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How do we teach arithmetic? Elements of *arithmetic for teaching* in Faria de Vasconcelos's handbook

Wagner Rodrigues Valente[1]

Jefferson dos Santos Ferreira[2]

ABSTRACT

This article presents the results of a research that focused on the grading, i.e., organisation in levels of knowledge, of the arithmetic teaching proposed by Faria de Vasconcelos's handbook "Como se ensina aritmética" (How to Teach Arithmetic). The theme proved to be important in view of the transformations of arithmetic to teach in primary school in times of so-called scientific pedagogy. Such an analysis was made from the perspective of a theoretical-methodological framework that takes into account the professional knowledge of teaching. As a result, there is a reorganisation of the arithmetic in primary education under the impact of experimental psychology with a statistical basis. Such transformations are shown more explicitly in the new grading of school education proposed by Faria de Vasconcelos.

KEYWORDS: Arithmetic for teaching. Scientific pedagogy. Faria de Vasconcelos.

Como se ensina aritmética? Elementos de uma aritmética para ensinar no manual de Faria de Vasconcelos

RESUMO

Neste artigo são apresentados resultados de uma pesquisa cujo foco tratou da graduação do ensino de aritmética proposta no manual *Como se ensina*

[1] Associate Professor II, Full Professor of the Department of Education of the School of Philosophy, Letters, and Human Sciences of the Federal University of São Paulo, *Campus* Guarulhos. http://orcid.org/0000-0002-2477-6677. *E-mail:* wagner.valente@unifesp.br.

[2]Doctoral degree student, Postgraduate Programme in Education and Childhood and Adolescence Education and Health of the Federal University of São Paulo, *Campus* Guarulhos. Orcid: http://orcid.org/0000-0001-7684-7716. Grant FAPESP. *E-mail:* jefferson.mat@hotmail.com.

aritmética, de Faria de Vasconcelos. O tema mostrou-se importante tendo em vista as transformações da aritmética a ser ensinada nos primeiros anos escolares em tempos da chamada pedagogia científica. Tal análise foi feita a partir da perspectiva de um referencial teórico-metodológico que leva em conta os saberes profissionais da docência. Como resultados, evidencia-se uma reorganização da aritmética do curso primário sob o impacto da psicologia experimental de base estatística. Tais transformações mostram-se de modo mais explícito na nova graduação do ensino escolar proposta por Faria de Vasconcelos.
PALAVRAS-CHAVE: *Aritmética para ensinar*. Pedagogia científica. Faria de Vasconcelos.

¿Cómo enseñar aritmética? Elementos de aritmética para enseñar en el manual de Faria de Vasconcelos

RESUMEN

Este artículo presenta los resultados de una investigación que se centró en la graduación de la enseñanza de la aritmética propuesta en el manual "Cómo enseñar aritmética", de Faria de Vasconcelos. El tema demostró ser importante en vista de las transformaciones de la aritmética que se enseñará en la escuela primaria en tiempos de la llamada pedagogía científica. Tal análisis se realizó desde la perspectiva de un marco teórico-metodológico que tiene en cuenta el conocimiento profesional de la enseñanza. Como resultado, hay una reorganización de la aritmética del curso primario bajo el impacto de la psicología experimental con una base estadística. Dichas transformaciones se muestran más explícitamente en la nueva graduación de la educación escolar propuesta por Faria de Vasconcelos.
PALABRAS CLAVE: Aritmética para la enseñanza. Pedagogía científica. Faria de Vasconcelos.

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Introduction

This article presents partial results of an ongoing doctoral study[3] that seeks to analyse processes and dynamics of preparation of the professional knowledge of pre-service arithmetic teachers who taught at Escola Normal de São Paulo, in times of what history of education calls scientific pedagogy. This research is part of a broad thematic project[4] that, supported by a teacher education framework from a socio-historical perspective, aims to investigate mathematics teachers' professional knowledge for a century (1890-1990) from a historical perspective. For this purpose, the project is divided into four thematic axes that guide master's, doctoral and post-doctoral research studies with themes related to each of the axes.

