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Solution or trap? Integrating courses as the place for Mathematics Education in undergraduate programs

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ARTICLE

Abstract: The research presented in this article aimed to discuss, in the context of undergraduate Mathematics courses, the so-called integrative disciplines, based on a qualitative curriculum analysis carried out within a Brazilian public university. It was found that, in the scenario analyzed, these subjects ended up becoming the *place* of Mathematics Education in these initial educational courses, which is potentially harmful because it conceals the failure to overcome training fragmentation and contributes to its intensification. It is argued that Mathematics Education and the dialog it promotes between specific and didactic-pedagogical education should not be restricted to integrating courses, but should be at the heart of the degree, with a view to qualifying education and bringing it closer to professional teaching practice for Mathematics teaching.

Keywords: Integrating Courses. Degree in Mathematics. Mathematics Education. Curriculum.

¿Solución o trampa? Disciplinas integradoras como lugar de la Educación Matemática en la licenciatura

Resumen: La investigación presentada en este artículo tuvo como objetivo discutir, en el contexto de los cursos de grado en Matemáticas, las llamadas disciplinas integradoras, a partir de un análisis curriculum cualitativo realizado en una universidad pública brasileña. Se observa que, en el escenario analizado, estas disciplinas terminaron convirtiéndose en el *lugar* de la Educación Matemática en estos cursos de formación inicial, lo que es potencialmente nocivo porque encubre la no superación de la fragmentación formativa y contribuye a su intensificación. Se señala que la Educación Matemática y el diálogo que promueve entre la formación específica y la didáctico-pedagógica no debe restringirse a las disciplinas integradoras, sino ser el núcleo de la titulación, con vistas a cualificar la formación y aproximarla a la práctica docente profesional para la enseñanza de las Matemáticas.

Palabras clave: Disciplinas Integradoras. Licenciatura en Matemáticas. Educación Matemática. Curriculum.

Solução ou armadilha? Disciplinas integradoras como o lugar da Educação Matemática na licenciatura

Resumo: A pesquisa apresentada neste artigo teve como objetivo discutir, no contexto das licenciaturas em Matemática, as chamadas disciplinas integradoras, com base em uma análise curricular qualitativa realizada no âmbito de uma universidade pública brasileira. Constata-se que, no cenário analisado, essas disciplinas acabaram se convertendo no *lugar* da Educação Matemática nesses cursos de formação inicial, o que é potencialmente nocivo por escamotear a não superação da fragmentação formativa e concorrer para o seu acirramento. Argumenta-se que a Educação Matemática e o diálogo por ela promovido entre a formação específica e didático-pedagógica não devem ficar restritos às disciplinas integradoras, mas constituírem o cerne da licenciatura, com vistas a qualificar a formação e aproximá-la da prática profissional

docente para o ensino de Matemática.

Palavras-chave: Disciplinas Integradoras. Licenciatura em Matemática. Educação Matemática. Currículo.

1 Introduction

In recent years, the discussion about the education of teachers who teach mathematics and about degree courses and their curriculum has advanced and taken on new contours, largely based on the results of research carried out in the field of Mathematics Education, a field essentially dedicated to issues of teaching and learning mathematics and, in particular, teacher education. Although progress can be seen in some aspects, in others it still seems to be going in circles, far from the transformations that are increasingly necessary and urgent.

In the second half of the last century, Lee Shulman (1986) brought light to the discussion about teachers' professional knowledge, establishing a theoretical framework for teaching by postulating pedagogical content knowledge (PCK) as the knowledge proper to this professional category. This paradigmatic shift helped differentiate the teaching profession from others, ratifying the need for training capable of mobilizing specific knowledge. Naturally, after Shulman, other scholars have deepened and complexified the discussion of knowledge specific to teaching, especially mathematics teaching, such as Ball, Thames and Phelps (2008), Carrillo-Yañez et al. (2018), among many others.

This theoretical contribution was part of the reforms to undergraduate courses, promoting, as early as the 1980s, the emergence of subjects of an integrative nature — in the sense of linking specific education to didactic-pedagogical education, not necessarily covering other elements, such as theory and practice —, impacting the initial education of teachers and thus constituting "a new model, which essentially continues to this day" (Moreira and David, 2018, p. 14). This set of *integrating courses* aimed to address teachers' specific knowledge, promoting dialog between specific and didactic-pedagogical education and thus overcoming the existing dichotomy. However, as Moreira and David (2018, p. 14) point out, "there is recognition that the introduction of integrating courses has not shown the expected results".

Since then, the configurations of undergraduate mathematics courses in Brazil have been discussed, under the aegis of CNE/CES Resolution n. 3/2003, which establishes the Curriculum Guidelines for undergraduate and graduate mathematics courses (Brasil, 2003). Even though they have introduced new discussions, theories and approaches to these courses, it is possible to see that the educational model used in Mathematics degrees, especially with regard to the integrating courses, is still far from being able to overcome the dichotomies and fragmentation of education that it proposes.

This analysis is backed up by an examination of the mathematics degree curricula at the State University of Ceará (Uece)[1], an exponent of teacher education in the state. Based on the analysis of the curriculum projects, a series of concerns and questions arise, revealing the need for reflection and dialog about the education undertaken at this higher education institution (HEI). For the purpose of this assessment, we are looking specifically at the layout and configuration of these curriculum components, with the aim of discussing, in the context of mathematics degrees, the so-called integrating courses, based on a curriculum analysis.

