In this context, teachers need to create an environment conducive to the expression of creativity, valuing trial and error, and encouraging students to share ideas, even if unusual. This identification of creative talents should be accompanied by activities that resonate with students' interests (Vieira, Possamai and Allevato, 2023).

According to the *Base Nacional Comum Curricular* (Brazil, 2018), the development of abilities goes beyond simply learning content, encompassing the capacity for abstraction, systemic thinking, creativity, curiosity and the ability to formulate multiple alternative solutions. Problem Solving also favors abilities such as teamwork, acceptance of criticism, critical thinking, communication and autonomous search for knowledge. From this perspective,

> In contrast to memorization exercises and mathematical tasks involving the mechanical application of formulas and algorithms, intellectually demanding tasks require the mobilization of more complex cognitive resources by students, thus implying the development of higher-order thinking abilities. In this sense, carrying out this type of task is an auspicious learning opportunity. (Vieira, Possamai and Allevato, 2023, p. 5).

This integration between Problem Solving and the development of creativity represents a promising path for teaching Mathematics, offering a solid foundation that not only facilitates the learning of content, but also prepares students for the future.

The following is a description of the educational practice, research methodology and analysis criteria used to analyze the problem.

3 Description of educational practice, research methodology and analysis criteria

The proposed problems were developed in the first semester of 2020 with morning classes (Food Engineering, Mechanics and Chemistry) and evening classes (Food Engineering, Civil Engineering, Mechanics, Production and Chemistry), following the steps of the Teaching-Learning-Assessment methodology in Mathematics through Problem Solving. At the beginning, the Numerical Calculus teacher introduced the researchers (P1, P2 and P3), who explained the proposal and the steps of the methodology to the students.

After this introduction, the students were organized into groups of 3 to 4 in Microsoft Teams, due to the pandemic, with the support of the researchers, to discuss and solve the problems. This configuration allowed the students to communicate to argue and present their ideas, receiving guidance in the virtual classroom. The researchers also acted as mediators of the discussions and in the construction of the research data.

As for the classification, the research is applied, aiming to generate content that addresses specific problems and their local contexts. It is also classified as qualitative, as it seeks to understand the relationship between the real world and the subjects involved. As stated by Bogdan and Biklen (1994), in qualitative research, the researcher is the main instrument for data collection, using observations and notes in the natural environment.

In addition, according to Kauark, Manhães and Medeiros (2010), the research is descriptive, focusing on the characteristics of phenomena or populations. The data include transcripts, field notes and other records, analyzed in their entirety, respecting the way they

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were constructed.

During problem solving, the focus was on recording and evaluating the students' reasoning and decisions, seeking to understand the meaning they attribute to the content. The inductive approach was central, allowing an analysis of the students' experiences (Bogdan and Biklen, 1994).

The research is also classified as action research, a cyclical process that improves practice through action and investigation. In this approach, researchers and students collaborate, aiming to improve teaching and learning. To this end, a four-step cycle was adopted, which includes planning, implementation, monitoring and evaluation (Tripp, 2005). Data construction was collaborative, involving both students and researchers.

In this context, analysis categories were developed that reflect characteristics relevant to the approach to the content, focusing on the role of the teacher, the problem itself and the engineering students. These categories were divided into three perspectives:

1) Looking at the teacher: mediator, motivator and questioner of the process.

2) Looking at the problem: starting point for learning, generator of concepts and procedures, with emphasis on practical knowledge.

3) Looking at the Engineering students: protagonism, involvement, participation, as well as creativity, reasoning, argumentation and discussion.

These perspectives constituted parameters in the elaboration and resolution of the problem, in addition to serving as criteria for analyzing the educational practice, ensuring that each aspect of the teaching-learning-evaluation process was considered in an integrated manner.

To this end, the data analysis is presented below.

4 Presentation of the research context and data analysis

It is worth remembering that, before the students began solving problem 3, they first solved problem 1, which aimed to recover the students' prior knowledge about the concept and calculation of the area of regular figures, in addition to their relationships for the generalization of the formula that calculates the area of trapezoids. Problem 2 sought, based on questions involving irregular regions, to provoke the generalization of the trapezoid method, which was later applied to problem 3, the focus of analysis in this article. From this, the students had the necessary support to apply their knowledge to problem 3, in which they had to calculate the area of the lake in Ramiro Ruediger Park, located in Blumenau (SC, Brazil).

Therefore, they collected data, distributing the measurement points around the lake and using an image from Google Maps, so that the height between these points was constant, for later use of the trapezoid method. Figure 1 shows the problem given to the students.

The knowledge that was intended for this problem to be constructed by the students was the organization of the data, considering that there is no curve related to a function on a Cartesian plane, given that only the image of the lake, with an indication of scale, was provided to the students. Thus, "this situation presupposes some obstacle that the subject must overcome, either because he needs to obtain new means to reach a solution, or because he must organize the means he already has in a different way." (Echeverría and Pozo, 1998, p. 20). To this end, the students had the time shown in Table 1 to complete the problem.

It should be noted that, in this context, the acronyms E, with a number in sequence, were used to describe the students participating in the research; GM, with a number in sequence, to represent the morning groups; and GN, with a number in sequence, to represent the evening groups.

