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Mathematical Creativity Tests for Basic Education: analysis in the field of Arithmetic and their potentialities

Abstract: Creativity is considered an important ability that contribute to the individual's integral formation, is valued in the job market and induces scientific/technological progress. In the mathematical context, it is defined as the ability to generate multiple solutions to problems. The objective of the study was to reflect on the potential of creativity in Mathematics test items for structuring pedagogical work in Basic Education. For this, research that applied specific tests were analyzed, considering the structure of the items and its content, to order to verify its potential for developing mathematical creativity. By results, the potencial of the items analysed were discussed, above all, as instruments for pedagogical work organization for stimulating the critical and creative potential in Mathematics of students.

Keywords: Mathematical Creativity. Creativity Tests. Mathematical Education.

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Article

Pruebas de Creatividad en Matemáticas para la Educación Básica: análisis de ítems en el campo de la Aritmética y sus potencialidades

Resumen: La creatividad es considerada una habilidad importante que contribuye al desarrollo integral del individuo, es valorada en el mercado laboral e induce al avance tecnológico/científico. En el contexto matemático, se define como la capacidad de generar múltiples soluciones a los problemas. Este estudio buscó reflexionar sobre el potencial de los ítems de pruebas de creatividad en Matemáticas para el trabajo pedagógico en la Educación Básica. Así, se analizaron investigaciones que realizó aplicaciones de pruebas, considerando la estructura de los ítems y contenidos involucrados, para visualizar potenciales para el desarrollo de la creatividad matemática. Como resultado, se discutieron potencialidades de los ítems, especialmente, como instrumentos organizativos del trabajo pedagógico para estimular el potencial crítico y creativo en Matemáticas de estudiantes.

Palabras clave: Creatividad en Matemáticas. Pruebas de Creatividad. Educación Matemática.

Testes de Criatividade em Matemática para a Educação Básica: análise de itens no campo aritmético e suas potencialidades

Resumo: A criatividade é apontada como habilidade importante para a formação integral do indivíduo; é valorizada no mercado de trabalho e impulsiona o avanço tecnocientífico. No contexto matemático, é definida como a capacidade de gerar múltiplas soluções para problemas. Assim, o objetivo do estudo foi refletir acerca das potencialidades de itens de testes de criatividade em Matemática para a estruturação de trabalho pedagógico na educação básica. Para tanto, analisaram-se pesquisas que realizaram aplicações de testes, considerando a estrutura dos itens e os conteúdos envolvidos, para visualizar possibilidades de desenvolver a criatividade matemática. Nos resultados, discutiram-se as potencialidades dos itens analisados, sobretudo como instrumentos para organizar o trabalho pedagógico dedicado a estimular o potencial crítico e criativo em matemática na Educação Básica.

Palavras-chave: Criatividade em Matemática. Testes de Criatividade. Educação Matemática.

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1 Introduction

Research on creativity in the educational field has expanded towards identifying, stimulating and evaluating students' creative potential, with a view to implementing pedagogical actions that enable the development of their problem-solving skill, whether through the production of different valid responses or through the use of varied strategies to obtain them (Gontijo, Fonseca and Vertuan, 2023).

This type of research is justified by the fact that, currently, creativity stands out as one of the human skills of great value for facing various contemporary challenges. This includes both those involving scientific and technological development and those involving the search for sustainable and ecologically appropriate solutions for the maintenance of the planet's ecosystems. Its importance for people's mental health is also highlighted.

Schools are a privileged space for the development of activities that enhance students' creativity. For this to happen, changes are needed in the teaching contract in force in many schools, in order to place the student as the protagonist of their formative process, exercising creative thinking in the process of constructing knowledge (Gontijo et al., 2018).

As the National Education Association (2012, p. 24), an American organization, points out, “in today's world of global competition and task automation, innovative capacity and creative spirit are becoming requirements for personal and professional success”. Vincent-Lancrin et al. (2020, p. 14) add that, “even if there were no economic argument, creativity and critical thinking contribute to human well-being and the proper functioning of democratic societies”.

Given the relevance of this subject in recent years, the Programme for International Student Assessment (PISA) included, in the test applied in 2022, the assessment of creative thinking skills (Brasil, 2022). The Organization for Economic Cooperation and Development (OECD), responsible for administering PISA, recently released the results report, with Brazil reaching 44th position out of a total of 56 countries (OECD, 2024).