In the first axis, the thematic projects address the *experts* and the production process of professional knowledge of the teacher who taught mathematics. In the second, the production of *mathematics to teach*. In the third, the production of *mathematics for teaching*. Finally, the fourth axis is linked to research on teachers who taught mathematics and the mathematics that had been taught.

This article is primarily part of the third axis, establishing a dialogue with the results of research from the other axes. Therefore, at this point, it is necessary to explain better the concepts that are fundamental to this text, mobilised as theoretical and methodological tools, which are, as mentioned above, the *mathematics to teach* and the *mathematics for teaching*. Actually, such concepts derive from appropriations of the notions of *knowledge to teach* and *knowledge for teaching*.

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[4] The thematic project is supported by FAPESP. For more information about the project go to: https://www.ghemat.com.br/projeto-de-pesquisa-fapesp.

The *knowledges to teach* is the result of "[...] complex processes that fundamentally transform knowledge to make it teachable" (HOFSTETTER; SCHNEUWLY, 2017a, p. 133). Those general cultural knowledges, common to all professions, have their origin in the disciplinary fields. Yet, the *knowledges for teaching* are specific to the teaching profession, so they characterise the teacher's work and cover a wide range of knowledges, being

> [...] mainly of knowledges about 'the object' of teaching and formative work (about the knowledges *to* be taught and about the student, the adult, their knowledges, their development, the ways of learning, etc.), about teaching practices (methods, procedures, devices, choice of knowledges *to* be taught, modalities of organisation and management) and about the institution that defines their professional field (study plans, instructions, purposes, administrative and political structures, etc.). As in every profession, those knowledges are multiform. (HOFSTETTER; SCHNEUWLY, 2017a, p. 134, emphasis added).

Based on those concepts of *knowledge to teach* and *knowledge for teaching*, Bertini, Morais, and Valente (2017) assume as theoretical hypothesis of research the existence of several mathematics, because they consider the mathematics of teaching and mathematics of education are different. In other words, it is not merely the same mathematics taught elementarily in school, and the one taught in an advanced way for qualification. We have two mathematics of a different nature. Thus, *mathematics to teach*, originating from the mathematical disciplinary field, undergoes complex transformations, which can be explained through the meanings given to it in school culture (JULIA, 2001) so that it constitutes an object of teacher teaching. It is elaborated

> [...] by historical processes, revealing this debtor knowledge, in every period, of the purposes attributed to the school, of the pedagogy reigning in a given school time, of the current conceptions about mathematics, among several other determinants. (VALENTE, 2019, p. 53)

Yet *mathematics for teaching* does not refer to knowledges that are common to several professions, but to knowledges that teachers use, that are specific to mathematics teaching, and, just as *mathematics to teach*, it is also the result of historical processes (VALENTE, 2019a). Such mathematics represents a knowledge about the object of teaching so that it is possible, according to conceptions of a given time, that the teaching profession takes place. In this sense, *mathematics to teach* and *mathematics for teaching* are articulated in each historical time.

The existence of those two mathematics refers historically to the relationships established between the educational field and the mathematical disciplinary field. Given the need for the schooling of knowledges, the dialogues between these fields elaborate *knowledges to teach* and *for teaching*.

Through the analysis of the articulations between the mathematical disciplinary field and the field of education, we have what can be called pedagogical waves, which represent historical moments that consolidate notions of teaching, learning, school purposes, among many other elements that are part of the dynamics and processes of schooling.

Thus, each pedagogical wave leaves its marks in the teaching of mathematics. One of them concerns the organisation of the knowledges. By organising knowledges in levels, which we will call from now on grading, we mean the process of organising the contents, in this case, mathematics contents, going through steps that indicate the teacher's teaching. Where to start, how to establish work steps, what connections data content should have with each other so that they can be considered requirements, etc. How should teaching progress?

