10.46551/emd.v9n17a15 ISSN 2526-6136

A creative geometric investigation in Middle School: the case of quadrilaterals

Abstract: The research aimed to analyze the mathematical creativity of students in Middle School in the production of different quadrilaterals in 3×3 grids, considering the fluency, flexibility and originality of the answers. Characterized as a qualitative approach research, it was observed that, in the answers of 12 students, although some showed the ability to generate varied solutions, there were difficulties in recognizing and using geometric transformations, such as rotations, reflections and translations. These results reveal the need for methodologies that encourage creativity and geometric thinking, for a deeper and more versatile learning. The research contributes to the debate on creativity in school Mathematics, fostering pedagogical practices that promote autonomy and critical thinking in the development of geometric skills and problem solving.

Keywords: Geometry. Quadrilaterals. Creativity. Middle School.

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Received • 30/10/2024
Accepted • 11/04/2025
Published • 27/05/2025

Article

Una investigación geométrica creativa en Secundaría: el caso de los cuadriláteros

Resumen: La investigación tuvo como objetivo analizar la Creatividad Matemática de estudiantes de los últimos años de la Educación Secundaría en la producción de diferentes cuadriláteros en cuadrículas de 3×3, considerando la fluidez, flexibilidad y originalidad de las respuestas. Caracterizada como una investigación de enfoque cualitativo, se observó que, en las respuestas de 12 estudiantes, si bien algunos muestran capacidad para generar soluciones variadas, existen dificultades para reconocer y emplear transformaciones geométricas, como rotaciones, reflexiones y traslaciones. Estos resultados revelan la necesidad de metodologías que fomenten la creatividad y el pensamiento geométrico, para un aprendizaje más profundo y versátil. La investigación contribuye al debate sobre la creatividad en la Matemática escolar, fomentando prácticas pedagógicas que promuevan la autonomía y el pensamiento crítico en el desarrollo de habilidades geométricas y la resolución de problemas.

Palabras clave: Geometría. Cuadriláteros. Creatividad. Años Finais. Enseñanza Fundamental.

Uma investigação geométrica criativa nos Anos Finais do Ensino Fundamental: o caso dos quadriláteros

Resumo: A pesquisa objetivou analisar a Criatividade Matemática de estudantes dos Anos Finais do Ensino Fundamental na produção de diferentes quadriláteros em malhas 3×3, considerando a fluência, a flexibilidade e a originalidade das respostas. Caracterizada como uma pesquisa de abordagem qualitativa, observou-se que, nas respostas de 12 estudantes, embora alguns mostrem capacidade de gerar soluções variadas, há dificuldades em reconhecer e empregar transformações geométricas, como rotações, reflexões e translações. Esses resultados revelam a necessidade de metodologias que incentivem a criatividade e o pensamento geométrico, para um aprendizado mais profundo e versátil. A pesquisa contribui para o debate sobre criatividade na Matemática escolar, fomentando práticas pedagógicas que promovam autonomia e pensamento crítico no desenvolvimento de habilidades geométricas e resolução de problemas.

Palavras-chave: Geometria. Quadriláteros. Criatividade. Anos Finais. Ensino Fundamental.

1 Introduction[1]

According to the *Base Nacional Comum Curricular* [National Common Curriculum Base — BNCC], "Geometry involves the study of a broad set of concepts and procedures necessary to solve problems in the physical world and in different areas of knowledge" (Brasil, 2017, p. 271). The *Parâmetros Curriculares Nacional* [National Curriculum Parameters — PCN] (Brasil, 1998) mention that it is a topic that students tend to be naturally interested in.

The teaching of geometric concepts in schools has not always received the attention it deserves. Classical works in Mathematics Education, such as Pavanello (1993) and Lorenzato (1995), have highlighted the neglect of Geometry since the 1990s. However, this scenario seems to be changing. In the introduction to the book *Laboratório de Ensino de Geometria* [Geometry Teaching Laboratory], Lorenzato (2012) highlights the resurgence of Geometry teaching and mentions the importance of experimentation through images or manipulable materials. Lorenzato (1995) highlights the importance of geometric concepts and states that

> without studying Geometry, people do not develop geometric thinking or visual reasoning, and without this ability, they will hardly be able to solve life situations that are geometrized; they will also not be able to use Geometry as a highly facilitating factor for understanding and resolving issues in other areas of human knowledge. Without knowing Geometry, the interpretative reading of the world becomes incomplete, the communication of ideas is reduced and the vision of Mathematics becomes distorted (p. 5).

The teaching of Geometry and Mathematics can be enhanced by being associated with the stimulation of creativity, giving students the possibility of thinking and making conjectures beyond pre-established algorithms. Boden (2004) conceptualizes *creativity* as the ability to produce ideas or artifacts that simultaneously present the characteristics of novelty, surprise and usefulness. The author, however, problematizes the criterion of originality by establishing an important theoretical distinction: historical creativity, which refers to genuinely unprecedented contributions in the context of human development, in contrast to psychological creativity, which concerns original cognitive processes for the individual in particular, regardless of their historical precedence.

According to Feldman, Csikszentmihalyi and Gardner (1994), *creativity* is a multifaceted concept, present in different contexts and interpreted in different ways. Currently, there is a growing appreciation of this skill in different fields of knowledge and social strata, with a strong call for the training of individuals capable of thinking innovatively and acting entrepreneurially in their areas of activity. This demand reflects the need to adapt to a world in constant transformation, in which the ability to create and reinvent becomes essential for individual and collective progress. According to Silver (1997),

> A new view of creativity has emerged from contemporary research—one that stands in sharp contrast to the genius view. This research suggests that creativity is closely related to deep and flexible knowledge in specific domains; it is often associated with long periods of work and reflection rather than with rapid and exceptional insight; and it is susceptible to instructional and experimental influences. The contemporary view of creativity also suggests that people who are creative in a domain demonstrate a creative disposition or orientation to their activity in that domain. That is, creative

[1] This article is part of an investigation prepared, developed and analyzed by the Geometry Study and Research Group (GEPGEO), associated with the Franciscan University (UFN).