It is hoped that these data can be used to promote public education policies that stimulate creative thinking among students around the world, as well as encourage a broader social debate on the importance and methods of supporting the development of creativity through education. Given the format of this assessment, PISA defined creative thinking as the “competence to engage productively in the generation, evaluation and refinement of ideas, which can result in original and effective solutions, advances in knowledge and impactful expressions of the imagination” (OECD, 2019, p. 7).

According to the *PISA 2021 Mathematics Framework (Draft)* (OECD, 2019), which details the Mathematics assessment in PISA, the major change in the latest edition compared to previous ones is the importance given to other elements that are present in contemporary times. Currently, thinking mathematically, combined with certain concepts that are not necessarily taught explicitly — they may even be concepts acquired through the students' own experiences — has proven to be essential in preparing young people to contribute to 21st century society. Citizens are expected to be engaged, creative and capable of making good decisions for themselves and for the group.

The debate about the importance of developing creativity and the role of schools in this process is not new. Researchers have been working in this field since the beginning of the 20th century. An important milestone in research occurred in the United States of America, when the president of the American Psychological Association, Joy Paul Guilford, in his annual opening speech at the association's work in 1950, proposed studying and measuring creativity as a human intellectual function (Urban, 1991). As a result, there was a considerable increase in the number of studies on creativity (Piirto, 2004).

Research on creativity, specifically in the field of Mathematics, also emerged at the beginning of the 20th century, with contributions from Henri Poincaré, and continues to this day. These studies are supported by researchers such as Balka (1974), Haylock (1987), Lee, Hwang and Seo (2003), Gontijo (2007), Carvalho (2015, 2019), Fonseca (2015, 2019), Kattou et al. (2013), Pitta-Pantazzi, Sophocleous and Christou (2013), among others. Many of them work with specific tests for creativity in Mathematics.

If tests of this nature contribute to identifying and evaluating how creative students can be in Mathematics, what potential do they present for pedagogical work in the daily classroom? Based on this question, the objective of this research was developed, which is to reflect on the potential of creativity test items in Mathematics for structuring pedagogical work with Basic Education students. Test items that require skills in the arithmetic field are highlighted, due to their scope and applications in all areas of Mathematics.

2 Theoretical framework

The teaching of Mathematics has two main objectives: to prepare individuals for citizenship and to prepare them for a career that is anchored in the areas of science and technology. To achieve these goals, it is necessary that the study of Mathematics be stimulated through approaches and examples that make it interesting and useful, considering the history of humanity and, at the same time, paying attention to the current world (D'Ambrosio, 2005). For the author, the teacher's mission is to organize teaching situations to achieve the greater objectives of Education, which are: "to enable each individual to reach their creative potential" and "to stimulate and facilitate common action, with a view to living in society" (D'Ambrosio, 2005, p. 37).

The National Council of Teachers of Mathematics (NCTM, 2000), in the *Principles and Standards for School Mathematics*, highlighted four elements to indicate the role of Mathematics in people's lives:

> *Mathematics for life.* Knowing mathematics can be personally satisfying and empowering. The underpinnings of everyday life are increasingly mathematical and technological. For instance, making purchasing decisions, choosing insurance or health plans, and voting knowledgeably all call for quantitative sophistication.
>
> *Mathematics as a part of cultural heritage.* Mathematics is one of the greatest cultural and intellectual achievements of humankind, and citizens should develop an appreciation and understanding of that achievement, including its aesthetic and even recreational aspects.
>
> *Mathematics for the workplace.* Just as the level of mathematics needed for intelligent citizenship has increased dramatically, so too has the level of mathematical thinking and problem solving needed in the workplace, in professional areas ranging from health care to graphic design.
>
> *Mathematics for the scientific and technical community.* Although all careers require a foundation of mathematical knowledge, some are mathematics intensive. More students must pursue an educational path that will prepare them for lifelong work as mathematicians, statisticians, engineers, and scientists. (NCTM, 2000, p. 4)

Considering the relevance of Mathematics in everyday life and the importance of promoting an education that awakens students' creative potential, the challenge is to promote a

Mathematics Education that incorporates these elements into the organization of schools' pedagogical work. From the point of view of Brazilian curricular guidelines, one of the general competencies of Basic Education, expressed in the Base Nacional Comum Curricular *[National Common Curriculum Base — BNCC]* (Brasil, 2017), is

> Exercise intellectual curiosity and use the approach specific to science, including research, reflection, critical analysis, imagination and creativity, to investigate causes, develop and test hypotheses, formulate and solve problems and create solutions (including technological ones) based on knowledge from different areas (p. 9).