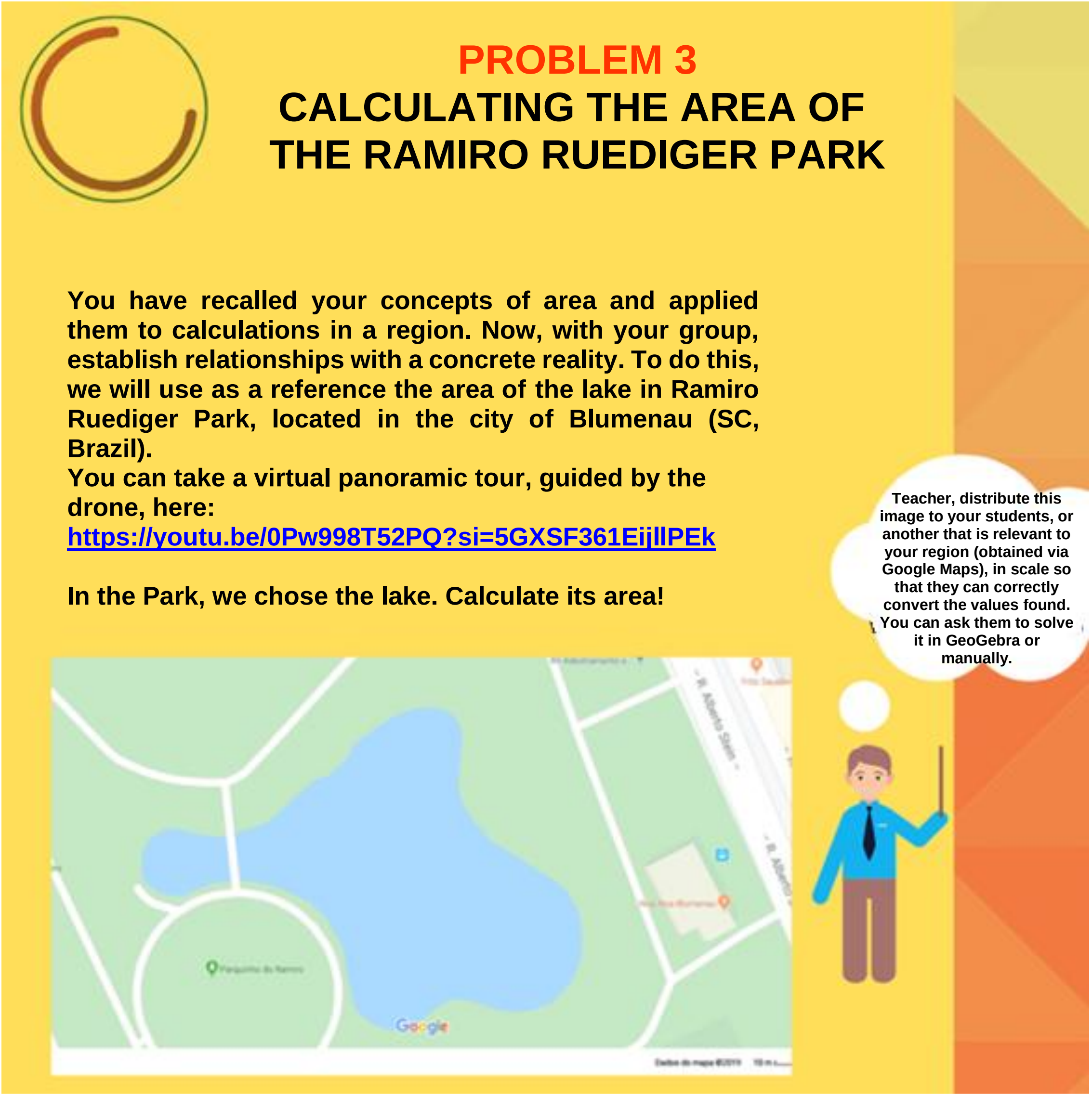

Figure 1: Problem 3 – Calculating the area of Ramiro Ruediger Park (Bertotti and Possamai, 2021, p. 9)

Table 1: Workload allocated to the problem

Activities	Morning and evening classes
Problem resolution	2 hours class
Discussion and formalization	1 hours class

Source: Own elaboration

Next, the problem-solving stage in groups is presented.

4.1 Problem-solving stage

To solve this problem, group GM1 chose to calculate the area of each square and add them together at the end, with the non-integers being *completed* with the other non-integers.

> (E1 speaking): To solve this problem, let's try a different approach and calculate the area of the lake using small squares?
>
> (E2 speaking): Maybe, then we can count how many squares fill the area of the park's lake. See

Figure 2, which shows E1 marking the points to count the squares.

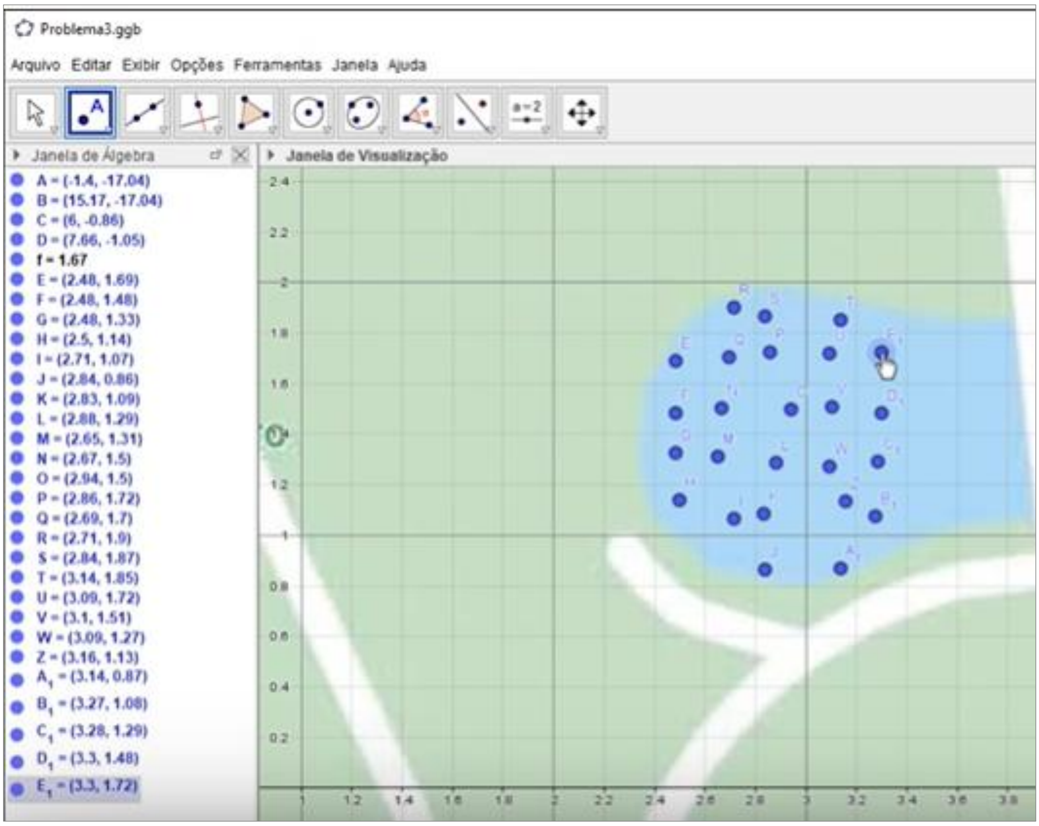

Figure 2: Demarcation of points – GM1 (Research collection)