> activity results from an inclination to think and behave creatively. This new view of creativity provides a much stronger foundation for constructing educational applications. Indeed, it suggests that a creativity-rich education should be appropriate for a broad range of students, not merely for a few exceptional individuals (p. 75-76).

For Gontijo (2006, p. 230), "pedagogical work that aims to promote creativity in Mathematics helps to overcome the anxiety involved in learning it, in addition to breaking down barriers that prevent success in this area". In this line, according to the Investment Theory in Creativity, defined by Sternberg (2006), creativity emerges from the dynamic interaction between six elements: intellectual ability, knowledge, thinking styles, personality, motivation and environment. Therefore, it is understood that creativity can be developed in the educational environment:

> Creativity, according to investment theory, is largely a decision. The decision-making view of creativity suggests that creativity can be developed [...] Creativity is as much a decision about and attitude toward life as it is a matter of ability. Creativity is often obvious in young children, but it may be difficult to find in older children or adults because their creative potential has been suppressed by a society that encourages intellectual conformity [...] We can teach students to think more creatively [...] Motivating this work is the belief that systems in many schools tend to favor children with potential in memory and analytical skills (Sternberg, 2006, pp. 90-93).

In the context of Mathematics Education, the predominant approach in studies on creativity conceives it as a capacity for divergent thinking, characterized by the production of multiple potential solutions to a given problem. This conception, initially proposed by Guilford (1956) and endorsed by Torrance (1977), Silver (1997), Leikin (2009), among others, is structured in four dimensions: fluency (number of ideas produced), flexibility (variety of categories or approaches), originality (degree of novelty of the answers) and elaboration (level of detail of the solutions). However, research in the field of Mathematics, in general, focuses only on the first three spheres.

Although it may seem subjective, the literature indicates ways of analyzing creativity in mathematical tasks. The assessment of Mathematical Creativity is made possible through open problem situations, with more than one solution. For Gontijo (2011, p. 3), "problem solving, problem formulation and redefinition are didactic-methodological strategies that enable the development and analysis of Mathematical Creativity". In this regard, it is possible to foster Mathematical Creativity through multiple solution tasks (Leikin et al, 2006; Leikin and Levav-Waynberg, 2007), so that this type of activity allows creativity to be assessed based on criteria such as fluency, flexibility and originality. These strategies are based on the theoretical premise that activities involving multiple resolution approaches enhance both the construction of mathematical knowledge and the development of creative thinking in Mathematics (Ervynck, 1991; Silver, 1997).

2 Quadrilaterals in the Base Nacional Comum Curricular

Given the above, considering the importance of geometric concepts — specifically, quadrilaterals — as well as the development of creativity, this research is justified, whose objective is to analyze the Mathematical Creativity of students in the Middle School through the production of different quadrilaterals in 333 grids, considering the fluency, flexibility and originality of the answers.

The concept of quadrilateral is already addressed from the Elementary and Middle School. Table 1 shows the objects of knowledge and skills of the BNCC related more specifically to this concept, with a view to presenting an overview of Elementary School.

Table 1: Quadrilaterals in the objects of knowledge and skills of the BNCC — Elementary School

Grade	Knowledge Objects	Skills
1st	Plane geometric figures: recognition of the shape of the faces of spatial geometric figures	(EF01MA14) Identify and name plane figures (circle, *square, rectangle* and triangle) in drawings presented in different arrangements or in the contours of faces of geometric solids.
2nd	Plane geometric figures (circle, *square, rectangle* and triangle): recognition and characteristics	(EF02MA15) Recognize, compare and name plane figures (circle, *square, rectangle* and triangle), through common characteristics, in drawings presented in different arrangements or in geometric solids.
3rd	Plane geometric figures (triangle, *square, rectangle, trapezoid* and *parallelogram*): recognition and analysis of characteristics	(EF03MA15) Classify and compare plane figures (triangle, *square, rectangle, trapezoid* and *parallelogram*) in relation to their sides (quantity, relative positions and length) and vertices.
	Congruence of plane geometric figures	((EF03MA16) Recognize congruent figures, using overlapping and drawings on *square or triangular grids*, including the use of digital technologies.
4th	Reflection symmetry	(EF04MA19) Recognize reflection symmetry in figures and in pairs of plane geometric figures and use it to construct congruent figures, using *grid meshes* and geometry software.
5th	Plane geometric figures: characteristics, representations and angles	(EF05MA17) Recognize, name and compare *polygons*, considering sides, vertices and angles, and draw them, using drawing material or digital technologies.
	Enlargement and reduction of *polygonal figures* in grid meshes: recognition of the congruence of angles and the proportionality of corresponding sides	(EF05MA18) Recognize the congruence of angles and the proportionality between the corresponding sides of *polygonal figures* in situations of enlargement and reduction in *grid meshes* and using digital technologies.

Source: Own elaboration based on the BNCC (Brasil, 2017, our emphasis)

Based on the objects of knowledge and skills identified in Elementary School, it is observed that the study of flat geometric figures is already explored in the 1st year with the relationship between the faces of geometric solids. From the 2nd grade onwards, square and rectangular figures are presented. From the 3rd grade onwards, the trapezoid and the parallelogram also come into focus. Otherwise, the most common quadrilaterals are addressed in the Elementary School. In the 5th grade, there is the study of polygonal figures, in which the notion of *quadrilateral* can be explored, although the term quadrilateral is not mentioned in either the objects of knowledge or the skills.