Discussing this issue, looking at a selection of real curricula inserted in a specific academic culture, is justified by the need to rethink the education processes in mathematics courses for the initial education of teachers. From the experience accumulated in this field and

[1] The State University of Ceará (Uece) offers five degree courses in Mathematics in various regions of the state: Fortaleza (Metropolitan Region), Aracati (East Coast), Limoeiro do Norte (Jaguaribe Valley), Quixadá (Central Hinterland) and Iguatu (Center-South). For more details on the Pedagogical Projects for these courses, see Melo and Sousa (2024).

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in research that shows this reality (Cyrino and Grando, 2022; Junqueira and Manrique, 2015; Manrique, 2009; Moreira, 2012; Moreira and Ferreira, 2021; SBEM, 2003; Zaidan et al., 2021), it is clear that the current models and educational practices are saturated. There is also a growing number of weaknesses and inconsistencies, which are leading to a crisis in undergraduate courses, particularly in the area of mathematics, and indirectly to an inability to improve teaching in basic education. The proposed reflection thus contributes to the nationwide discussion on what initial teacher education is desired and what the possibilities are for its realization.

In order to achieve this goal, this paper is structured as follows: after this introduction, the context, methodology and panorama of the research that gave rise to the proposed debate are presented; then, the theoretical discussion is deepened, with the aim of producing new meanings about the integrating courses in Mathematics degree courses, the main objective; finally, the paper closes with some considerations.

2 Discussion based on a curriculum analysis: the case of UECE

The discussion presented originates in the context of the research entitled *The initial education of mathematics teachers at UECE: curricula, trainers and education practices on the agenda*, which has been underway since the beginning of 2024 and is coordinated by the first author. Its aim is to investigate this teacher education from the perspective of contemporary discussions in Mathematics Education, especially those produced within the scope of WG07: Education of Teachers who Teach Mathematics of the Brazilian Society of Mathematics Education (SBEM).

In the first stage of this research, the aim is to investigate, by means of a qualitative documentary analysis (Fiorentini and Lorenzato, 2012; Ludke and André, 1986; Stake, 2011), documents that express the curricula of the five degree courses in Mathematics at this university, in order to understand the presence of Mathematics Education as a field of knowledge in these curricula and courses. Details of this analysis can be found in Melo and Sousa (2024) and Melo and Taveira (2024) and, in summary, below.

At first glance, the documents reveal outdated Course Pedagogical Projects (PPC) — one of them even dating back to 2008 —, with equally outdated legal foundations — most referring to CNE/CP Resolution n. 1/2002 (Brazil, 2002), and others to CNE/CP Resolution n. 2/2015 (Brazil, 2015); but none referring to CNE/CP Resolution n. 2/2019 (Brazil, 2019)[2] —, reflecting a fragmented formative project and "a hodgepodge of discourses, that is, a combination of dissonant conceptions in the same document, represented, among others, by the conceptual foundation that points to [so-called] updated aspects of the literature in contrast to the curriculum disposition, representing variations of the '3+1' model" (Melo and Sousa, 2024).

Regarding the presence of Mathematics Education in these curricula, it was concluded that there is a lack of understanding that goes beyond Mathematics Teaching, i.e. the didactic and methodological aspects. There is a lack of a perspective that goes beyond this dimension, contemplating the complexity of teaching practice, reflection on the educational process itself, evolving to the constitution of the identity and professional development of the future teacher and also a vision of Mathematics Education "as a scientific field responsible for elaborating, discussing and problematizing the ways in which mathematical knowledge is produced and presented, with its own epistemologies, approaches and definitions" (Melo and Taveira, 2024, p. 10).

[2] Regarding the curriculum adaptation of Mathematics degree courses to the 2019 DCN and its effects, it is worth checking out the discussion and criticism promoted by Zaidan et al. (2021), the result of extensive research with Pedagogical Projects of Mathematics degree courses in Brazil.

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This Mathematics Education bias can be seen in specific curriculum components, the so-called integrating courses, created with the purpose — mentioned nominally in some of the documents analyzed — of overcoming the dichotomies that have been present in Mathematics degree courses for decades. In the curricula observed, these subjects are called *Teaching Laboratory of...* or just *Teaching of...*, and are complemented by some field of Mathematics, such as Geometry, Algebra, Trigonometry, among others.

From the analysis as a whole, two findings are possible. The first is that there is a recognition, at least conceptual, present in the introductory parts of the documents — in which their foundations are presented), that Mathematics Education is an area that *encompasses* mathematical content subjects and *educational courses*, which indicates that it is responsible for integrating both education — specific and didactic-pedagogical. An example of this is the passage in one of the PPCs analyzed, which highlights the objective of "extinguishing the dichotomy that exists today between mathematical content courses and education courses. *Making courses appear that strengthen the relationship between these two areas that today form a larger area called Mathematical Education"* (Uece, 2011, p. 6, emphasis added).

The second conclusion, although it partly contradicts the previous one, is that Education is concretized in the so-called *teaching courses*, which are consolidated as its main representatives, in charge of building the *bridge* between the areas of Pure Mathematics and didactic-pedagogical education. This finding is cause for concern, because in this way,

> the praxis promoted by/within Mathematics Education would be reduced to the dimension of teaching, as he explicitly states when referring to "teaching courses". [...] the literature and accumulated experience in the area show that, in practice, there has been an *outsourcing of responsibilities*: the so-called "pure mathematics" courses are not concerned with didactic-pedagogical aspects and neither are the didactic-pedagogical courses concerned with specific aspects of mathematics, since there are integrating courses to fulfill this function (Melo and Sousa, 2024, p. 8-9, emphasis added).