The students had doubts about the result they found (2,910 m²), because when they searched online, they found that the area of the lake in Ramiro Ruediger Park was often cited as 4,000 m². However, this value includes the area of the deck over the lake. Furthermore, in a conversation parallel to the problem solving, the students commented: *"This type of problem is not at all similar to the ones we solve in class. The problems in class always had some function for us to integrate. In this one, we needed to collect the data and, from there, decide how to calculate the area"*. This comment highlights how the problem-solving experience does not follow a traditional linearity (Allevato, 2005).

Therefore, the ability to approach problems in an unconventional way is fundamental for the development of students' creativity. As stated by Bicer et al. (2020, p. 458): "the time and effort to develop their creativity in general will not be mobilized unless teachers provide experiences that offer cognitively demanding educational opportunities".

Furthermore, although the members of GM1 did not solve problem 3 using the trapezoidal method, they managed to obtain a result close to what was expected, since, by calculating the area of the lake using this method, they arrived at an approximate value of 2,890 m². In this sense, Allevato (2005, p. 89) states that

> there is no linearity in the way mathematical content is approached. Students' ideas follow paths that are particularly different from the linear sequence characteristic of traditional classes. The nodes may be directly connected to each other or by passing through other nodes. Thus, there is no single, determined path that connects them.

With the exception of GM6, which solved the problem using Simpson's method (also a

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numerical integration method), the other groups, GM12, GN1, GN2, GN3 and GN6, used the trapezoidal method to solve the problem. In this sense, the arguments and discussions of GN1 stand out, as they, like the other groups, reached a consensus on a solution to problem 3:

> (E20 speaking): I was thinking about sharing this image, but I don't think it will be necessary.
>
> (E19 speaking): Notice that in this GeoGebra image we already have the dashed lines superimposed over the lake region. So we should start tracing the demarcations of the trapezoids around the lake, right?
>
> (E18 speaking): We also need to see if our interval will be even or odd... so we have to measure the length of the lake.
>
> (E21 speaking): And don't forget that the heights of the trapezoids need to follow the same pattern too.
>
> (E20 speaking): Maybe we could divide their height every two or three squares, what do you think? How many squares are there in this region of the lake, did anyone count? (E19 speaking): Let me see: 5, 10, 15, 20, 25, 30, 35, 36, 37, 38... that's it, there are 38 little squares.
>
> (E20 speaking): 38 is even.
>
> (E21 speaking): That's it, so the height could be every two little squares.

Figure 3 shows the beginning of the demarcation of points around the lake, every 2 squares, carried out by the group.

> [At this point, E19 asks what the blank space indicated in Figure 3 is]
>
> (E21 speaking): It's a bridge! There's a bridge over the lake.
>
> (E19 speaking): But do we consider this part?
>
> (E21 speaking): Yes, since the lake passes under the bridge, we should consider it.
>
> (E19 speaking): Right, true.
>
> (E19 speaking): Someone remembers to write down the scale measurement, that 20.01m is equivalent to 1.68.
>
> (E20 speaking): Is it 1.68cm?
>
> (E19 speaking): This is the measurement that GeoGebra adopts as a standard, I don't think we know if it's cm or mm. But that doesn't matter, because in the end, the values will be converted and this unit 'disappears'.
>
> (E20 speaking): True, it's just a reference for the scale.

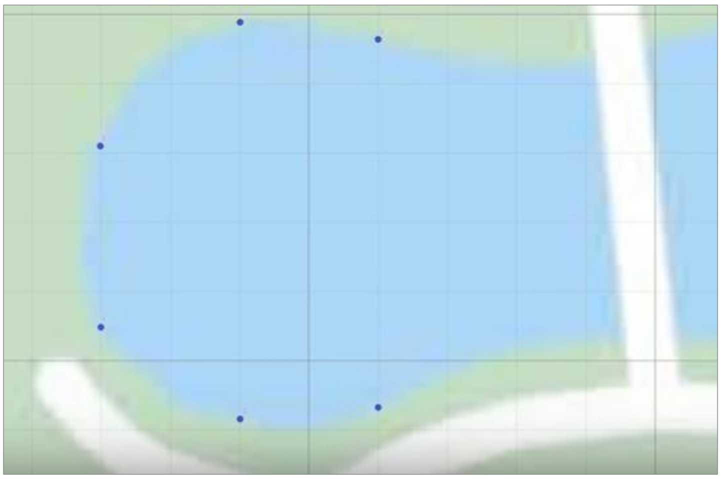
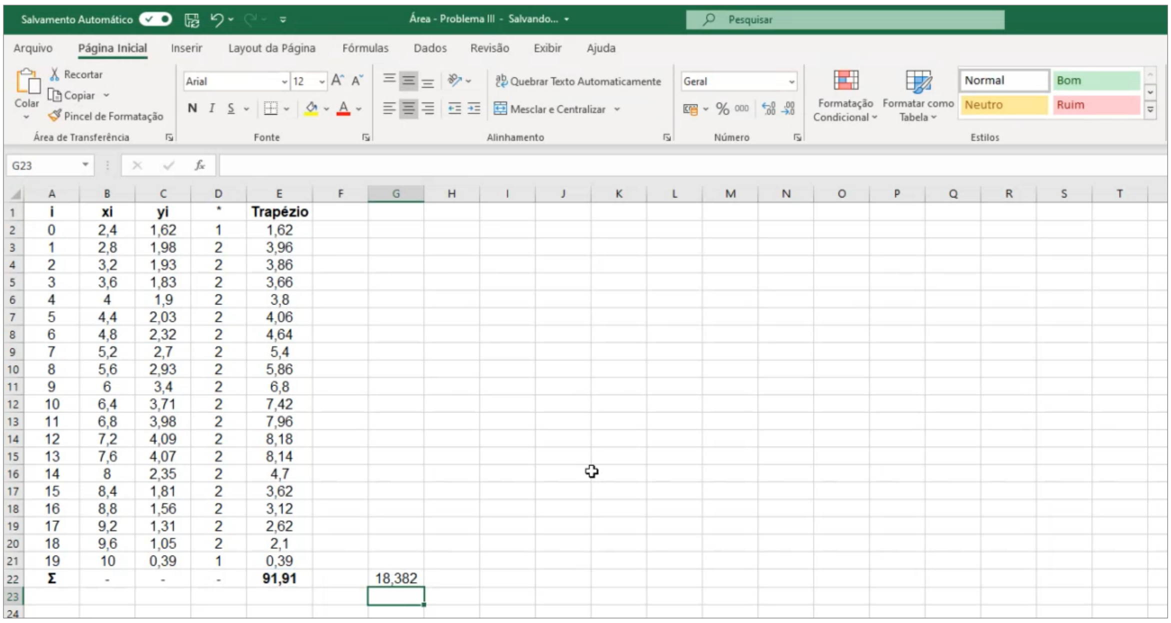