It is interesting to highlight the evidence of relating flat geometric figures to spatial geometric figures, as well as the use of different resources such as square and triangular grids, digital technologies and drawing material.

The term *quadrilateral* is initially mentioned in the 6th grade of Middle School, then in the 8th grade, as shown in Table 2.

Table 2: Quadrilaterals in the objects of knowledge and skills of the BNCC — Middle School

Grade	Knowledge Objects	Skills
6th	*Polygons:* classifications according to the number of vertices, the measurements of sides and angles and the parallelism and perpendicularity of the sides	(EF06MA18) Recognize, name and compare *polygons*, considering sides, vertices and angles, and classify them as *regular and non-regular*, both in their representations on the plane and on the faces of polyhedra.
		(EF06MA20) Identify characteristics of *quadrilaterals*, classify them in relation to sides and angles and recognize the inclusion and intersection of classes between them.
	Construction of similar figures: enlargement and reduction of flat figures in *grids*	(EF06MA21) Construct similar *flat figures* in situations of enlargement and reduction, using *grids*, Cartesian planes or digital technologies.
7th	Translation, rotation and reflection symmetries	(EF07MA21) Recognize and construct *figures* obtained by translation, rotation and reflection symmetries, using drawing instruments or dynamic geometry software and link this study to flat representations of works of art, architectural elements, among others.
	Regular polygons: square and equilateral triangle	(EF07MA27) Calculate measurements of internal angles of *regular polygons*, without using formulas, and establish relationships between internal and external angles of polygons, preferably linked to the construction of mosaics and tiling.
8th	Congruence of triangles and demonstrations of properties of *quadrilaterals*	(EF08MA14) Demonstrate properties of *quadrilaterals* by identifying the congruence of triangles.
	Geometric constructions: angles of 90°, 60°, 45° and 30° and *regular polygons*	(EF08MA15) Construct, using drawing instruments or dynamic geometry software, medians, bisectors, angles of 90°, 60°, 45° and 30° and *regular polygons*.
	Geometric transformations: translation, reflection and rotation symmetries	(EF08MA18) Recognize and *construct* figures obtained by compositions of geometric transformations (translation, reflection and rotation), using drawing instruments or dynamic geometry software.
9th	*Regular polygons*	(EF09MA15) Describe, in writing and through a flowchart, an algorithm for constructing a *regular polygon* whose side measurement is known, using a ruler and compass, as well as software.

Source: Own elaboration based on the BNCC (Brasil, 2017, our emphasis)

Although Table 2 identifies only two skills that mention quadrilaterals, there are other skills in which this concept can be explored, such as those that mention polygons or flat figures.

As in the Elementary School, in Middle School, relationships are made with the faces of polyhedrons, as well as the use of different resources such as grids, Cartesian planes, digital technologies, drawing instruments, relationships with works of art, architectural elements, mosaics, tiling, dynamic geometry software, rulers and compasses, among others.

Another aspect that can be observed in the skills refers to the relationships between figural registers and natural language. According to Duval (2009), in Geometry, these two types of registers are fundamental and need to be mobilized simultaneously in an interactive manner. For example, the skills mention *recognizing* and *naming*, that is, a conversion from natural language to figural and vice versa.

It is worth noting that one of the general competencies of Basic Education — competencies that are interrelated and unfold in the didactic treatment proposed for the three stages of Education — emphasizes the importance of exercising intellectual curiosity, mentioning, among other examples, creativity:

> Exercise intellectual curiosity and use the approach specific to science, including research, reflection, critical analysis, imagination and *creativity*, to investigate causes, develop and test hypotheses, formulate and solve problems and create solutions (including technological ones) based on knowledge from different areas (Brasil, 2017, p. 9, our emphasis).

These aspects related to creativity were already mentioned previously in the PCN, both for the Elementary and Middle School (Brasil, 1997, 1998). The documents indicate that work with Mathematics should enable students to be able to

> questioning reality by formulating problems and trying to solve them, using logical thinking, *creativity*, intuition, critical analysis skills, selecting procedures and checking their suitability (Brasil, 1997, p. 7, our emphasis).
>
> The teaching of Mathematics will make its contribution as methodologies are explored that prioritize the creation of strategies, verification, justification, argumentation, critical thinking, and favor *creativity*, collective work, personal initiative and the autonomy of developing confidence in one's own ability to know and face challenges (Brasil, 1997, p. 31, our emphasis).

Thus, this work addresses the study of quadrilaterals, seeking to favor the development of creativity, considering the use of 3×3 dotted grids and the possibility of multiple solutions for the same task, in order to analyze how Mathematical Creativity can be perceived in the answers produced by the participants.

3 Creativity in Mathematics classes

There is no consensus on what Mathematical Creativity is, so according to Mann (2005), there are more than one hundred definitions on the subject. Thus, Mathematical Creativity can also be defined "as the process of understanding problems or gaps in information, forming hypotheses, testing and modifying these hypotheses, and communicating the results. This process can lead to any of many types of products — verbal and nonverbal, concrete and abstract" (Torrence, 1977, p. 6).

Among the main criteria for evaluating creative solutions in problem-solving in Mathematics, the most common in research in the area focus on fluency, flexibility, and originality (Guilford, 1956; Torrence, 1977; Silver, 1997; Leikin, 2009, 2013; Vale, Pimentel, and Barbosa, 2018).