In this sense, in the analysis, we were struck by the fact that, more alarming than the lack of a consistent understanding of Mathematics Education — which covers the complexity and breadth of this field of knowledge and education for teaching practice, and which is the central axis of these initial educational courses, especially at Uece — is the supposed overcoming of the dichotomies of Mathematics degrees (Melo and Sousa, 2024). Thus, as announced in the documents, the issue of educational fragmentation and the polarization between specific and didactic-pedagogical education, as well as between theory and practice, is overcome. However,

> although it aims to overcome the historical dichotomy between specific and pedagogical knowledge, by focusing only on didactic aspects of teaching mathematics, this restricted understanding of mathematics education, fundamentally translated into laboratory or "Teaching of" courses, contributes to the deepening of this dichotomy, explicitly from the documentary analysis. This is because considering that we have overcome anachronistic views that have been exhaustively refuted by studies and research over the last few decades, which promote fragmentation in education, prevents us from understanding this issue as latent and still present. *This ends up masking a reformulated continuity of the educational model commonly known as 3+1* (Melo; Taveira, 2024, p. 10, emphasis added).

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The scenario that can be observed, especially in view of the outdated curriculum, is that there is no prospect of valuing Mathematics Education in the undergraduate courses at the IES under analysis. On the contrary, there is a tendency for this area to be confined to the integrating subjects that further fulfill the hours of Practice as a Curriculum Component (PCC), as will be discussed below. Thus, in the dispute for space in the curricula of these courses (Arroyo, 2013), there is a kind of trap that captures this field of knowledge, limiting it to the didactic aspect of teaching work and confining it to the scope of the integrating courses. These subjects are therefore configured as the place of Mathematics Education in the curriculum of the Mathematics degree course, an aspect that deserves a more in-depth analysis, which will be addressed below.

3 Integrating courses as the place for Mathematics Education in undergraduate programs: solution or trap?

In order to continue with the debate, we need to start by establishing an understanding of curricula and degree courses in mathematics. To begin with, if we consider that "curriculum is much more than a selection of content and that such a selection of content says a lot about who you want to train, we can think of curriculum discussions as a powerful instrument in the manufacture of desirable courses in contemporary times" (Pacheco, Ruidiaz and Silva, 2024, p. 6, emphasis in original). In this case, desirable math teachers to meet the demands of 21st century education and schools, which implies their knowledge, education and practice.

With this in mind, a degree in mathematics needs to offer a curriculum, articulated around specific knowledge, that expresses the profile of Mathematics teacher required today. Never losing sight of the fact that, as with other courses — Engineering, Medicine, Dentistry, among others —, the degree is also a professional course, i.e. it aims to train a mathematics education professional (Fiorentini and Oliveira, 2013). Thus, we agree with the authors when they state that

> the education of mathematics teachers should be guided by the different social practices of mathematics educators; adopt, in initial education, practices and projects in which undergraduates can integrate, contrast, problematize and investigate the relationships between their undergraduate mathematics education, their didactic-pedagogical education related to the content, and the complexity of school practices. (Fiorentini and Oliveira, 2013, p. 918).

Overcoming the transmissive idea of knowledge, from the perspective of Banking Education that Paulo Freire (2005) has long pointed out, and understanding the undergraduate degree as a space for learning the profession, "the undergraduate degree needs to be seen as a port of passage and initiation into the process of investigating pedagogical practice in Mathematics, a fundamental condition for promoting their professional autonomy and their own professional development throughout their career" (Fiorentini, 2005, p. 113). This is because, if you consider initial teacher education as a time/space for learning the profession, all the knowledge listed and articulated in the curriculum contributes (or at least should contribute) to this education, i.e. it should occupy a place in the set of professional knowledge mobilized, directly or indirectly, in teaching practice.

However, what has been observed in recent decades, as Moreira and David (2018, p. 15) state, is that

> the construction of substantive links between education and practice is seen as a task to be carried out basically outside mathematical education. The latter is

> fundamentally responsible for deepening the disciplinary component of teaching knowledge, which usually means going beyond the school form of mathematical knowledge, presenting the trainee with the "advanced and profound" form of this knowledge, in other words, academic mathematics.

However, there is a generalized difficulty in systematizing and articulating the theoretical knowledge mobilized throughout the course — not only that related to Academic Mathematics, in the terms of Moreira and David (2018), but also knowledge of a different nature — which makes this education fragmented and far from what is proposed. It can be said, although it is complicated to discuss knowledge from different epistemes in this way, that it is as if different and apparently disconnected "tools" were given to teachers in education, without an instruction manual or guidance on how to use them together, in order to carry out the desired work.

For a long time, this disarticulation was attributed to the fact that undergraduate courses were basically formed by the simple juxtaposition of specific and didactic-pedagogical knowledge — a model referred to as 3+1 —, leaving it up to the future teacher to relate and mobilize this knowledge concretely and together. It is worth remembering what Fiorentini (2005, p. 107) says, that "both groups of specific and didactic-pedagogical courses train the future teacher pedagogically and mathematically".