Figure 3: Point demarcation – GN1 (Research collection)

This group, GN1, chose to make a relationship between the abscissa and ordinate axes when collecting data. For example: for any marked point, this corresponds to 2.4 on the abscissa axis for two images (ordinate axis). The students in this group, throughout the resolution, were considering only the value of one image for each value of x, that is, they were disregarding the difference between the images for calculation purposes, as shown in Figure 4.



i	xi	yi	*	Trapézio	F	G
0	2,4	1,62	1	1,62		
1	2,8	1,98	2	3,96		
2	3,2	1,93	2	3,86		
3	3,6	1,83	2	3,66		
4	4	1,9	2	3,8		
5	4,4	2,03	2	4,06		
6	4,8	2,32	2	4,64		
7	5,2	2,7	2	5,4		
8	5,6	2,93	2	5,86		
9	6	3,4	2	6,8		
10	6,4	3,71	2	7,42		
11	6,8	3,98	2	7,96		
12	7,2	4,09	2	8,18		
13	7,6	4,07	2	8,14		
14	8	2,35	2	4,7		
15	8,4	1,81	2	3,62		
16	8,8	1,56	2	3,12		
17	9,2	1,31	2	2,62		
18	9,6	1,05	2	2,1		
19	10	0,39	1	0,39		
Σ	-	-	-	**91,91**		18,382

Figure 4: Data collection – GN1 (Research collection)

However, E20 noticed this interpretation error during data collection.

(E20 speaking): Guys, we need to calculate another table to get the result.

(E21 speaking): What do you mean?

(E20 speaking): Think for a moment about the y values we collected.

(E21 speaking): Oh, sure. We didn't get all the parts of y corresponding to x. Only the value at the top.

(E20 speaking): That's right, we also need to get the other image of x. Then, with that, we make the difference of the values found, otherwise we would be considering the green area of the graph as well, understand?

(E19 speaking): I didn't understand what you meant, could you explain it to me again?

(E20 speaking): Go to the figure so I can explain it better. See that, looking at point 2.4 (abscissa), we only consider the upper value of the point as f(x), that is, we are also including the green area in this calculation. We need to consider with this point 2.4, the value of the lower (ordinate) image, too. Make the difference between the two images so that we only have the value of the lake area.

(E18 speaking): I understand what you are saying. In all calculations in which the green area is present in the demarcation of the points, is that it?

(E19 speaking): Sure, sure, it makes sense, now I understand the reasoning.

Figure 5 represents the basis of the previous dialogue.

In this way, the students solved the problem through correct reasoning that surprised the researchers by relating the x-axis to the y-axis to find the area value. This is because, to solve this problem, it was only necessary to have as data the value of the height measurement (x-axis) and the base (difference between the two points on the y-axis) of each trapezoid.

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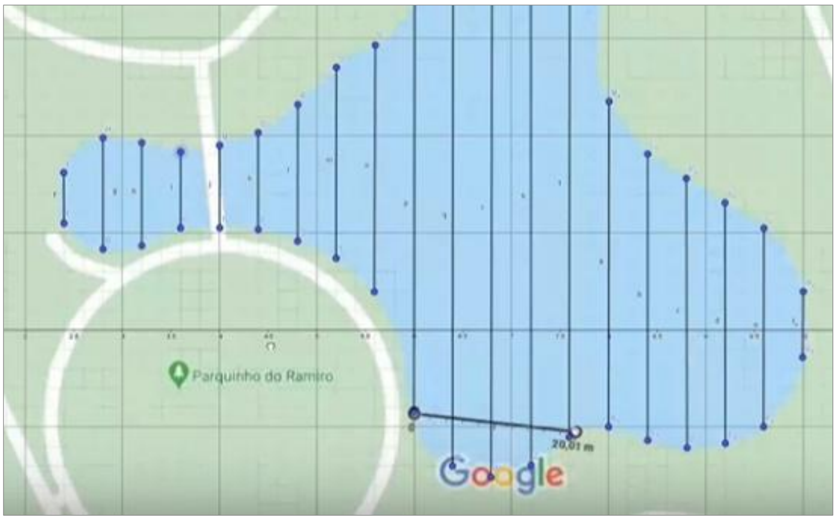


Figure 5: Trapezoids in the lake – GN1 (Research collection)

The students, based on this chosen path for resolution, were able to argue and debate other issues, such as, for example, what, based on the trapezoidal method, should be considered to obtain only the area value of the lake in Ramiro Ruediger Park, disregarding the other factors. These findings are in line with what Cai and Lester (2012, p. 152) describe:

> The problem-solving learning environment provides a natural setting for students to present multiple solutions to a problem to their group or class and to learn mathematics through social interactions, i.e., negotiation, and shared understanding. Such activities help students clarify their ideas and gain different perspectives on the concept or idea they are learning.