Thus, we should emphasise, for example, that it is not:

> [...] the teacher's pleasure to linger on a theme, on a subject; or even to minister only certain content that, in a particular, individual way, the teacher considers important. Since the

> graduated school, at the end of the nineteenth century, the programmes indicate when, during the course, some content should be taught, i.e., they establish a *grading*, a movement that indicates how teaching should advance. (VALENTE, 2019a, p. 55, author's emphasis)

Grading mathematics teaching is an ingredient of the teachers' work. A given grading in mathematics teaching is a teacher's work tool. It governs the sequence of classes, themes of some content, exercises, problems, etc. In those terms, the teaching profession does not dispense with a grading. Also, we should mention that a given grading is directly linked to how a conception of teaching and learning of students is consolidated at a given time. Such notion is typical of a given pedagogical wave, of a given pedagogy that is established as hegemonic in a specific period of the history of education. These considerations lead us to think that grading as a tool of the teaching profession is an element of *knowledge for teaching*. Specifically, for what is being studied in this article, grading refers to the constitutive element of an *arithmetic for teaching*.

Thus, this work sought to analyse how knowledge teaching grading of arithmetic was proposed by Faria de Vasconcelos' handbook *Como se ensina aritmética*. Which elements in this handbook can we take as those of an *arithmetic for teaching*?

Choosing the handbook

Previously, we discussed how grading the teaching of a given knowledge is related to the pedagogical wave in force. Hence, this work focuses on the grading of the arithmetic teaching in the movement that became known in Brazil as New Education[5], in a strand known as scientific pedagogy.

[5]A deepening of the theme of the New Education in Brazil, as well as its most diverse strands, can be read in the book *Brasil arcaico, Escola Nova: ciência, técnica e utopia nos anos 1920-1930* (Archaic Brazil, New Education: Science, Technique and Utopia in the years 1920-1930) authored by researcher Carlos Monarcha (2009).

As stated before, this study is part of a broad project, dialoguing with research from other researchers equally interested in studying mathematics present in teaching and teacher education. For this text, we turn our attention to teacher education, searching for elements of an *arithmetic for teaching*. Thus, we dialogue with the results of Pinheiro's doctoral research (2017). This author showed how scientific pedagogy transformed school arithmetic in Brazil.

Based on the experimental psychology of statistical measurement, characterised mainly by the application of psychological and pedagogical tests (SOARES, 2014; BASSINELO, 2014, and VALENTE, 2014), scientific pedagogy was the reference basis for the construction of a *tailored arithmetic* made known in textbooks (PINHEIRO, 2017). An *arithmetic to teach*.

Given these results of Pinheiro's research (2017) on the categorisation of an *arithmetic to teach*, the issue of how teachers should be trained to teach this knowledge remained. Specifically, what *arithmetic for teaching* should the teacher form so that he/she is qualified to teach a *tailored arithmetic*?

To answer this question, this research chose to analyse the pedagogical handbooks.

The pedagogical handbooks as sources for the analysis of elements of *arithmetic for teaching*

The pedagogical handbooks are important instruments to address teacher education historically. History of education researchers show that they can assist with the interpretation of the pedagogical discourses (BOTO, 2018).

In the context of the development of the broader thematic project, some works have already used pedagogical handbooks as an analytical means to understand the knowledge put into teaching and teacher education in given historical times. As an example, we can cite Maciel's (2019) doctoral thesis which, with a time frame covering from 1880 to 1920, sought to characterise

elements of the professional knowledge of the teacher who teaches mathematics in pedagogical handbooks.

Maciel's (2019) analysis showed, among other things, the objectification directives of mental calculus, mainly in the pedagogical waves known as traditional school and intuitive method. Thus, based also on these results and in view of a subsequent pedagogical wave – called New Education - in this article the focus of research turns to an important aspect of this pedagogical wave known as scientific pedagogy, because, as previously highlighted, Pinheiro's research (2017) showed how this pedagogy was inserted in Brazilian education, resulting in a *tailored arithmetic*, an *arithmetic to teach*. What *arithmetic for teaching* was elaborated so that teaching was possible? We will seek to answer the question in a restricted way, analysing a pedagogical handbook.