Despite the evidence given to creativity in the BNCC, the document does not present an understanding of what constitutes this construct, nor does it indicate elements that may favor development in the school environment (Fonseca and Gontijo, 2020). Noting this gap, some historical and conceptual elements of research on creativity in Mathematics will be briefly described.

Studies related to mathematical creativity began to emerge at the beginning of the 20th century, through the contributions of Henri Poincaré (Hadamard, 1954; Sriraman, 2004; Poincaré, [1904] 1995). Poincaré (1995) observed that the mind works in two ways: unconsciously and consciously. Conscious work leads to unconscious work — the work that points to solutions after a long period of rest. From this, the author sought to better understand the habits of mathematicians at the time and published, in the French journal *L'Enseignement Mathématique*, a questionnaire with 30 items, the objective of which was to identify the process of creation in Mathematics of mathematicians and which factors contributed to this process (Hadamard, 1954).

Poincaré (1995) concluded that there are two types of mathematicians: analysts, guided by logic, and geometers, guided by intuition. For him, both types of mind are necessary for the advancement of science; however, he admits that intuition is a more prominent element in the creative process in Mathematics, and each person has a different level of intuition (Poincaré, 1995).

Based on these reflections, other scholars have endeavored to understand these processes. Jacques Hadamard (1954), for example, inspired by the works of Wallas (1926) and Poincaré ([1904] 1995), sought to delve deeper into the aspects related to the creative process in Mathematics, describing it in four stages: preparation, incubation, illumination (or insight) and verification.

Haylock (1987), in turn, presented creativity in Mathematics as the ability to formulate problems, invent theorems, deduce formulas and original methods to solve problems. Lev-Samir and Leikin (2013) defined creativity in Mathematics as the cognitive ability to generate different solutions to the same problem. Kattou et al. (2013) associated mathematical creativity with the ability to solve inductive and deductive problems and the ability to understand the differences and similarities between mathematical objects/elements.

In order to investigate the relationships between mathematical skill — a multidimensional construct that takes into account quantitative, causal, spatial, qualitative, and inductive/deductive skills — and creativity in Mathematics — defined as a domain-specific characteristic —, Kattou et al. (2013) administered tests to students aged 9 to 12, in order to analyze and understand how these two elements interact.

Three models were established: in the first, mathematical ability and creativity in Mathematics would be correlated. In the second model, mathematical ability would be a subcomponent of creativity in Mathematics, and in the third, creativity in Mathematics, together with other abilities — spatial, quantitative, qualitative, causal, and inductive/deductive —, would be a subcomponent of mathematical ability (Kattou et al., 2013).

Using confirmatory factor analysis, latent class analysis and analysis of variance, they considered the third model as the most appropriate, placing creativity in Mathematics as a prerequisite for the development of mathematical ability and illustrating, through data analysis, that students with higher scores on mathematical ability tests were also the most creative (Kattou et al., 2013).

Some authors have highlighted that the automation of students' behavior in relation to mathematical problems can hinder the development of creativity, as well as the little time dedicated to solving these problems (Almouloud, 2007; Mann, 2005; Harpen and Sriraman, 2013; Vasconcelos, 2002).

Gontijo (2007), considering the diversity of concepts in the literature about what characterizes creativity in Mathematics, proposed a definition with the intention of merging aspects previously proposed by other researchers. For the author, creativity in Mathematics is the

> skill to present numerous possibilities of appropriate solutions for a problem situation, so that they focus on distinct aspects of the problem and/or different ways of solving it, especially common ways (originality), both in situations that require the resolution and elaboration of problems and in situations that require the classification or organization of objects and/or mathematical elements according to their properties and attributes, whether textually, numerically, graphically or in the form of a sequence of actions (Gontijo, 2007, p. 37).

In order to stimulate and assess creativity in Mathematics, different strategies have been proposed for the development of school activities, with three types predominating: problem solving, problem elaboration and problem redefinition (Gontijo, 2007; Haylock, 1997).