In group GM12, researcher P1 intervened in the middle of a conversation to check how the students were thinking.

> (E15 speaking): I have a question regarding problem 3. Can we only use trapezoids to fill the lake region? Couldn't I make some large squares and just outline them with trapezoids to speed up the process?
>
> (E16 speaking): We need to use only one type of figure, since the method is specific to trapezoids. Remember the generalization of the formula we came up with?
>
> (E15 speaking): Yes, but they all need to be the same size when we apply them?
>
> (E16 speaking): I believe we need to standardize the same height, changing only the measurements of the bases of the trapezoids.
>
> [At this point, the researcher enters the group to check how the students are thinking].
>
> (P2 speaking): I noticed that you are on the right track, dividing the figure into several trapezoids, but check if they are the same height... remember the trapezoid formula that you generalized?
>
> (E15 speaking): Yes, it's just that it's not quite ready yet! Now we are studying which would be the best trapezoid to fill the area of this region, some type that is not too small, which would require us to take many measurements later, and not one that is too large, which would not be close to the intended area value. We were even discussing this before, we can't make a big square and go around it with trapezoids, right?
>
> (P2 speaking): You can make it with trapezoids, all of the same height. But remember that they don't all have to be the same, that is, with the same base. Look at the photo I sent in the chat (read Figure 6).
>
> (E15 speaking): Oh, I understand. It's just the width that can vary. Thank you very much!

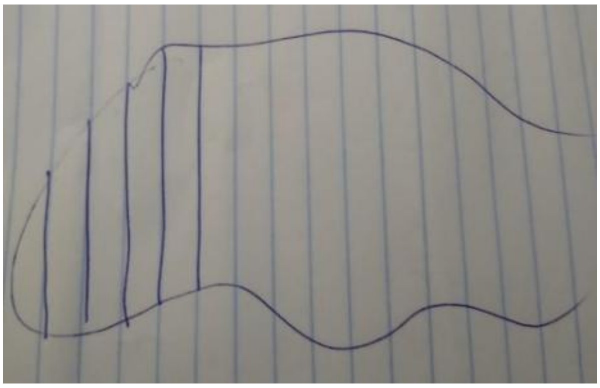


Figure 6: Representation of the lake by P2 (Research collection)

In this sense, it is clear that P2's comment merely confirmed the answer regarding the question that E15 had directed to E16 about whether the trapezoids needed to all be the same size.

These discussions among the students and also with the researcher lead to reflection on how much information to provide to the students.

> In problem-solving teaching, one of the most perplexing dilemmas is how much to tell students. On the one hand, telling students reduces their thinking. Students who perceive that the teacher has a preferred method or approach are extremely reluctant to use their own strategies. Students also will not develop self-confidence or problem-solving abilities by listening to the teacher explain his or her thinking. On the other hand, saying too little can sometimes result in students stumbling and waste valuable class time. (Van de Walle, 2009, p. 75)

Thus, the author argues that the amount of information to be given to students should allow the situation to remain challenging for them and still allow them to reflect and remain focused on finding a solution, without reducing the problem to a mechanical procedure. As stated by Echeverría and Pozo (1998, p. 64-65), "discussion with peers forces students to make explicit and justify how they understand a task, the tools and techniques with which they seek to approach it, the objective they set for themselves by using each of these techniques, and the order in which they will use them".

This help may also involve clarifying some nomenclature or handling some resource used in the solution, as happened in two of the groups evaluated, GN3 and GN6, who had initial difficulties in handling some GeoGebra tools. As a result, one of the members of the groups mentioned asked the researcher for help in the virtual classroom. The GN3 transcript shows this moment.

> (E27 speaking): P1, could you join our individual chat to clarify some doubts related to GeoGebra? [in the virtual classroom]
>
> (P1 speaking): Good evening, group! What is your question? [in the virtual classroom]
>
> (E27 speaking): Good evening! We are having difficulty measuring (reading) the scale value indicated on the graph.
>
> (P1 speaking): You will need to go to the 3rd window, at the top of GeoGebra, and select the segment option, in the red arrow below this little window.
>
> (E27 speaking): Oh, right. I was only able to select the straight option before, because I had not seen the other options through this little arrow.
>
> (P1 speaking): That's right, in each option in these windows there are others that can be used.

> Now try to measure the scale value with this segment.
>
> (E27 speaking): Perfect, it worked, 1.68 is equivalent to 20.01m. Another question, to divide the lake with the trapezoids, do we use this same straight line segment?
>
> (P1 speaking): That's right, the aim is to measure the distance from one point to the next point on the base.

Furthermore, at this moment of researcher mediation, another question was raised by the same group:

> (E27 speaking): Yes, I understand... and how many trapezoids are needed to calculate the area of the lake with good precision, considering a standard height?
>
> (P1 speaking): That's up to you. You can choose a height of 1, 2, 3, 4... squares, but 2 squares are enough to have a good precision in the result.
>
> (E27 speaking): Yes, I had already understood the idea that the smaller the height of the trapezoid, the better the precision... I only asked this because of the time we had to solve this problem.
>
> (P1 speaking): Don't worry about that, the important thing is for you to present the reasoning you had to solve the problem. The final systematization of the work can be handed in by the next class.

This discussion took place with the students sharing the GeoGebra screen, which they were using to solve the problem, as shown in Figure 7.

Figure 7: Screen sharing – GeoGebra (Research collection)

This screen sharing tool allows students to quickly resolve difficulties and build knowledge in a context other than face-to-face teaching (Garcia, 2011). In this sense, it is possible to encourage group work and collaboration, which are essential elements for the problem-solving stage.