Fluency refers to the ability to generate several different solutions to the same task. This skill can be developed by seeking out as many different ideas as possible. Ideas are often associated with each other, and the more someone dedicates themselves to a topic, the more fluent they become. According to Vale, Pimentel and Barbosa (2018), fluency is essential, as the first step to solving problems or creating something creative is having a wide range of ideas to choose from.

Vale, Pimentel and Barbosa (2018) highlight that teaching practices commonly encourage students to seek only one correct answer, instead of exploring multiple possibilities. This practice limits those who do not feel the need to propose more than one solution and restricts the development of their creativity. According to Silver (1997), the use of open-ended or less restricted tasks in their resolution can encourage students to think of different solutions, stimulating fluency and, consequently, creativity.

Even though not all ideas generated are necessarily relevant, this process is important because it is related to flexibility. As Silver (1997, p. 77) states, “students can not only become fluent in generating multiple problems from a given situation, but they can also develop creative flexibility by generating multiple solutions to a given problem”.

Flexibility is the ability to think in different ways to generate different perspectives on the same problem, and is fundamental in problem-solving from a creative perspective (Vale, Pimentel and Barbosa, 2018). Thinking flexibly allows the problem to be analyzed from different angles, thus facilitating the creation of connections between different areas of knowledge and the subject's previous concepts.

For Silver (1997), flexibility in problem-solving and formulation is revealed through the different ways that students use to solve, express or explain a problem. In other words, solutions can be categorized based on the different processes or content used to achieve them.

Originality is the ability to think beyond the obvious, in an unusual way, so that the ideas developed are generally new and unique in problem-solving (Silver, 1997). Among the skills described, this is the most difficult to develop, but it can be improved through tasks that stimulate creativity.

Based on Vale, Pimentel and Barbosa (2018), it can be said that, although originality, by definition, involves the creation of new ideas, schemes and solutions, it can be evaluated relatively among a group of subjects. In other words, a student who presents a creative solution that is unique in relation to the group in which he or she is inserted can be considered as someone who has developed originality. According to Leikin (2013), creativity is seen as the process of developing unique and original ideas and, therefore, is considered by many to be the main factor of creativity.

Mathematical Creativity is one of the main characteristics of Advanced Mathematical Thinking, manifesting itself in the ability to establish mathematical objectives and identify the intrinsic relationships between them (Ervynck, 1991). Therefore, Mathematics teaching should provide students with opportunities to develop their creativity through interactive activities that stimulate open-ended thinking.

Leikin (2013) reinforces the importance of Mathematical Creativity by stating that

> in a rapidly changing world, in which technological and scientific advances change social networks and the lives of individuals, creativity is needed both to adapt to this changing world and to continue these advances. Mathematical creativity is a specific type of creativity whose importance is obvious. On the one hand, the advances in different branches of mathematics, which research mathematicians bring to life, reflect the human intellect. On the other hand, mathematics is one of the central scientific areas that allows sustaining social scientific and technological progress in a variety of areas, offering scientists and high-tech specialists a powerful apparatus and models for the analysis of situations, prognoses and processes (p. 386).

Given the relevance of Mathematical Creativity in the development of human reasoning, it is important to incorporate educational practices that can favor the creation of creative solutions for different types of tasks. Given the understanding of the concepts of fluency, flexibility and originality, it is possible to develop and evaluate pedagogical practices that can promote Mathematical Creativity in students, according to the objective of this research.

4 Methodological and development aspects of the task

This research adopts principles of a qualitative approach, since, according to Garnica (2020), some of the characteristics of this type of research are

> (a) the transience of its results; (b) the impossibility of an a priori hypothesis, whose objective of the research will be to prove or refute; (c) the non-neutrality of the researcher who, in the interpretative process, uses his/her perspectives and previous experiential filters from which he/she cannot free himself/herself; (d) that the constitution of his/her understandings occurs not as a result, but in a trajectory in which these same understandings and also the means of obtaining them can be (re)configured; and (e) the impossibility of establishing regulations, in systematic, previous, static and generalist procedures (p. 96).

Furthermore, the methodological procedures used included a case study, as this is an empirical investigation that seeks to examine a phenomenon in its real-life context in depth, to encompass important contextual conditions that were not clearly evident (Yin, 2010). The author also highlights that the case study is appropriate in three conditions: (1) when the research questions are of the *how* or *why* type; (2) when it concerns contemporary social phenomena; and (3) when there is no need to control the behavioral events of the subjects involved in the investigation.

Thus, to produce the data, the responses of 12 students from the 6th and 9th grades of Middle School from a municipal public school in the central region of Rio Grande do Sul, Brazil, were considered. These subjects were considered because the activity includes the concept of quadrilateral, which should be addressed throughout the Middle School, as established by the BNCC. The understandings of Borba, Almeida and Gracias (2018), supported by Confrey (1998), who defend the importance of listening to the voice of students, understanding their reasoning and their mathematical elaborations, were also considered.

The task developed with the students was adapted from Millington (2008) and reorganized based on discussions in a research group by the authors of this article and other participants in the group. The activity protocol initially presents examples of quadrilaterals in a 2×2 grid and in a 3×3 grid, as well as some explanations and examples of the representations that are considered and those that are disregarded, that is, quadrilaterals that are translations, rotations or reflections. After these explanations, the students are asked: "Create different quadrilaterals in the 3×3 grid and justify why they are different".

To complete the task, the students had a total of one hour and were able to use the ruler to make their constructions. However, before starting the task, the teacher gave a brief verbal explanation of what was required.