The choice of a handbook for this theoretical exercise went mainly through two stages: in the first one, we searched for papers that dealt with teacher education handbooks in the period studied, in which the pedagogical wave of the New Education was in force. The second stage, in turn, included official documents that ratified the recommendation to use handbooks, or to a certain extent, appropriated them as proposed by Chartier (2003). We believe that this procedure endorsed those handbooks, considered important for their circulation and use in teacher education within the time frame outlined by the doctoral project, which presents its partial results here.

In this process of choosing handbooks, Faria de Vasconcelos's name stood out as far as arithmetic based on scientific pedagogy is concerned, as shown by Marques (2013; 2018). On the other hand, it is a work referenced in teacher education programmes of the Instituto de Educação de São Paulo in the 1930s. The choice of the handbook *Como se ensina aritmética* is also related to the fact that it was used to substantiate the writing of *A construção científica dos programas* (1934) by José Ribeiro

Escobar[6], a teacher at Escola Normal de São Paulo. Escobar, just one year after Faria de Vasconcelos's publication, transcribed five sections that emphasised psychology and the renewal of the teaching of arithmetic.

Faria de Vasconcelos: *Como se ensina aritmética?*

Faria de Vasconcelos's handbook, as highlighted above, was chosen because it is a representative of scientific pedagogy, an approach that the author made clear already in the preface of the work by stating that:

> The way to deal with the problems, the conclusions reached, the recommendations made, are based on the results of the most recent investigations carried out in the field of didactics, renewed by psychology and scientific experimentation. (VASCONCELOS, 1933, p.9)

In the section *A psicologia e a renovação do ensino de aritmética* (psychology and the renewal of the teaching of arithmetic of the handbook), transcribed by Escobar (1934), the affiliation of the work to the scientific pedagogy is reinforced when it reads that the teaching

> [...] has been improved and renewed, even in some aspects, mainly thanks to the intervention of the three factors: use of psychology data, research practice and application of mental and pedagogical measures. (VASCONCELOS, 1933, p. 17-18).

Thus, once again, an emphasis is placed on psychology and tests as an essential factor for the development of the teaching of arithmetic. According to Vasconcelos, to progress in this teaching "[...] it is also necessary, besides the psychological knowledge of the student, the logical knowledge of the discipline and the processes of teaching" (1933, p. 18).

6 José Ribeiro Escobar graduated from Escola Normal de São Paulo in 1903, and taught arithmetic and algebra between 1921 and 1929, when he was sent on a mission to the state of Pernambuco to help the primary education reform (CAMPOS, 2018). Back in São Paulo in the early 1930s, he became head of the programmes and textbooks division of the general board of education of the state of São Paulo, where he wrote *A construção científica dos programas*.

From this quotation, we can observe that between the lines of the author's speech the relationship between disciplinary field and education sciences is explicit, as discussed by Hofstetter and Schneuwly (2017b). The teacher must master the disciplinary content, at the same time that he/she must be aware of the students' psychological aspects.

Thus, the reading of Vasconcelos' teaching handbook (1933) allows us to understand how scientific pedagogy affected arithmetic teaching. In other words, how this pedagogical wave transformed teaching and teacher education in relation to mathematics.

The manual gives us a sense of how psychology, especially experimental psychology of statistical measurement, imposes its logic on this organisation that is related to the grading of teaching. As previously mentioned, a given grade is implicitly linked to a conception of teaching and learning of students. The way to grade is closely associated with this notion. In the period we studied, scientific pedagogy, in which statistically-based experimental psychology predominated, commanded this grading, justified how teaching pace should occur, i.e., the step-by-step teaching work with school contents, with school arithmetic.

In previous studies, it has been possible to highlight this appropriation of psychology and statistics in arithmetic for teacher education, which was obtained as a result of studies conducted by Ferreira and Valente (2020). In this work, when analysing changes in the professional knowledge of the teacher who taught arithmetic, the authors, based on scientific pedagogy, through Aguayo's (1936) publication, concluded that there was a serialisation of the exercises that were the results of psychological tests, among other things. In this case, a scientific pedagogy referencing the teaching work for arithmetic teaching is evidenced.