Still regarding the reported transcription, it was once again noted, as in problems 1 and 2, that students were concerned about delivering the problem solution on time. Therefore, it is important that, if the group is unable to completely finish solving the problem in the proposed time, the teacher encourages them to at least present the reasoning that developed in this process, since it is from this that the result is reached. In other words, with the reasoning formed, students are able to formalize it, in the form of a report, at a time other than the classroom.

Next, the steps following the problem solving are discussed.

4.2 Steps after problem resolution

During the plenary session and the search for consensus, it became clear that, based on the groups' presentations and their written records, the students were able to calculate the area of the lake in Ramiro Ruediger Park.

With the exception of groups GM1 and GM6, which, respectively, calculated the area of the lake using their own method and Simpson's method, the other groups chose to solve the problem using the trapezoidal method.

During GM1's presentation in the plenary session, researcher P2 encouraged the students to also solve this problem using the method they had developed in class. Figure 8 shows the group's speech and presentation.

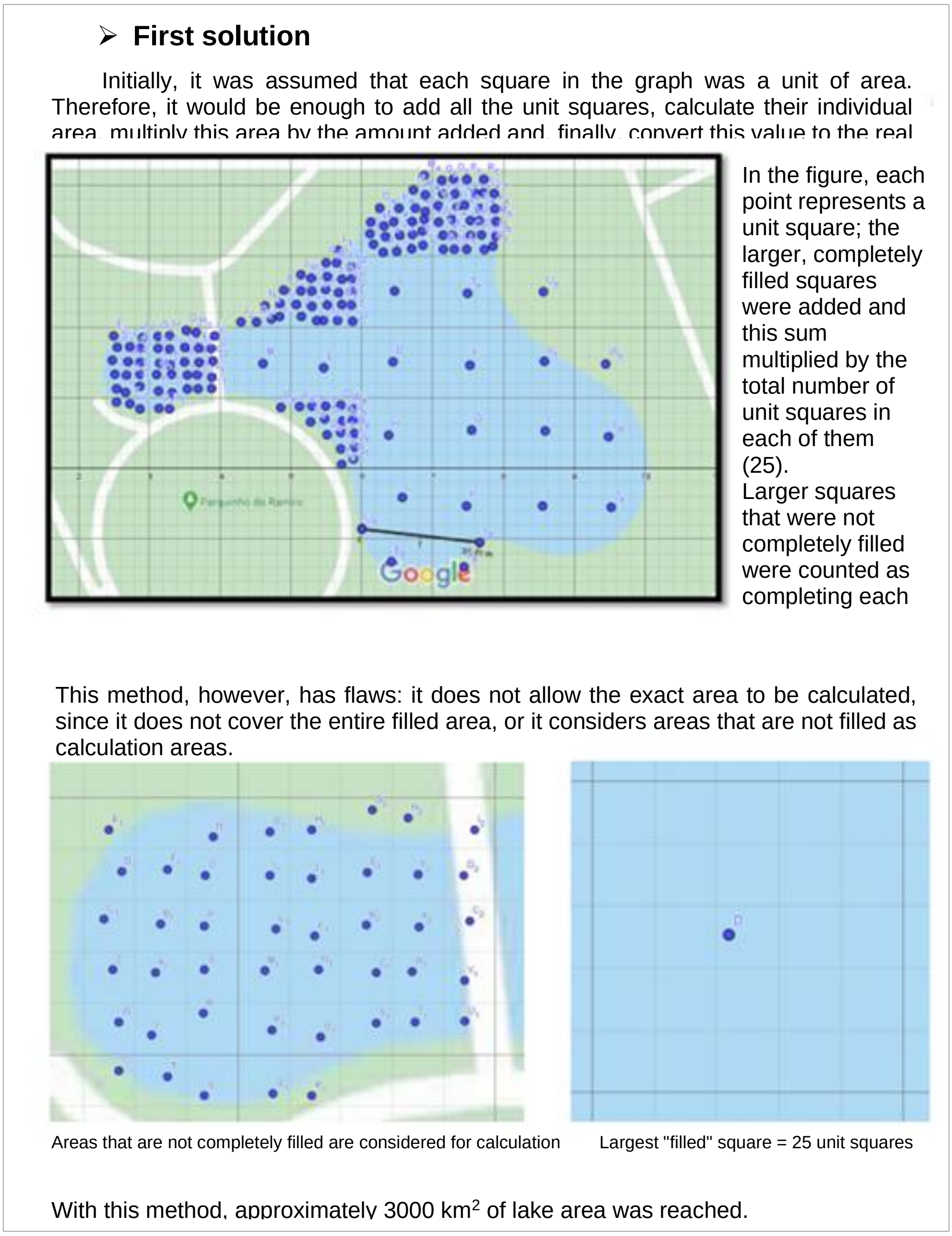


Figura 8: Primeira resolução do GM1 (Acervo de pesquisa)

After this presentation, E1 comments on his work and P2 intervenes in this process:

(E1 speaking): We are not sure if this method is correct, if it is really valid. Therefore, we would like to wait for the discussion in class to then formalize this in a better formatted file and send it to you.

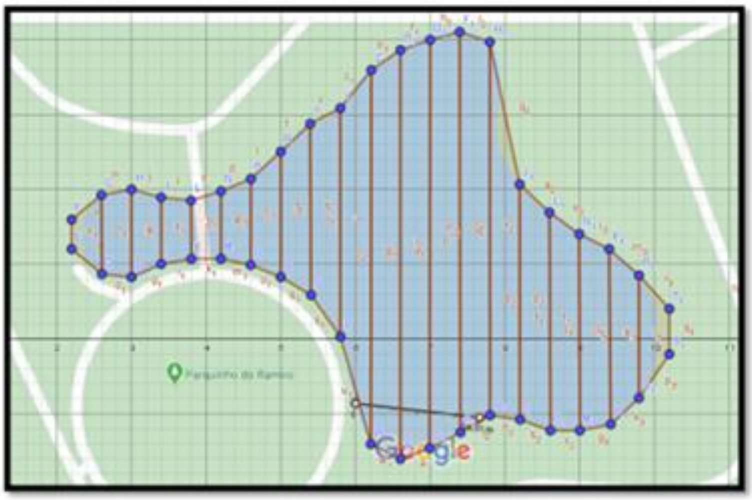
(P2 speaking): Your method is adequate, because you made approximations that also reached the expected result. You can and should present this solution in the report. But we also propose a challenge to you: include, along with this solution, the resolution via the trapezoidal or Simpson method. After that, find the area and compare it with this first solution presented by you.