Problem solving occurs with the introduction of open problems in the school environment, that is, problems that do not require the direct application of an algorithm or formula and that allow the student to arrive at several answers. Problem elaboration occurs through formulation, resolution and improvement, that is, the individual is able to recognize problems in questions that involve mathematical situations and is able to express them in an improved way. The redefinition of a problem can be explained as an activity in which the student redefines the elements of a situation in terms of their mathematical attributes, that is, it encourages the student to look at the problem from another perspective, finding creative solutions (Gontijo, 2007; Haylock, 1997).

Another important factor in relation to creativity in Mathematics is assessment. This can be done in everyday school life, by analyzing students' productions in order to identify those that represent innovative or unusual ways to find a solution to a problem. It can also be done by applying structured tests. However, tests are, by their very nature, intended to identify talented students or those with high abilities in the field of Mathematics, and can even be applied in regular classes in schools in order to identify characteristics of creative thinking — fluency, flexibility and originality of thought — in students' productions.

In this context, several tests have been created in different countries to assess creativity in Mathematics (Balka, 1974; Carvalho, 2015; Gontijo, 2007; Haylock, 1987, 1997; Kattou et al., 2013; Lee, Hwang and Seo, 2003; Fonseca, 2015).

The tests seek to assess: fluency of thought, which represents the number of different ideas generated that are appropriate solutions to the proposed problems; flexibility, which is related to the number of different categories in which the generated solutions can be classified; and originality of thought, which refers to the unconventionality of the ideas generated, that is, appropriate original solutions that stand out from the group of solutions presented by the group of students in a class under investigation (Gontijo, 2007).

Associated with the analysis of the characteristics of creative thinking — fluency, flexibility and originality of thought —, Balka (1974) established some criteria to evaluate creativity in Mathematics: the formulation of hypotheses, whose purpose is to evaluate mathematical situations; the evaluation of unusual mathematical ideas based on reflection on their implications in mathematical situations; the perception of problems based on mathematical situations to the point of formulating new questions that make it possible to answer them; the elaboration of specific problems through a general mathematical problem; the breaking of the static mental framework as the individual seeks solutions to different mathematical problems; and the elaboration of models that aim to solve mathematical situations.

Com o intuito de aperfeiçoar os critérios de avaliação da criatividade em Matemática, Leikin, Levav-Waynberg e Guberman (2011) desenvolveram um modelo para pontuar o desempenho dos alunos nos testes com problemas de múltiplas soluções, considerando a originalidade, a fluência e a flexibilidade das respostas obtidas. As autoras consideraram as possibilidades de pontuação a partir do modo de resolução dos problemas e tamanho/natureza dos grupos em questão.

Gontijo (2007), after structuring a test with 6 questions, also sought to classify the solutions presented and decided to measure fluency through the “number of solvable mathematical problems”; flexibility through the “number of categories constituted based on the number of semantic relations involved in each answer”; and originality through the “relative rarity of the problems proposed” (p. 157-158).

Furthermore, Carvalho (2015) proposes a structure to calculate a coefficient related to creativity in Mathematics. The creation of instruments to measure creativity in Mathematics is important to define strategies aimed at teachers, so that they seek ways of working that stimulate the creative potential of their students (Gontijo et al., 2018).

The use of open-ended problems to stimulate and assess creative thinking in Mathematics aims, among other factors, to develop students' procedural fluency. This fluency refers to the

> skill to apply procedures efficiently, flexibly, and accurately; transfer procedures to different problems and contexts; build or modify procedures from other procedures; and recognize when one strategy or procedure is more appropriate to apply than another (NCTM, 2023, p. 1).

NCTM is based on four principles that are considered necessary actions to ensure that every student has access to and develops procedural fluency. These principles apply to the entire K-12 curriculum, including basic facts, whole numbers, multidigit numbers, and rational numbers. They also apply to other procedures throughout the curriculum, such as comparing fractions, solving proportions or equations, and analyzing geometric transformations. The four principles are:

- Conceptual understanding should precede and coincide with procedural instruction.
- Procedural fluency requires a repertoire of strategies.

- Basic facts should be taught using number relationships and reasoning strategies, not memorization.
- Assessment should be responsive to the components of fluency and student characteristics (NCTM, 2023, p. 2).