In order to overcome this dichotomization and fragmentation of education, integrative courses emerged, which would be dedicated to bridging these knowledge poles. Notably, however, this proposal failed, as evidenced by the amount of research and discussions that still attest to the recurrence of this issue and the need to move forward. One example, among many, is the document entitled *Subsídios para a discussão de propostas para os cursos de licenciatura em Matemática* [Subsidies for the discussion of proposals for undergraduate courses in Mathematics], published by the SBEM in the context of an effervescent discussion following the publication of CNE/CES Opinion n. 1.302/2001 (Brazil, 2001) and the imminent publication of CNE/CES Resolution n. 3/2003, which establishes the Curriculum Guidelines for Mathematics courses (Brazil, 2003). It states that

> the Mathematics Degree Course must be conceived as an initial education course in Mathematics Education, *in a configuration that breaks down the dichotomy between pedagogical knowledge and specific knowledge and the dichotomy between theory and practice.* The identity of degree courses is obviously based on mathematical knowledge, viscerally linked to pedagogical and historical treatment, which will shape a "Mathematics" that is different from the merely formalized and technical. The establishment of this identity requires a rethink of the education of teacher educators and special care in the choice of professionals who work in degree courses, in the sense that they are committed to the pedagogical project of these courses (SBEM, 2003, p. 4, emphasis added).

This document, which discusses the panorama of degree courses at the beginning of the century, points out, among other things, the problems to be tackled in initial education, which have existed for at least 20 years. These problems include: "The almost total disconnect between mathematical knowledge and pedagogical knowledge and between theory and practice" and "the treatment of pedagogical content that is decontextualized and devoid of meaning for future mathematics teachers, thus failing to win students over to its importance" (SBEM, 2003, p. 6).

It should be noted that, in the wake of this debate, the nature of the specific and didactic-

pedagogical knowledge is not necessarily questioned, i.e. what knowledge (or content) should be worked on — a discussion raised by Cristovão *et al.* (2023) — but how to relate them, with a view to mobilizing the pedagogical knowledge of the content, as stipulated by Shulman (1986), which is specific to the teacher and necessary for their practice. Manrique (2009, p. 523) has already announced that one of the central problems of this initial education is "the disarticulation between specific and pedagogical knowledge, which is worked on in a decontextualized way, without meaning for future teachers, thus failing to win students over to its importance in their future teaching activities".

The author also points out that, "with a view to integrating pedagogical aspects and the specific area, *the courses in the area of Mathematics Education seem to favor this integration and are considered to be integrating and interdisciplinary courses*" (Manrique, 2009, p. 530-531, emphasis added). The reader will notice that the literature in Mathematics Education itself understands this field as interdisciplinary and integrative, capable of promoting dialogue between the areas of education, and we don't disagree with this. The issue, in this understanding, is that this dialog takes place within the scope of Mathematics Education, which should be at the heart of the degree in Mathematics, not necessarily in specific curriculum components, but throughout the course. This point of view will be discussed below.

The fact is that, even though they are defended and considered in this way, the integrating courses have apparently not succeeded in overcoming the training dichotomy, i.e. they have not fulfilled the thankless mission of solving a historical problem generated by two areas that are almost diametrically opposed. In the words of Moreira and Ferreira (2021, p. 7),

> the fundamental idea of the teacher education model, based on the formula Bachelor's Degree + Didactics (essentially, what to teach + how to teach), seems to be so ingrained in the actors of the professional teacher education process and in the curriculum scenario of Bachelor's Degrees, that the assessment that it is possible to integrate two instances of education, designed, from the outset, to develop in an essentially dichotomous way, is naturalized.

However, without a deep understanding of the disarticulation that operates, integrated courses attack the effects, but not the causes of the problem, and this is essentially where they become a trap. They seek — and, in many cases, have even succeeded satisfactorily — to promote dialogues and reflections on the meeting of specific and didactic-pedagogical knowledge, but, as they are restricted to their own curriculum components, they do not properly involve specific and didactic-pedagogical education, which continues to operate and reproduce a pedagogical tradition. This tradition, "despite the advances in research in Mathematics Education, has meant that school practices seem to have evolved little" (Fiorentini, 2005, p. 111).

At the end of the day, "without a specific questioning of the basic logic of the '3+1' model and with the addition of integrating courses, the discussion about the curriculum structure of undergraduate courses basically turns to a space dispute between 3 + 1" (Moreira and Ferreira, 2021, p. 7). In other words, not only has the dichotomy of mathematics degrees not been overcome, but a trichotomy has been produced — in which specific knowledge, didactic-pedagogical knowledge and *mathematics education* knowledge compete for space in the curriculum, in the form of courses — which is how this *third way* can be called. In this respect, Moreira and Ferreira (2021, p. 7-8) point out that

> in a perhaps surprising way, "pedagogism" and "contentism" also showed a facet of complicity, in that the question of the autonomous development of

> mathematical education and didactic/pedagogical education was not touched upon. Every man for himself, as we might say today. The smokescreen that covered up this untouched isolation of each of these two areas of teacher education was the block of "integrating" courses. Basically, the dispute took place under a veiled consensus: for mathematicians, mathematical education; for educators, didactic and pedagogical education. Let the integrating courses "manage" to integrate what was practically non-integrable. *The essence of the "3+1" remains: a conception of teacher education in which mathematical knowledge is selected and worked on from an internal logic of mathematics, tacitly postulating the attribution of a professional (teaching) sense to this disciplinary knowledge, a sense that should be developed in other instances of the educational process. Thus, the inclusion of integrating courses can be seen as a palliative that doesn't touch the heart of the matter* (emphasis added).

The product of this configuration is a course structure that encompasses three independent, relatively autonomous blocks, which are juxtaposed to fulfill the curriculum workload, but which delegate to the graduate the task that the course is unable to fulfill: "organizing training knowledge into an organic, consistent and instrumental body of knowledge for teaching practice in mathematics" (Moreira, 2012, p. 1141). All of this under the pernicious guise of the supposed overcoming of dichotomies, when, in fact, we are moving towards their intensification. After all, the supposed overcoming of a problem makes it impossible to effectively overcome it: there is no need to think of solutions to problems that have already been *solved*. Thus, we continue to remedy the effects, not the causes.