It should be noted that this task is an activity in which it is necessary to redefine the problem, that is, to take it and turn it upside down (Sternberg and Grigorenko, 2003). For Gontijo (2006), redefining a problem generates multiple possibilities for representing a situation. There are 16 possible solutions, organized into six categories, as illustrated in Figure 1.

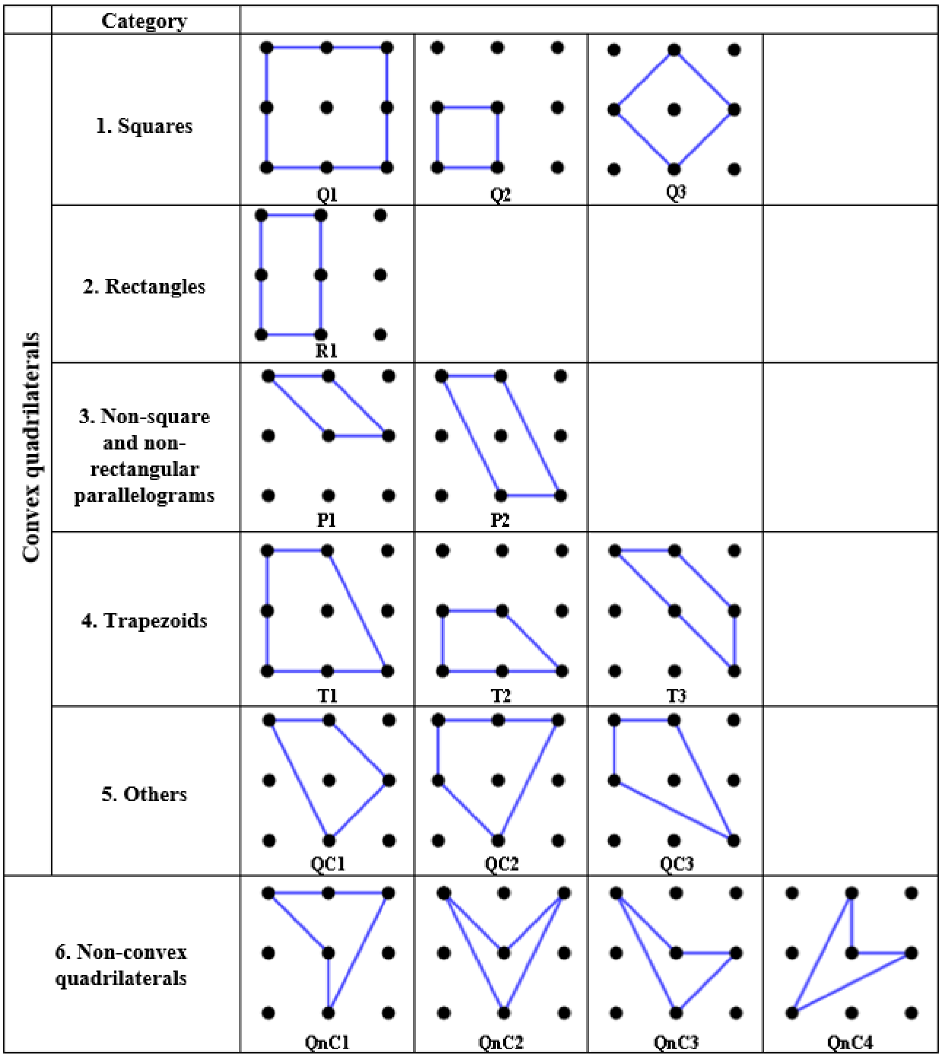

Figure 1: Possible solutions organized into categories (Own elaboration based on solutions presented in Millington, 2008)

The analysis of the responses is carried out as follows: initially, the solutions presented by each student will be quantified and the total number of distinct solutions quantified. From this, the first analysis criterion is listed: *C1: Does the student identify rotations, translations or reflections of the same figure?*. Then, considering the distinct solutions of each student, the number and which quadrilaterals were produced are identified, in addition to their respective category. In this way, the aim is to verify the presence or absence of evidence related to the second criterion: *C2: Does the student show evidence of being creative or not?*.

In total, 12 students attempted to represent the 16 possible solutions of the proposed task. To measure the creativity index, the Leikin model (2009) was taken into account, which makes it possible to evaluate Mathematical Creativity in a numerical value that reflects how developed the fluency, flexibility and originality of these students are based on their responses. As discussed previously, for Silver (1997), fluency is developed by generating multiple ideas and responses to a problem, exploring different possibilities. Flexibility is enhanced by proposing new solutions after creating at least one initial alternative. Originality is stimulated by seeking multiple solutions and creating an innovative idea. In numerical terms, these components can be measured as follows.

Fluency (Flu or *n)* is often assessed by the number of appropriate ways generated to solve a problem, reflecting both the pace of solution and the transitions between different approaches. In a written test, a student's fluency is identified by the number of appropriate solutions in his or her individual solution space (Leikin, 2013). In this research, students' fluency will be measured by the total number of quadrilaterals submitted as solutions to the proposed task.

Ao avaliar a *flexibilidade (Flx)*, consideraram-se diferentes grupos de abordagens para resolver uma situação problema. Duas soluções são classificadas em grupos distintos quando utilizam estratégias baseadas em representações, propriedades — teoremas, definições ou construções auxiliares — ou ramos da Matemática diferentes (Leikin, 2013). No caso da presente pesquisa, esses grupos distintos são os tipos de quadriláteros possíveis, como quadrados, retângulos, paralelogramos, trapézios, outros quadriláteros convexos e, por fim, quadriláteros não convexos.