In order to focus the analysis undertaken here, Arithmetic was chosen — an area of Mathematics that explores numerical operations and is important due to its elementary nature, since its rules and principles are necessary in all other areas of Mathematics, as well as in everyday life. The BNCC highlights that

> Mathematics is not limited to the quantification of deterministic phenomena — counting, measuring objects, quantities — and calculation techniques with numbers and quantities, as it also studies the uncertainty arising from random phenomena. Mathematics creates abstract systems that organize and interrelate phenomena of space, movement, shapes and numbers, associated or not with phenomena of the physical world. These systems contain ideas and objects that are fundamental for understanding phenomena, constructing meaningful representations and consistent arguments in the most varied contexts (Brasil, 2017, p. 265).

The document proposes dividing Mathematics into thematic units, the first of which is called Numbers. This unit seeks to work on the concepts of Arithmetic, aiming at the development of numerical thinking and the construction of the notion of number — considered fundamental in Mathematics, as well as in other areas. These notions are expanded and deepened in situations involving the other thematic units: Algebra, Geometry, Quantities and Measurements, and Probability and Statistics (Brasil, 2017).

In addition, this thematic unit favors interdisciplinarity and involves cultural, social, political, psychological and economic dimensions. These are issues that promote the development of students' personal and social skills, who become capable of building contexts for the applications of mathematical concepts (Brasil, 2017).

Based on this theoretical framework, it was possible to map items from some tests designed to assess creativity in Mathematics — developed by researchers from different countries (Akgul and Kaveci, 2016; Balka, 1974; Carvalho, 2015; Dunn, 1975; Fonseca, 2015; Gontijo, 2007; Gontijo; Fleith, 2010; Harpen and Sriraman, 2013; Kattou et al., 2013; Kim, Cho and Ahn, 2003; Lee, Hwang and Seo, 2003; Leikin and Lev, 2007; Livne, Livne and Milgram, 1999; Nikmah, 2017; Pitta-Pantazzi, Sophocleous and Christou, 2013; Silver and Cai, 1996; Walia and Walia, 2017). It was also possible to classify them, using a methodology detailed in the next section.

3 Methodology

In order to identify creativity tests in Mathematics in academic productions, we chose to carry out a mapping as a methodological strategy for the development of this work. To this end, we consulted the Virtual Library of Research on Critical and Creative Thinking in Mathematics[1], which brings together a broad bibliographic and audiovisual collection on the subject, available on the PI Group portal — *Research and Investigations in Mathematics Education*.

According to Fiorentini et al. (2016), research mapping is a process of surveying and

[1] Disponível em: https://bit.ly/pensamentocriticoecriativoemmatematica.

describing research information on a specific field. In this study, we collected items present in tests that assess creativity levels in Mathematics, analyzing creative thinking skills. The observed tests were developed by researchers who applied them to groups of students at a certain school level and reported their experience in articles, dissertations, and theses. The items of some tests were adapted from older tests, which explains why some similarities were found between research conducted in different countries, considering specific school levels.

In order to select these items, the original creations and the adaptations were considered to be the same items, which did not prevent us from finding variety in the studies that were available. The items were selected from a total of 17 works that contained references to tests — 12 articles, 2 dissertations and 3 theses — published in Germany, Brazil, South Korea, the United States, the Netherlands, India, England and Turkey.

Initially, the items were categorized according to the Mathematics topics addressed in their formulations, placing them in one of the following areas: Geometry, Algebra and Arithmetic. They were then classified according to the strategies for developing creativity in Mathematics: problem solving, problem elaboration and redefinition of a problem. In parallel, the potential uses of these items for pedagogical practice in the classroom were highlighted. It should be noted that, in this article, items that assess creative thinking in Mathematics in the field of Arithmetic will be discussed.

4 Results and analyses/discussions

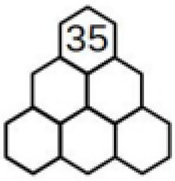
In the 17 studies analyzed, 39 items were mapped in total, 16 in the area of Arithmetic, 6 in the area of Algebra and 17 in the area of Geometry. Regarding the strategies for developing Creativity in Mathematics, the items are divided into 8 for Problem Solving, 10 for Problem Elaboration and 21 for Problem Redefinition.