The case of Uece is emblematic in this sense: the curriculum components of *Teaching Laboratory of...* or *Teaching of...* are created, giving the impression of attending to the teacher's didactic education, which, if it happens, happens in an isolated and fragmented way (not to mention aspects other than the didactic ones). In this context, the specific courses — an expression which, in our view, is inappropriate to refer to academic Mathematics courses, as it suggests that the others are, in contrast, generalist — are exempt from dealing explicitly with the teaching of the related content or, even more so, the contextualization of that knowledge in the complexity of the teacher's work.

For example, by creating the course of *Geometry Teaching Laboratory*, the teacher of Euclidean Geometry, Analytical Geometry and other courses in this field of Mathematics feels unobliged to try to relate the specific knowledge of Geometry worked on in these courses with the teaching education of the undergraduate student, since this purpose falls organically to the integrating course. "In other words, learning and learning to teach are not worked on in an integrated way, which would allow for a better articulation between theory and practice" (Melo and Taveira, 2024, p. 7), in addition to reinforcing the supposed identity between Mathematics Education courses and Mathematics teaching, in its purely didactic-methodological bias.

Another serious aspect noted in the case of this university — and which may be reflected in similar cases — is the fact that, with regard to the legally established 400-hour workload of Practice as a Curriculum Component, established by law, in CNE/CP Resolutions n. 2/2015 and n. 2/2019 (Brazil, 2015, 2019), it was decided to create specific components that would encompass this quantity, instead of distributing it among the components already existing in the course, thus reinforcing the separation between theory and practice (Melo and Taveira, 2024). Thus,

> in terms of the curriculum dispute, there were "two birds with one stone", as the popular saying goes: in addition to relegating Mathematics Education to

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> the space of the integrating courses, these courses were also responsible, as a rule, for covering the workload of Practice as a Curriculum Component (PCC) required by the Curriculum Guidelines. As a result, the components and total workload of other areas of the curriculum remained "untouched" and were able to maintain their usual functioning and status quo in initial education (Melo and Sousa, 2024, p. 9).

The trap into which Mathematics Education falls in this curriculum configuration is that it takes care of this dialog between the areas, which continue to go practically one way and little (or nothing) else. In other words, by focusing all its efforts on didactic-pedagogical education in the treatment of specific content, there is often no time and human material left over to deal, in the field of Mathematics Education, with other aspects involved in the complexity of teaching in Mathematics, such as the mobilization of other knowledge, in-depth knowledge of school reality and Mathematics curricula, self-training, identity mobilization and the beginning of professional development, among others.

The curriculum analysis of UECE's Mathematics degrees points in this direction, when it is noted that "there is no consistent indication of an ideology about Mathematics Education, only a sign that it is up to this field of knowledge to promote dialogue between the historically hegemonic areas in mathematics teacher education" (Melo and Sousa, 2024, p. 9). In this scenario, the math teacher is left *wiping the ice*, trying to patch up an education that has been fragmented for a long time, and not really engaged in a broad, integrated education project, aligned with contemporary discussions on teacher training, and which articulates the math teacher's professional knowledge with a view to their future practice.

In Melo and Sousa (2024), there is evidence of this scenario based not only on the curriculum configuration and the syllabuses of the courses related to the area of Mathematics Education, but also on the last competition for permanent teachers at this institution, specifically the degree requirements for applying to the Mathematics Education/Mathematics Teaching sector — note that the very name of the sector already shows its bias. It is clear, therefore, "what is expected of a mathematics educator at UECE: to work primarily on the teaching approach, associating specific didactics and methodologies with mathematical content (of Basic Education, not those of Higher Education)" (Melo and Sousa, 2024, p. 11).

It may not be out of place to remind you at this point that, in mathematical terms, in the language of set theory, Mathematics Education is not the simple intersection between the sets of Education and Mathematics, nor is it the union of both, but, so to speak, it is another set (inter and multidisciplinary), which contains these two (even intersected) and other elements involved in the complex phenomenon of learning and teaching mathematics. This is the understanding of Fiorentini and Lorenzato (2012, p. 5), when they conceptualize Mathematics Education as "a praxis that involves the mastery of specific content (mathematics) and the mastery of pedagogical ideas and processes related to the transmission/assimilation and/or appropriation/construction of school mathematical knowledge".

This can also be seen in the SBEM document (2003, p. 20), which states that, as it is not the sum of Mathematics and Education courses, Mathematics Education is configured as a new synthesis, "which incorporates epistemological, philosophical, historical, psychological, political, methodological and cultural dimensions into the education curriculum in the search for a better understanding of the processes of teaching and learning Mathematics, as well as its social and political role".

Given this understanding, it is clear that limiting both the conception and practice of Mathematics Education to integrative courses is to consider it only an element of the initial training of Mathematics teachers, when, in fact, it is the guiding thread of this course that trains

for the practice of Mathematics Education. As discussions at the beginning of this century already pointed out,

> the identity of the courses is built on elements that make up the process of building professional knowledge, such as: *linking academic education with professional practice, emphasizing didactic-pedagogical knowledge of the mathematics to be taught and encouraging, during the degree course, investigative practices that promote the articulation between theory and practice.* These elements should be reflected in the definition of the course's objectives, in the choice of educational content, in the methodological approach, in the creation of different times and spaces for students to experience, in the relationships between teacher educators and teachers in education, in the dynamics of the classroom, in the assessment process (SBEM, 2003, p. 13, emphasis added).