Leikin (2013) suggests using a decimal base to assess a student's flexibility through their responses: $Flx_i = 10^1 = 10$ for the first appropriate solution. For each consecutive solution, there are different scores:

- $Flx_i = 10^1 = 10$, if the solution belongs to a group of solutions different from those to which the previous solution(s) belongs — category of quadrilaterals;
- $Flx_i = 10^0 = 1$, if the solution belongs to one of the groups previously used, but presents a clear minor distinction — another quadrilateral of the same category;
- $Flx_i = 10^{-1} = 0{,}1$, if the solution is almost identical to one of the solutions already presented — reflection, rotation or translation of the same quadrilateral.

A student's total flexibility score on a problem is the sum of his or her flexibility across the solutions present in his or her individual solution space, given by the equation $Flx = \sum_{i=1}^{n} Flx_i$, where *n* is the fluency score.

According to Leikin (2013), the proposed decimal basis for scoring flexibility reflects both the product and the problem-solving process. For example,

> if the total flexibility score for a solution space is 31.2, we know that it includes 3 solutions that belong to different solution groups, 1 solution that uses a solution strategy from one of the previous groups exhibiting a small but essential difference, and 2 solutions that repeat the previous ones (Leikin, 2013, p. 392).

To quantify *originality (Or)*, a relative assessment was used, based on the convention of a solution from a specific group of students with a similar educational background. To do this, the individual solution spaces of each participant were compared with the collective solution space of the reference group, calculating the percentage *(P)* of students who produced a given solution (Leikin, 2013). The following criteria were used to define the values in decimal form:

- $Or_i = 10^1 = 10$ for an unconventional solution. This type of solution is usually produced by less than 15% of the students in a given group (P < 15%);
- $Or_i = 10^0 = 1$ for model-based solutions or solutions that imply a learned solution strategy. The frequency of this type of response is usually between 15% and 40% (15% ≤ P < 40%);
- $Or_i = 10^{-1} = 0{,}1$ for conventional or algorithmic solutions. This type of solution is usually presented by more than 40% of the students (P ≥ 40%).

A student's total originality score, based on his/her answers, is given by the sum of the values of each valid solution presented (fluency value n), according to the equation $Or = \sum_{i=1}^{n} Or_i$.

Leikin (2013) explains this *P*-value range by providing an example of how to interpret an individual's originality score:

> A total originality score of 21.3 means that the evaluated solution space includes 2 perception-based/unconventional solutions, 1 solution that is partially unconventional, and 3 algorithmic solutions. The decision about the 15% and 40% thresholds for the different levels of originality was based on previous experiments. We also compared the results of written tests with students' performance in individual interviews and in class discussions. We found that, in the written tests, these percentages (15% and 40%) correspond quite accurately to the different levels of originality of the solutions produced and presented both during the interviews and in the class discussions (Leikin, 2013, p. 392-393).

Finally, *creativity (Cr)* can be calculated from these values, being given by the sum of the products of the flexibility and originality values of each solution presented by the student, as defined by Leikin (2013) in the equation $Cr = \sum_{i=1}^{n} Flx_i \times Or_i$.

Based on these values, it was possible to quantify the creativity index of the participants from the quadrilaterals formed in the 3×3 grid, aiming to assess the degree of familiarity of the students with the classifications of quadrilaterals and their possible transformations, such as reflections and rotations. These skills are expected to be developed in Elementary School, according to the BNCC (Brasil, 2017).

5 Results and discussions

The 12 participants, identified by the letters A to L, received 20 grids to present the 16 different possible solutions to the task: representing quadrilaterals in 3×3 grids. Table 3 shows the distribution of responses among the students.

Table 3: Presentation of data, according to solutions presented by students

Student	Quantity of quadrilateral	Quantity of distinct quadrilaterals	Solutions presented															
			Q1	Q2	Q3	R1	P1	P2	T1	T2	T3	QC1	QC2	QC3	QnC1	QnC2	QnC3	QnC4
A	20	13	X	X	X	X	X		X	X	X	X	X		X	X	X	
B	9	6	X	X	X	X			X	X								
C	6	6	X	X	X	X				X		X						
D	19	13	X	X	X	X	X		X	X	X	X	X		X	X	X	
E	9	9	X	X	X	X			X	X	X	X	X					
F	9	9	X	X	X	X	X		X	X			X		X			
G	15	15	X	X	X	X	X	X	X	X	X	X	X	X	X		X	X
H	13	13	X	X	X	X	X	X	X	X	X	X	X	X		X		
I	14	9	X	X	X	X			X	X	X		X	X				
J	18	10	X	X	X	X	X		X	X			X		X	X		
K	18	14	X	X	X	X	X	X	X	X	X	X	X			X	X	X
L	12	9	X	X	X	X			X	X		X	X	X				

Source: Research data

Only five students (C, E, F, G and H) followed the instructions to present only distinct quadrilaterals as answers to the task. This is an indication that the others did not pay attention to the support material or were unable to identify that some of their answers were the same after being rotated and/or reflected and/or translated. This is possible evidence of a group deficit in relation to skill EF08MA18, which is related to recognizing figures from geometric transformations (Brasil, 2017).

Since the participants are students in the 6th and 9th grades of Middle School, there is a certain deficit in skills that should have already been developed in the Elementary School (1st to 5th grade), as provided for by the BNCC, especially in skills EF03MA16, EF04MA19 and EF05MA18, linked to the recognition of similarities and differences between polygons.

Among the responses, it was noted that quadrilaterals QnC4, P2, QC3 and QnC3 — considered more original — were the least frequent in the students' responses, with P values of 16.7%; 25% and 33.3%, respectively. Meanwhile, quadrilaterals Q1, Q2, Q3, R1 and T3 appeared in the responses of all students (P = 100%). This result was expected, given that some of them were present in the support material, which possibly influenced the students to present them as solutions. Although this choice is not wrong, it does not effectively contribute to the assessment of students’ creativity.