Since the analyses of this study focus on the Arithmetic category, items that require respondents to have problem-solving skills involving calculations with different types of numbers, to develop problems whose solution results from performing arithmetic operations and to reorganize numbers according to their characteristics and/or attributes were included.

The items involving problem solving are presented below.

Table 1: Problem Solving

Look at the number at the top of the pyramids. All the cells should contain only one number. Each number in the pyramid can be calculated by performing the same operation on two numbers that appear below it. Fill in the cells of the pyramid, keeping the number 35 at the top. Try to find as many solutions as possible.



Source: Kattou *et al*. (2013, p. 172).

Table 2: Problem Solving

Two odd numbers add up to 20. What could these numbers be?

Source: Bishop (1968) *apud* (Dunn, 1975, p. 330)

Table 3: Problem Solving

This activity consists of performing operations involving only the number 4. You must use four number 4s, performing mathematical operations between them. The result of these operations must

also be equal to 4. Try to find the largest number of solutions, including all of the following arithmetic operations: addition, subtraction, multiplication, division, square root, factorial, etc. It is not necessary to use all of the operations in each solution presented.

Source: Livne, Livne e Milgram (1999, p. 237-238)

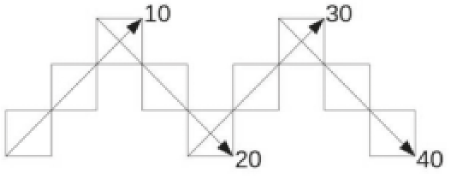
Table 4: Problem Solving

Knowing that $(p+q)(r+s) = 36$, find p, q, r and s.

Source: Bishop (1968) *apud* Dunn (1975, p. 330)

Table 5: Problem Solving

All cells must be filled with a number. Numbers must not be repeated. To fill the cells below, it is necessary to consider that the first diagonal must result in 10, the second in 20, the third in 30 and the last in 40, not cumulatively. You can choose to use any mathematical operation to be performed between the cells to obtain the results of the diagonals, such as +, -, *, ÷, log etc. However, you must use a single mathematical operation for each proposed solution. Find as many solutions as possible.



Source: Fonseca (2015, p. 59)

The following items refer to problem formulation.

Table 6: Problem Formulation

Imagine that you are at school recess. There are some children running around, others jumping rope, others painting. There are also children buying snacks and other situations that you can imagine. Thinking about this scenario, create as many math problems as you can. Create really interesting problems. You will be given enough lines so that you can create as many problems as possible. Also try to create original problems, that is, think of problems that your classmates could not imagine.

Source: Carvalho (2015, p. 108)

Table 7: Problem Formulation

Ask questions like the example below, which require you to find two unknowns. Make sure that the information provided in your question is adequate to find the desired unknowns. Check the problem for accuracy. You do not need to write the answer to your question. If the space provided below is not sufficient, you can use the back of the paper. Example: Ali's age is three times Ahmet's age. The sum of their ages is 48. How old is each of them?

Source: Akgul e Kahveci (2016, p. 63)

Table 8: Problem Formulation

Create problems whose solutions follow the same arithmetic operations. Write as many problems as you can. First, decide on the solution, and then define the appropriate questions for the solution.

Source: Akgul e Kahveci (2016, p. 63)

Table 9: Problem Formulation

Develop three different questions that can be answered based on the following information: Paul, James and Anthony were returning home by car after a trip. Anthony drove 140 km more than James. James drove twice the distance traveled by Paul. Paul drove 90 km.

Source: Akgul e Kahveci (1996, p. 525, com adaptações para o contexto brasileiro)

Table 10: Problem Formulation

Consider the following situation: The car lease would cost him R$ 900.00 per month for 18 months and the car would have to undergo a service every 3,000 kilometers driven, at a rate of R$ 120.00 per service. Develop as many problems as you can using the information presented. It is not necessary to solve the problems that you have developed.

Source: Dunn (1975, p. 329, com adaptações para o contexto brasileiro)

Table 11: Problem Formulation

Write questions with 20 cents as the answer.

Source: Akgul e Kahveci (1975, p. 330)

The following are the items that require problem redefinition.

Table 12: Problem Redefinition

List all the things that could happen under the following mathematical situation: using a base-14 numbering system instead of our traditional base-10 numbering system.