All the arguments in this discussion point to a common horizon: *overcoming the dichotomy(ies) in undergraduate courses will not happen within specific spaces in curriculum components, which keep the logic in full operation, but through a reformulation of the model, which changes its structures and is reflected in the structuring of the educational project and the concepts that support it.* In other words, the *dialog* between specific education and didactic-pedagogical education, which is of interest to Mathematics Education, should not be confined, delegated or relegated to the scope of the integrating courses. Instead, it should be at the heart of every educational course that is really interested in educational mathematics teachers.

Although this is a recurring discourse, it should be emphasized that

> it's not a question of weakening the content [or mathematical education], but of teaching what is really relevant and has meaning and sense for the math teacher's education, ensuring not only that they learn, but that this knowledge becomes part of their practice (SBEM, 2003, p. 25).

After all, as Fiorentini (2005) said, both specific courses train pedagogically and didactic-pedagogical courses train mathematically; and, going further, Mathematics Education courses, whether they have a didactic or theoretical approach, train mathematically and pedagogically - this is how the teacher's specific professional knowledge is considered. The fact is, as I've said elsewhere

> to continue the dispute for space within this model is to postpone the necessary change in these courses, which should be aimed at both qualifying teacher education and mobilizing professional knowledge with a view to bringing students closer to their future practice (Melo and Sousa, 2024, p. 11).

This requires the community involved in initial education — trainers, managers, curriculum developers, teachers in education and those already trained — to design a broad and integrated education project that signals and makes this perspective effective from the outset, as there is no longer any patch possible in the current model that leads to what is advocated in research on the initial education of mathematics teachers.

Finally, it should be noted that this is a much more political (and socio-cultural) challenge than a conceptual one (Peralta, 2019). In terms of scientific production and discussion, as Moreira and Ferreira (2021, p. 27) point out, “we can say that we currently have some theoretical directions that could help build a path to eventually overcoming a mathematics

teacher education project that has historically proved problematic". However, "defensible and consistent theoretical directions are not enough. We need to consider the power relations and social legitimacy granted to the different groups that take part in curriculum decisions, on the basis of which degree courses are structured" (Moreira and Ferreira, 2021, p. 27).

This is why the problem and possible suggestions have been pointed out for decades, but with little real progress. In these scenarios of change, it has to be taken into account that the people involved in reformulating curriculum and implementing initial education courses have, in most cases, been trained in the model they want to overcome. It is therefore expected that these individuals will give up the security they have in knowing this model, or even the knowledge they acquired during this education, in the name of something that is being developed and which has the potential to contribute to better education. Of course, reality is not so categorical, so *this or that*, but full of specificities and complexities that add even more nuances to the process of change.

As people concerned with curriculum issues in Mathematics Education, especially with the curriculum(s) of mathematics degree courses, it is up to us to continue producing views and arguments on the issue, finding (or producing) "paths, loopholes, shortcuts, tactics, creatively and collaboratively traced in order to question and resist, defending what we know about training teachers, what we know about mathematics education in favor of social justice" (Cyrino and Grando, 2022, p. 13). 13), i.e. a concern with and action against the eradication of social inequalities and injustices (Taveira, 2024). In addition, it is extremely necessary to occupy the political spaces of educational and curriculum decisions, strengthening the collective around the interests to which it is dedicated — the qualification of teaching and learning in Mathematics — and, consequently, the education initial and continuing of the teachers who teach it.

Considering that the aim of this discussion has been fulfilled, we move on to the final considerations.

4 Final considerations

Based on the discussion, the main thesis is that, in the scenario under analysis, i.e. in the case of UECE — which can, after due reflection, be generalized to other scenarios —, in addition to not having produced the expected results (Moreira and David, 2018), the integrating courses — even though positive effects can be linked to them — ended up *capturing* Mathematics Education and became *the place* of this field in the Mathematics degree.

This is potentially harmful, since i) it conceals the failure to overcome the fragmentation of training represented, among other things, by the dichotomy of specific training versus didactic-pedagogical education, and even contributes to its intensification; ii) it relegates Mathematics Education, as a field of knowledge, and its knowledge to these spaces, instead of making it permeate the entire initial teacher education course, as a multidisciplinary area capable of effectively integrating specific, didactic-pedagogical and practical knowledge, among others; and iii) compromises the educational practice of mathematics teachers who, restricted to these curriculum components, have to deal with multiple aspects of the teacher's education in this space and in these possibilities, such as didactic education, theoretical education in mathematics education, training for research, among others.

We can therefore conclude that Mathematics Education and the dialog it promotes between specific education/specific knowledge and didactic-pedagogical education/didactic-pedagogical knowledge should not be restricted to the scope of the integrating courses, but should be at the heart of the educational course, running through it from end to end. Only in this way, in a broad and integrated educational project, will it be possible to think of teacher

education for mathematics teaching that is also integrated.

It is well known that many studies point to the shortcomings and inadequacy of today's mathematics degree model for education teachers to meet contemporary demands. Little has been discussed, however, about what other education is necessary and possible, given the scenario of extensive scientific production in the field of Mathematics Education, and, even more so, what education is desired for these future teachers from the perspective that "bringing the issue of desire into the curriculum debate can be a powerful tool for denaturalizing paths taken so long ago and for inventing other curriculum" (Pacheco, Ruidiaz and Silva, 2024, p. 8).