(E1 speaking): Okay, we will solve the problem using one of these methods as well and forward it to you.

So, in Figure 9, the group's second resolution is presented after P2's mediation.

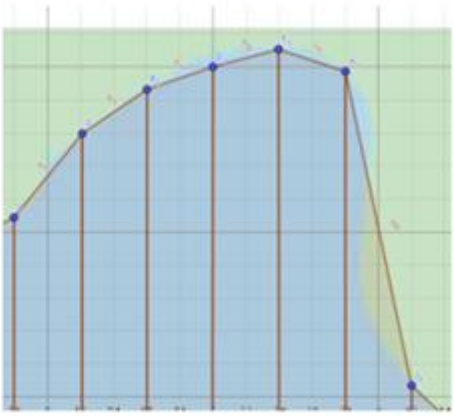
➢ **Second resolution**

For greater precision, the polygon method can be used, inserting trapezoids in the area to be calculated, determining their individual areas, adding them and converting the final value to the real number.



On the side, a figure of the area of the lake covered by the trapezoids. For calculation purposes, they all have the same height (2 unit squares), so Simpson's Rule can be used to calculate their area.

The graphical sum of the areas of the trapezoids is equivalent to 21 area units. It is known that 20.01m is equivalent to 1.61 area units; thus, it is concluded that 21 area units are equivalent to approximately 3243m². This value still has an error, since some areas (see below) are not covered by the trapezoids, and some areas without water body filling are marked incorrectly, as in the figures below.



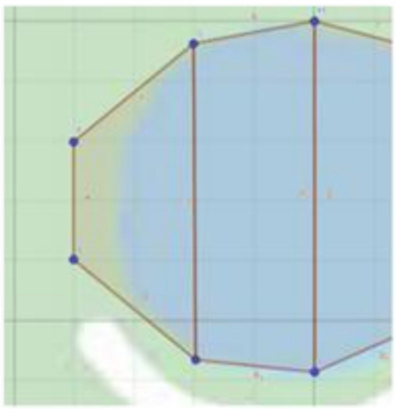


Figure 9: Second resolution of GM1 (Research collection)

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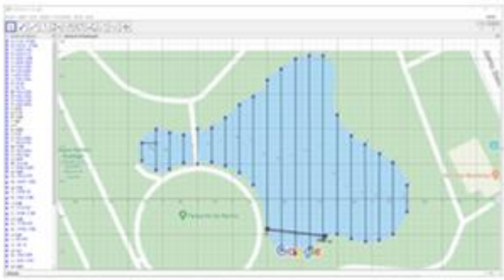
The value found by the group of 3243 m² was greater than the expected value of 2900 m², and this situation is mainly due to the previous way in which the measurement points were distributed around the lake. As can be seen in the previous image (right), in Figure 9, GM1 began marking the points while still in the green area. Therefore, this additional area ended up influencing the expected area value for the lake; however, the important thing is the detailing, in the report, of the information that led to the error. In it, the group explained the reason for the inconsistencies in the area found in relation to the expected one (observed in the plenary session).

The other groups, GM4, GM6, GM7, GM12, GN1, GN2, GN3 and GN6, managed to find a value close to the expected one for the area, as shown in the statement by the member of group GN3:

> (E27 speaking): We divided the lake into 19 trapezoidal segments in order to calculate the most approximate area possible of the lake in Ramiro Ruediger Park. Then, based on the equation $A_{Ta} = \frac{h}{2}[b_1 + 2(B_1 + B_2 + B_3) + \cdots + B_{n-1} + b_n]$, we built a table to better represent the results.

Figure 10 illustrates the students' calculation procedure, which led them to find an area value of approximately 2879m².



i	Base	*	Trapézios	Trapézios na Escala
0	0,51	1	0,51	6,074457
1	1,17	2	2,34	27,871038
2	1,04	2	2,08	24,774256
3	0,8	2	1,6	19,05712
4	1	2	2	23,8214
5	0,98	2	1,96	23,344972
6	1,37	2	2,74	32,635318
7	1,94	2	3,88	46,213516
8	2,53	2	5,06	60,268142
9	4,26	2	8,52	101,479164
10	5,16	2	10,32	122,918424
11	5,56	2	11,12	132,446984
12	5,48	2	10,96	130,541272
13	5,25	2	10,5	125,06235
14	3,4	2	6,8	80,99276
15	2,94	2	5,88	70,034916
16	2,76	2	5,52	65,747064
17	2,47	2	4,94	58,838858
18	2,04	2	4,08	48,595656
19	0,76	1	0,76	9,052132
€	_	_	101,57	1209,769799

		h	Somatório Trapézios
A=	(h/2)*Somatório	2,382	1209,77 u.a
escala	11,9107		**Resultado da Área**
			2879,252 m²

Figure 10: GN3 Resolution (Research Collection)

All these discussions on problem solving and reflection on the processes used in learning highlight the fundamental role of creativity in the educational context. As Gontijo (2007) argues, creativity in mathematics is linked to the ability to present original and varied solutions to a problem. This diversity of approaches enriches the teaching-learning process, allowing students to become critical and autonomous thinkers. Furthermore, according to Possamai and Silva (2020, p. 8), when students engage in dialogue, this process

> tends to, in addition to developing learning, improve students' abilities that

> will be useful in all areas and for life, such as: creativity in the search for a solution to the proposed problem; criticality when analyzing their procedure and its result, as well as those of their colleagues; power of argumentation to present their proposal to the detriment of others; autonomy in the search for a solution; and, finally, the ability to work collaboratively, presenting proposals, discussing possibilities and accepting other alternatives when they are coherently presented.