Source: Balka (1974, p. 149)

Table 13: Problem Redefinition

Use the numbers below to create groups of four numbers that have common characteristics. Use your imagination to create the groups and explain why you put the numbers in the same group. Create as many groups as you can. 2 3 4 5 7 9 10 15 21 25 28 49

Source: Pitta-Pantazi, Sophocleous e Christou (2013, p. 213)

Table 14: Problem Redefinition

Write three problems that can be solved with the operation 51 ÷ 4 and that have three different correct solutions, equal to: a) $12\frac{3}{4}$ b) 13 c) 12

Source: Simon (1993, apud Pitta-Pantazzi *et al.*, 2012, p. 203).

Table 15: Problem Redefinition

Consider the integers from 2 to 16 (including 2 and 16) and write the various subsets that you can establish involving these numbers, indicating the rule for the formation of each of them, that is, indicating the characteristics that the numbers have and that make them able to be in the same subset.

Source: Gontijo (2007, p. 91)

Table 16: Problem Redefinition

The first item on the test asks the student to use the symbols +, -, ×, ÷¸, and ( ), if necessary, to write as many true equations as possible with three given numbers in a given order and an equal sign. This seems like an attempt to translate into mathematical terms the standard divergent thinking item, but in verbal form, "How many ways can you use a brick?" If the three given numbers were 2, 3, and 8, then possible answers include: a) $2^3 - 8 = 0$ b) $2 \times 3 + 8 = 14$ c) $\frac{2}{3} \div 8 = \frac{1}{12}$

Source: Dunn (1975, p. 328)

In Gontijo (2007) there are some examples of answers given by students who took the Creativity in Mathematics test developed by the author. The items found in the test (Gontijo, 2007) were presented in this article as item 3 of the Problem Solving category (Table 3), item 4 of the Problem Elaboration category (Table 9) and item 4 of the Problem Redefinition category (Table 15).

Item 3 (Table 3) presented 59 answers and, in it, different frequencies were reported for each of them. The most frequent answer was $\frac{4}{4} \cdot \sqrt{4} \cdot \sqrt{4} = 4$ —including variations that do not change the expression —, having appeared 64 times. Other common answers among students were:

- $\sqrt{4} + \sqrt{4} + 4 - 4 = 4$;
- $4 - \sqrt{4} + 4 - \sqrt{4} = 4$;
- $\frac{4 \cdot \sqrt{4}}{4} + \sqrt{4} = 4$.

On the other hand, 28 unique responses were obtained, such as:

- $\frac{\sqrt{4}^{\sqrt{4}}}{4} \cdot 4 = 4$;
- $\left(\frac{4!}{4!}\right)^4 \cdot 4 = 4$;
- $4^{\frac{\sqrt{4}}{4}} + \sqrt{4} = 4$;
- $\frac{4!}{4} - \left(\sqrt{\sqrt{4}} \cdot \sqrt{\sqrt{4}}\right) = 4$.

Based on these examples, it can be seen that the most frequent responses use the four basic operations and focus on using more trivial inverse elements, while the unique responses explore different numerical representations and their relationships, demonstrating greater creativity. It is worth mentioning that the variety of arithmetic operations that can be used can contribute to the development of procedural fluency highlighted by the NCTM (2023).

Therefore, in the school setting — specifically in the classroom environment — the use of this item can become a provocative element for the development of fluency, flexibility and originality of thought. This is because it encourages students to produce different responses, to use different strategies/categories to solve them and to formulate unusual responses when compared in each group.

Additionally, recognizing the difficulty that many students may initially have when in contact with the aforementioned item, teachers can make use of what Bezerra, Gontijo and Fonseca (2021, p. 94) called creative feedback, that is, feedback "intended to develop creative potential". One possibility is to create questions that encourage students to remain in constant cognitive movement, promoting their engagement in the proposed activities. An example of how this type of approach can be materialized was presented by Fonseca and Gontijo (2020).

As for item 4 of the Problem Elaboration (Table 9), this presented 72 responses and fewer repetitions in relation to the previous item. The most frequent response was *"How many km did Tiago drive?"*, followed by *"How many km did Antônio drive?"*. Presenting intermediate frequencies, responses such as *"Who of the three drove the most?"*, *"How many km did the three travel together?"* and *"How much more did Antônio drive than Paulo?"* were obtained.