In this context, we are experiencing a crisis in mathematics degrees which, as Antonio Gramsci (2000, p. 311) said, "consists precisely in the fact that the old is dying and the new cannot be born; in this interregnum, a wide variety of morbid symptoms appear". It is therefore necessary to develop and implement educational models that take into account what has been investigated and concluded to be good education for mathematics teachers, which mobilizes their professional knowledge, contributes to the (re)constitution of their teaching identity and enables their insertion and professional development.

There is an urgent need to direct research results towards proposing educational models — including considering new legal guidelines to support them —, understanding that Mathematics Education is the field that has dedicated itself to better understanding and discussing teaching, teaching and learning mathematics, especially in Basic Education. To do this, it is necessary to overcome the internal theoretical differences, not by homogenizing the theoretical lenses, which would be counterproductive, but by making them dialogue, understanding that they are all aimed at qualifying the teaching and learning of mathematics for the new generations.

At the same time, it is necessary to strengthen political engagement and struggle in order to occupy the spaces for discussing teacher education for mathematics teaching, which has been dominated by other sectors and other educational projects. If we consider education to be a political act, we must also take the discussion of curricula as a space for political debate, since "it is not just a matter of listing purely technical or scientific arguments" (Moreira and Ferreira, 2021, p. 28), but of creating the material conditions for these arguments to be transformed into public education policies, for example.

In addition to strengthening research and proposals in this area, this means endorsing the work of the SBEM, an entity that represents this field of knowledge and those who are part of it, as well as WG19: Mathematics Education, of the National Association of Graduate Studies and Research in Education (ANPEd), among others, providing greater visibility and, consequently, more space for this agenda in the discussion of proposals for teacher education and Mathematics curriculum.

Conflicts of Interest

The authors declare no conflicts of interest that could influence the results of the study presented in the article.

Data Availability Statement

The data collected and analyzed in the article will be made available upon request to the authors.

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References

ARROYO, Miguel Gonzáles. *Currículo, território em disputa.* Petrópolis: Vozes, 2013.

BALL, Deborah Loewenberg; THAMES, Mark Hoover; PHELPS, Geoffrey. Content knowledge for teaching: what makes it special? *Journal of Teacher Education*, v. 59, n. 5, p. 389-407, nov. 2008. https://doi.org/10.1177/0022487108324554

BRASIL. Ministério da Educação. Conselho Nacional de Educação. Parecer CNE/CES n. 1.302, de 6 de novembro de 2001. Diretrizes Curriculares Nacionais para os Cursos de Matemática, Bacharelado e Licenciatura. Brasília: Diário Oficial da União, 5 mar. 2002, Seção 1, p. 15.

BRASIL. Ministério da Educação. Conselho Nacional de Educação. *Resolução CNE/CES n. 2, de 18 de fevereiro de 2003*. Estabelece as Diretrizes Curriculares para os cursos de Matemática. Brasília: Diário Oficial da União, 25 fev. 2003, Seção 1, p. 13.

BRASIL. Ministério da Educação. Conselho Nacional de Educação. Resolução CNE/CP n. 2, de 1 de julho de 2015. Define as Diretrizes Curriculares Nacionais para a formação inicial em nível superior (cursos de licenciatura, cursos de formação pedagógica para graduados e cursos de segunda licenciatura) e para a formação continuada. Brasília: Diário Oficial da União, 2 jul. 2015, Seção 1, p. 8-12.

BRASIL. Ministério da Educação. Conselho Nacional de Educação. *Resolução CNE/CP n. 1, de 18 de fevereiro de 2002*. Institui Diretrizes Curriculares Nacionais para a Formação de Professores da Educação Básica, em nível superior, curso de licenciatura, de graduação plena. Brasília: Diário Oficial da União, 9 abr. 2002, Seção 1, p. 31.

BRASIL. Ministério da Educação. Conselho Nacional de Educação. Resolução CNE/CP n. 2, de 20 de dezembro de 2019. Define as Diretrizes Curriculares Nacionais para a Formação Inicial de Professores para a Educação Básica e institui a Base Nacional Comum para a Formação Inicial de Professores da Educação Básica (BNC-Formação). Brasília: Diário Oficial da União, 10 fev. 2020, Seção 1, p. 87-90.

CARRILLO-YAÑEZ, José; CLIMENT, Nuria; MONTES, Miguel; CONTRERAS, Luis C.; FLORES-MEDRANO, Eric; ESCUDERO-ÁVILA, Dinazar; VASCO, Diana; ROJAS, Nielka; FLORES, Pablo; AGUILAR-GONZÁLEZ, Álvaro; RIBEIRO, Miguel; MUÑOZ-CATALÁN, M. Cinta. The Mathematics Teacher's Specialised Knowledge (MTSK) model. *Research in Mathematics Education*, v. 20, n. 3, p. 236-253, 2018. https://doi.org/10.1080/14794802.2018.1479981

CRISTOVÃO, Eliane Matesco; FERREIRA, Ana Cristina Ferreira; BARBOSA, Cirléia Pereira; COUSA, Flávia Cristina Figueiredo; PAIVA, Maria Auxiliadora Vilela; ZAIDAN, Samira. Ensaio sobre a formação matemática do futuro professor de Matemática pautada nos conhecimentos matemáticos próprios da docência. *Espaço Plural*, v. 19, n. 39, p. 86-108, 2023.