However, quadrilateral QC1 was also in the material, but four of the twelve students did not represent it as a solution. This fact is another indication that the students were not paying as much attention to the initial explanations and the support material, revealing that attention, reading and interpretation can hinder the development of mathematical reasoning.

Based on the analysis of the responses of the 12 participating students, it was possible to measure the fluency, flexibility and originality indexes, in order to calculate the creativity index, using the equations presented by Leikin (2013), as illustrated in Table 4.

Table 4: Student indices

Student	Fluency *(Flu)*	Flexibility *(Flx)*	Originality *(Or)*	Creativity *(Cr)*
A	13,00	67,70	1,30	8,77
B	6,00	33,30	0,60	3,33
C	6,00	42,00	0,60	5,20
D	13,00	67,60	1,30	7,76
E	9,00	45,00	0,90	5,50
F	9,00	63,00	0,90	6,30
G	15,00	69,00	4,20	10,60
H	13,00	67,00	3,10	9,50
I	9,00	45,50	1,80	4,55
J	10,00	64,80	1,80	6,48
K	14,00	68,40	3,20	9,64
L	9,00	45,30	1,80	6,43
Média	**10,50**	**56,55**	**1,79**	**7,01**

Source: Research data

In this table, it can be seen that only five (A, D, G, H and K) of the twelve students presented fluency rates above average. For this value, only the responses of single quadrilaterals were considered, discarding repetitions through reflections or rotations. Student G was the one who presented the most valid responses, lacking only one (QnC2) for the total of 16, while students B and C (Figure 2) presented only 6.

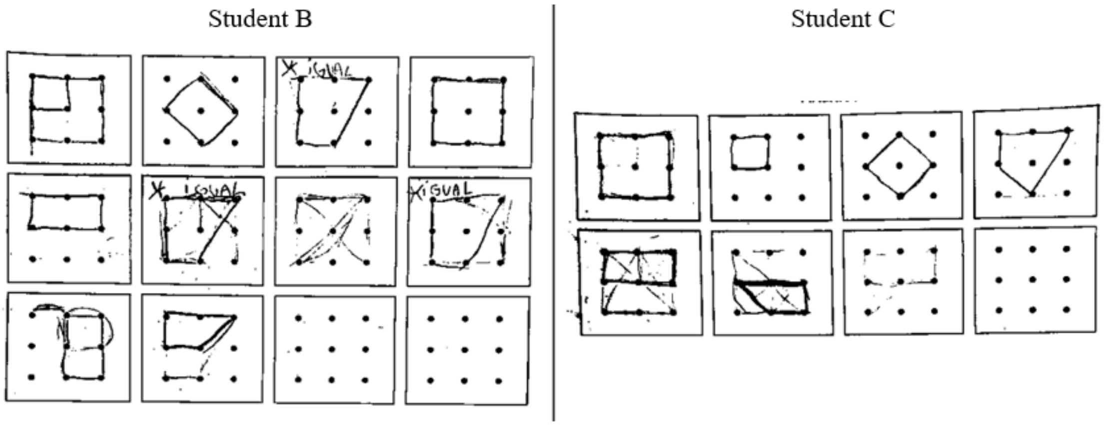

Figure 2: Presentation of data, according to solutions presented by students B and C, respectively (Research data)

Another aspect observed refers to the difficulty in handling the ruler for the constructions. Even though they were asked to use it to make the constructions, student B did not do so. Student C tried to use it, as can be seen in some of the constructions, but was still unsuccessful.

Using the example in Figure 2, it can be seen that Student B presented T1 three times as a response to the task and R1 twice, repeating it by means of a rotation. The act of using the solution more than once without realizing it has a direct impact on the individual's flexibility, being counted as 0.1 for each repetition.

According to Leikin (2013, p. 392), “a score of 0.1, which is a negative power (-1) of 10, reflects the students' lack of critical thinking, which is essential for mental flexibility, and the inability to recognize the two solutions produced as being identical”.

In terms of flexibility, it was noted that Student B, despite presenting the lowest *Flx* value among his colleagues, did not repeat his solutions through transformations, while Student J (*Flx* = 64.80), of his 18 valid solutions (quadrilaterals), eight are repetitions with rotations, translations and reflections. However, the latter covered the six defined quadrilateral categories, with at least one solution of each (Figure 3), while Student B covered only three of them (Figure 2).

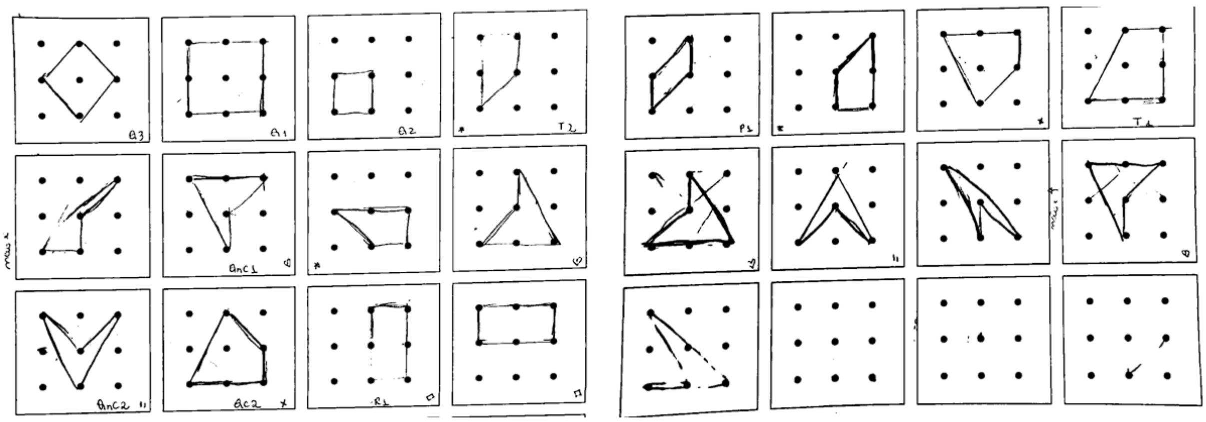

Figure 3: Presentation of data, according to solutions presented by students J (Research data)