Regarding the unique answers, a total of 52 were obtained and, as an example, it was observed: *"If Antônio returned twice as fast as Tiago, who arrived home first?", "Knowing that the average speeds of Paulo, Tiago and Antônio were, respectively, 45km/h, 60km/h and 120m/h, who arrived home first?", "What is the ratio between the distances of Antônio and Tiago?"* and *"If the car, before the trip, had 15,000 km driven, how much would it have after the trip?".*

It was noted that the most frequent answers were prepared in a simpler way, without many details. On the other hand, the unique answers were more specific and require more critical analysis.

Item 4 of Problem Redefinition presented 86 answers. The most frequent classifications involved even numbers (highest frequency), odd numbers, prime numbers, multiples and divisors of 3, multiples and divisors of 4, multiples and divisors of 5 and perfect squares. The lowest frequencies, however, brought less conventional classifications and which, at times, involved more than one process, such as: *"divided by 2 have an even result", "numbers written with 4 letters", "multiple of 3 and 5 simultaneously", "roots of the equation* $x^2 - 4x + 3 = 0$*"* and *"The next number is always the sum of the two previous ones".*

5 Summary of results

Based on the analysis of the items, it was found that the Problem Solving items can be adapted to the classroom through activities that require multiple solutions or a single solution achieved through several strategies. Furthermore, because it is a type of item that can be evaluated as right or wrong — even if it has different solutions — it can be applied in assessment or exhibition activities so that students can compare their results, stimulating and, simultaneously, evaluating their creativity.

In parallel, the Problem Elaboration items are adaptable to the students' reality and do not necessarily make judgments about right or wrong, which gives the category more freedom in its proposal and implementation for both the teacher and the students. The activities can be carried out outdoors or in spaces outside the classroom, so that students can explore objects or situations present in their daily lives that sometimes go unnoticed, identifying Mathematics in the real world. Furthermore, it can be related to the Problem Solving category, since, once the problems are ready, students can exchange them and solve the ones that were created by their classmates.

Finally, the Problem Redefinition items, which were the majority among those mapped, can be used in the classroom so that students can look at problems from different points of view, encouraging divergent thinking. These items can be used in activities that involve playful objects, since they facilitate the visualization of new perspectives; group activities, since they generate the discussion of ideas; or even in the presentation format, so that students can share their thoughts.

In short, the items do not need to be applied directly in the classroom, but they have the potential to inspire teachers to generate new activities, whether in written format or through dynamics. These items can even help teachers find inspiration to structure entire workshops, according to the model presented in Fonseca and Gontijo (2021).

6 Final considerations

The aforementioned items can be used in classroom practices, in activities that can guide students into their own daily lives. An example of this is item 1 of the Problem-Solving section, in which students, instead of just imagining themselves in the school playground, can actually be taken outside the classroom to explore the school spaces and create problems based on their reality.

In addition, it gives teachers freedom to use different types of activities according to what the school proposes. These activities can range from simple proposals, which only require individual reflections or group discussions, to more complex activities, which involve the preparation of work for presentation or which require written assessments.

One of the limitations identified in the research was the small number of items found, since many authors were inspired by previous works when developing their tests. As a result, even though they have an extensive personal archive of works that include Creativity in Mathematics tests, a recurrence of similar items was observed, with specific adaptations or, in some cases, identical.

In addition, a shortage of Algebra items was noted when compared to Arithmetic and Geometry items, which limited observations around this thematic area — considered essential for the construction of algebraic thinking. Regarding the item categories, few Problem Solving items were identified when compared to Problem Elaboration and Redefinition. This occurs because Elaboration and Redefinition are categories that offer more freedom to teachers in their proposals and to students in their responses.

In this context, future research can be undertaken regarding the effective use of these items in the classroom. The development of new items, the adaptation of existing ones and their selection in a balanced quantity allow for implementation in Basic Education classes through activities integrated into the students' routine. With this, it becomes possible to observe the results obtained and analyze how the development of creative potential can contribute to learning in Mathematics.

Considering that creativity is gradually beginning to be incorporated into the school environment, this type of research on item mapping and analysis of its use is relevant to present this skill. In addition, they exemplify how creativity can be applied based on items from tests published by researchers in the field. This type of study also offers valuable guidance for educators seeking to implement Creativity in the context of Mathematics, which is still considered an uninteresting subject when taught in an obsolete way and in contexts of little use in the reality of the 21st century (D'Ambrosio, 2005).

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