CYRINO, Márcia Cristina de Costa Trindade; GRANDO, Regina Célia. (Des)construção curricular necessária: resistir, (re)existir, possibilidades insubordinadas criativamente. *REMat*, v. 19, p. 1-25, 2022. https://doi.org/10.37001/remat25269062v19id728

FIORENTINI, Dario. A formação matemática e didático-pedagógica nas disciplinas da licenciatura em Matemática. *Revista de Educação*, n. 18, p. 107-117, jun. 2005.

FIORENTINI, Dario; LORENZATO, Sérgio. *Investigação em Educação Matemática:* percursos teóricos e metodológicos. 3. ed. rev. Campinas: Autores Associados, 2012.

FIORENTINI, Dario; OLIVEIRA, Ana Teresa de Carvalho Correa. O lugar das matemáticas na licenciatura em Matemática: que matemáticas e que práticas formativas? *Bolema*, v. 27, n. 47, p. 917-938, dez. 2013. https://doi.org/10.1590/S0103-636X2013000400011

FREIRE, Paulo. *Pedagogia do Oprimido.* 42. ed. Rio de Janeiro: Paz e Terra, 2005.

GRAMSCI, Antonio. *Cadernos do Cárcere*. v. 3. Tradução de Carlos Nelson Coutinho. Rio de Janeiro: Civilização Brasileira, 2000.

JUNQUEIRA, Sonia Maria da Silva; MANRIQUE, Ana Lúcia. Reformas curriculares em cursos de licenciatura de Matemática: intenções necessárias e insuficientes. *Ciência & Educação*, v. 21, n. 3, p. 623-635, jul. 2015. https://doi.org/10.1590/1516-731320150030007

LUDKE, Menga; ANDRÉ, Marli Eliza Dalmazo Afonso. *Pesquisa em Educação*: abordagens qualitativas. São Paulo: EPU, 1986.

MANRIQUE, Ana Lúcia. Licenciatura em Matemática: formação para a docência × formação específica. *Educação Matemática Pesquisa*, v. 11, n. 3, p. 515-534, set./dez. 2009.

MELO, Carlos Ian Bezerra; SOUSA, Francisco Edisom Eugenio. Concepções sobre Educação Matemática nos documentos curriculares das licenciaturas em Matemática da Universidade Estadual do Ceará (UECE). In: *Anais do 9º Seminário Internacional de Pesquisa em Educação Matemática*. Natal, 2024, p. 1-15.

MELO, Carlos Ian Bezerra; TAVEIRA, Flávio Augusto Leite. Compreensões de Educação Matemática em um documento curricular da UECE. In: *Anais do 6º Simpósio Internacional de Pesquisa em Educação Matemática.* Campina Grande, 2024, p. 1-12.

MOREIRA, Plínio Cavalcante. 3+1 e suas (in)variantes: reflexões sobre as possibilidades de uma nova estrutura curricular na licenciatura em Matemática. *Bolema*, v. 26, n. 44, p. 1137-1150, 2012. https://doi.org/10.1590/S0103-636X2012000400003

MOREIRA, Plínio Cavalcante; DAVID, Maria Manuela Martins Soares. *A formação matemática do professor:* licenciatura e prática docente escolar. 2. ed. 2. reimp. Belo Horizonte: Autêntica, 2018.

MOREIRA, Plínio Cavalcante; FERREIRA, Ana Cristina. A formação matemática do professor da Educação Básica: das concepções historicamente dominantes às possibilidades alternativas atuais. *Perspectivas da Educação Matemática*, v. 14, n. 35, p. 1-30, 2021. https://doi.org/10.46312/pem.v14i35.13262

PACHECO, Débora Reis; RUIDIAZ, Paola Amaris; SILVA, Marcio Antonio. Currículos de Matemática e seus desejáveis personagens: da falta à produção. *Bolema*, v. 38, p. 1-19, 2024. https://doi.org/10.1590/1980-4415v38a230058

PERALTA, Deise Aparecida. *Habermas e as professoras e professores de Mmatemática:* vislumbrando oásis. Curitiba: Appris, 2019.

SBEM — Sociedade Brasileira de Educação Matemática. *Subsídios para a discussão de propostas para os cursos de licenciatura em Matemática:* uma contribuição da Sociedade

Brasileira de Educação Matemática. Brasília: SBEM, 2003.

SHULMAN, Lee S. Those who understand: knowledge growth in teaching. *Educational Researcher*, v. 15, n. 2, p. 4-14, 1986. https://doi.org/10.3102/0013189X015002004

STAKE, Robert E. *Pesquisa qualitativa:* estudando como as coisas funcionam. Tradução de Karla Reis. Porto Alegre: Penso, 2011.

TAVEIRA, Flavio Augusto Leite. Curriculum, Curricular Justice, and Mathematics Education: a political manifesto. *Revista Internacional de Pesquisa em Educação Matemática*, v. 14, n. 2, p. 1-13, 2024. https://doi.org/10.37001/ripem.v14i2.3947

UECE — Universidade Estadual do Ceará. Faculdade de Filosofia Dom Aureliano Matos. *Projeto Pedagógico:* Curso de Licenciatura em Matemática. UECE: Limoeiro do Norte, 2011.

ZAIDAN, Samira; FERREIRA, Ana Cristina; DE PAULA, Enio Freire; SANTANA, Flávia Cristina de Macedo; COURA, Flávia Cristina Figueiredo; PEREIRA, Patrícia Sandalo; STORMOWSKI, Vandoir. *A licenciatura em Matemática no Brasil em 2019:* análises dos projetos dos cursos que se adequaram à Resolução CNE/CP 02/2015. Brasília: SBEM, 2021.