Regarding originality, students G, K and H stand out, with *Or* values = 4.20; 3.20 and 3.10, respectively. These are the participants who presented the most original answers. This high index is good from an individual point of view, showing that these students thought of different solutions from the others. However, when looking at the group, it is observed that the other students presented common solutions, especially students B, C, E and F.

Similarly, these students stand out positively (G, K and H) and negatively (B, C, E and F) in relation to the creativity index, given the strong influence that originality has. Based on the creativity values *(Cr)* achieved, it is possible to see how much the students understood the task, as well as their performance in Mathematics. Leikin (2013) states that

> we hypothesize that in the fluency-flexibility-originality triad, fluency and flexibility are dynamic in nature, while originality is a "gift". We demonstrate that originality appears to be the strongest component in determining creativity. The strength of the relationship between creativity and originality can be seen as validating our model, being consistent with the view of creativity as the invention of new products or procedures. At the same time, our studies demonstrate that this view is true for both absolute and relative creativity. Based on the research findings, we hypothesize that one way to identify students with talent in mathematics is through the originality of their ideas and solutions (p. 396).

After constructing the quadrilaterals, students were asked about the number of quadrilaterals and why they were different: *After creating, indicate the number of solutions found, justifying why the quadrilaterals you obtained are different from each other*. Table 5 presents the answers obtained.

Table 5: Presentation of data, according to solutions presented by students

Student	Quantity of quadrilaterals	Quantity of distinct quadrilaterals	Response description
A	20	13	I made 20, they are different because some are in different corners of the grid.
B	8	6	6, they are different shapes and sizes.
C	6	6	6, because some are bigger and others smaller.
D	19	13	Quantity 17. They are different from each other, but the number of sides is the same. Some of them are: triangle, square and rectangle.
E	9	9	I made nine and they are different because they are not the same.
F	9	9	I found 9 solutions. They are different because the shapes are not the same, some are bigger, others smaller.
G	15	15	Some are smaller, others are pointy, bigger.
H	13	13	13, they are different because I did not follow the same path for all of them.
I	14	9	15. They are different because they have different features.
J	18	10	15, the segment and position of each one are different.
K	18	14	I made 16, they are not the same because they are different shapes and sizes.
L	12	9	I made 12, they are different because, even though they have four sides, there are different shapes.

Source: Own elaboration using research data

It is worth noting that the question asked may not have been clear enough for the students, or perhaps it would have been necessary to provide more details or examples about what was expected as an answer. For example, it could have been suggested that the students mention the types of quadrilaterals they found. Only one student (D) mentions square and rectangle, but at the same time, he wrongly mentions triangle. In general, the answers are vague and do not provide sufficient arguments about the differences between the solutions presented. This may be due to a gap in skill EF06MA20 of the BNCC, which refers to the ability to "identify characteristics of quadrilaterals, classify them in relation to sides and angles, and recognize the inclusion and intersection of classes between them" (Brasil, 2017, p. 303).

It is clear that the students are able to convert the representation in Natural Language to Figural when they are asked to produce the figures. However, when asked to observe the figural representations more carefully and expose written differences between them, even though they are all quadrilaterals, this does not happen. In this sense, Duval (2009) highlights the importance of the relationship between these two types of record when geometric concepts are addressed, as well as the importance of double conversion.

6 Final considerations

The objective of the research was to evaluate the creativity of students in Middle School in the production of different quadrilaterals in 3×3 grids, considering the fluency, flexibility and originality of the answers. The results reveal that, although the students demonstrated the ability to propose multiple solutions, there is a tendency for standardized, even repeated, answers based on geometric transformations such as rotations, reflections and translations, revealing little creativity in this group of students.

Based on the discussions, the need for teaching methodologies that stimulate creative reasoning is emphasized, allowing students to feel encouraged to explore alternative solutions, in addition to pre-established algorithms. Creativity in Mathematics is an interesting way to develop not only specific problem-solving skills, but also to foster innovative and adaptable thinking. These are important skills for Mathematics, Education and society as a whole.

Based on this study, we intend to conduct other investigations that promote creativity. One of them seeks to extend the present investigation to a larger number of students in the final years of elementary school and high school students. Another one deals with the elaboration of a sequence of creative tasks involving geometric concepts (Silva et al., 2024) in which it is necessary to redefine a problem, generating multiple possibilities of representing a situation, according to Gontijo (2006). Furthermore, the doctoral research of the first author aims to investigate the role of creativity in the development of Geometric Visual Spatial Thinking in undergraduate students in Mathematics.

By fostering the development of a more dynamic and creativity-oriented teaching of Mathematics, this work reinforces the importance of preparing students to face mathematical situations in a fluent, flexible and original way, enriching their learning experience and their integral education.

Acknowledgements

The research was developed with support from the Brazilian Federal Agency for Support and Evaluation of Graduate Education (CAPES) – Financing Code 001.

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