Therefore, by fostering an environment in which problem solving is central, educators not only stimulate critical thinking but also encourage students to explore their mathematical creativity, aligning with the view of Bicer et al. (2020) and Vieira, Possamai and Allevato (2023) on the importance of cognitively challenging experiences that favor the construction of meaningful conceptual understandings.

In this vein, Leal Junior and Onuchic (2015, p. 959) emphasize that “creativity acts as an inherent power in the agencies and events in the thinking of students, who, when faced with problems, will seek ways to solve them”. Dedication to details during problem solving also demonstrates students’ interest in this process, as well as

> although it is not a question of reducing school problems to the format of everyday tasks and situations, it seems that for students to face school tasks as real problems, they need to be related to the contexts of interest to the students or, at least, adopt an interesting format, in the literal sense of the term. (Echeverría and Pozo, 1998, p. 42).

Due to the students' clear understanding of how to calculate the lake's area using numerical methods, researcher P1 only highlighted, in the formalization stage, some important steps that led to the solution of the problem, such as: (i) scale measurement, in which 1.67 represents 20.01 m; (ii) the height of the trapezoid of 0.2 (for those who used this measurement) represents 2.38 m of the real scale and (iii) the area is obtained by generalizing the trapezoid method, as found in problem 2 — in general, the height value is divided by two, being multiplied by the sum of all the bases with their respective coefficients.

Therefore, the final considerations regarding the data analysis are presented below.

5 Discussion and considerations

When analyzing the results obtained with the proposed problem, it is observed that it acted as a generator of concepts and procedures, based on the students' prior knowledge and, frequently, supported by group discussions, in which different resolution strategies were presented. In this context, Cavalcanti (2001, p. 121) states that “accepting and analyzing the different resolution strategies as valid and important stages in the development of thought allows learning through reflection and helps the student to have autonomy and confidence in their ability to think mathematically”.

As for student engagement, it was possible to note that they were actively involved when relating the practical engineering problem with the application of the trapezoidal method, developed from problems 1 and 2. This connection facilitated the construction of new concepts and procedures during the search for a solution to the problem presented.

This scenario shows that engineering education, as in other areas of knowledge with a professional focus, should not be restricted to problems disconnected from the knowledge construction process and the practical context of future professionals. The post-university reality demands that undergraduates face and solve a variety of problems and situations, since the role of the engineer essentially involves solving challenges. Therefore, it is essential that they are prepared to learn new content and improve their abilities, based on the experiences acquired throughout their academic training.

However, the intention is not to reduce mathematics to a merely utilitarian function, suggesting that only application problems should be addressed. The crucial thing is that the questions presented, whether in the context of mathematics or in the context of students' professional practice, are challenging and thought-provoking.

In this sense, the Teaching-Learning-Assessment methodology in Mathematics through Problem Solving is a valuable opportunity for students to improve their collaborative work abilities. In order for the group to be able to solve the problem, it is necessary for the members to imagine, reason, structure, debate and exchange ideas in a respectful and constructive manner. It is worth noting that the idea that everything is completely correct or that there is only one answer is not central to this methodology. On the contrary, the process of constructing knowledge implies that students may make mistakes, which results in disagreements during discussions, leading to enriching debates. In this debate environment, the opportunity for groups to speak and listen to each other's opinions promotes consensus regarding the proposals presented, mediated by the teacher. This prevents a student from pointing out a colleague's mistake, minimizing insecurities and opening space for new challenges and discussions.

Therefore, it is essential that some points are implemented during educational practice: (i) the teacher must pay attention to the students, demonstrating confidence in their ongoing learning process; (ii) a dialogical relationship must be established between teacher and student, allowing the student to express their ideas; (iii) the assessment must be formative, encouraging reflection and analysis on what was produced, promoting self-assessment; (iv) the focus must be on the development of processes and not only on final results, avoiding judgments; (v) students must be encouraged to organize and detail their reasoning clearly when solving problems; (vi) the assessment must be timely for learning and the development of abilities.

Furthermore, teachers must be patient in their efforts to continually improve their classroom practice and help their students become professionals with the abilities needed in the post-university world. Problem Solving, combined with Creativity, undoubtedly contributes significantly to this transformation.

The educational practice provided moments of reflection, creativity and mathematical understanding, especially in the relationship between theory and practice, as witnessed by the students:

> Testimony 1: "From what I saw in our group, we feel more comfortable doing what was proposed, because there is no pressure of 'silence, we are in class'. We went to a separate group and there we discussed the work in complete freedom, something that, in class, has often been the subject of 'external conversation', even when we were talking about something related to the subject".
>
> Testimony 2: "One of the biggest benefits for me was having seen in more depth where I will use the calculus learnings in my profession".
>
> Testimony 3: "It made me reflect and challenge myself more with the content, presenting a range of problems different from those shown in class, making the discussion richer".

It is worth noting that this study contributes to research in Mathematics Education by reinforcing Problem Solving as a methodological approach capable of promoting active learning and the development of fundamental mathematical abilities. The research shows how interaction between students and the formulation of diverse strategies broaden conceptual understanding, strengthening argumentation and creativity. In addition, the results obtained highlight the importance of pedagogical practices that articulate theory and application, providing support for future research that explores new forms of teaching that value experimentation and the collective construction of knowledge. In this way, the study dialogues with contemporary research on Mathematics teaching and offers paths for methodological innovations that favor learning with understanding.

In conclusion, it is important to highlight that the researchers involved in this research share the same sentiment expressed by Onuchic and Allevato (2011, p. 82), when they state that "teachers who teach this way get excited and do not want to go back to teaching in a traditional way. They feel gratified when they realize that students develop understanding based on their own reasoning".

To this end, it is suggested that, in future research, new educational practices be developed and analyzed within the context of Problem Solving and Higher Education, since there is a lack of studies in this area of knowledge.

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