This study, when considering Faria de Vasconcelos's handbook, points to the analysis of a new empirical material, through which the systematisation of what we can characterise in the future as a grading process of mathematical knowledge, in the scientific pedagogy time frame,

one of the strands of the New Education continues at a broader level. How this occurs in Vasconcelos's handbook is the subject of this study, as previously said. Let us begin, therefore, more specific analysis of the grading proposed in this handbook.

The rhetoric of Vasconcelos's work is based on the need for scientific work in education, something specific to the discourse that supports scientific pedagogy. Thus, the pedagogical work needs to take into account the results of scientific research. For example, the need for those investigations, as Faria de Vasconcelos indicates, concerning the teaching of numbers and their learning, was in the fact that it

> [...] depends primarily on the child's preparation, their development degree, their interest in the number and the knowledge they have acquired from it. If the child, as the authors of the research we will describe state, has a functional understanding of small numbers, and can communicate with other children and adults on the basis of a common knowledge of small quantities, then we have the indication that the child is prepared for teaching of the number and something different from this delays their knowledge. (VASCONCELOS, 1933, p. 41-42)

Taking into account this need to know where to start teaching numbers, one can take as an example the relationship between the investigations promoted by scientific pedagogy and this teaching. From the research conducted by Buckingham and Maclatchy, the author points out that they

> [...] conducted an extensive investigation of an individual nature, which was carried out with 1,356 children from 17 cities and villages and some rural districts. In their investigation, they used six 'tests.' (VASCONCELOS, 1933, p. 42)

Based on those tests, one of the conclusions reached by the authors, according to Faria de Vasconcelos's report, was that six-year-old children started school with considerable knowledge of the numbers already. Take, for example, mental calculations, in which they concluded:

> Memory counting: 90 % of children count at least up to 10 and 60% up to 20; the typical child (average) counts up to 27 or 28; 1 in 8 counts up to 100. Half the children count tens to 40, while only a quarter count that way. (VASCONCELOS, 1933, p. 42-43)

In the excerpt above, we have an example of how scientific pedagogy using tests indicated the students' level of knowledge statistically, in this case, in relation to mental counting. The results obtained by the tests, by showing the extent to which students can count mentally, give the elementary-school teacher references as to where to start their work. Knowledge, therefore, will be available to the teacher for the exercise of his/her craft. Thus, scientific pedagogy changes elements of the teacher's professional knowledge.

The application of tests also indicates a sequence of combinations of objects that takes into account the frequency of students' hits. Another example of tests applied by Buckingham and Maclatchy and presented by Faria de Vasconcelos refers to the combination of verbal problems related to the addition operation:

> [...] the children were examined through ten verbal problems to see if they knew some sum combinations. The combinations used in this test were as follows: 5 +1, 7+1, 1+9, 4+4, 1+6, 5+2, 8+2, 4+5, 5+3 and 3+5, which corresponded respectively to the percentages: 71.5, 63.9, 48.5, 36,9, 48,5 43.8, 43.6, 21.8, 31.8, 26.9. Only 7% of the children answered them all correctly. (VASCONCELOS, 1933, p. 44)

Another investigation highlighted by Faria de Vasconcelos was carried out by Frank Leslie Clapp[7], author who, as Almeida and Leme da Silva (2014) show, was appropriated in Brazil by Alfredina de Paiva e Souza[8], from the Instituto de Educação do Rio de Janeiro. The authors

[7] According to Almeida and Leme da Silva (2014), Frank Leslie Clapp was a "psychologist and lecturer of education at the University of Wisconsin, Madison" (ALMEIDA; LEME DA SILVA, 2014, p. 53).

[8] Alfredina de Paiva e Souza worked at the Instituto de Educação do Rio de Janeiro where "[...] she was the head teacher of the teaching practice section and taught calculus. Besides being a lecturer and full professor at the Institute, she conducted research and experimental work on mathematics teaching, supported by international studies. Thus, she produced teaching handbooks,

point out that based on Clapp's tests, regarding numerical combinations, teacher Alfredina applied a series of tests in eleven schools in Rio de Janeiro to 1,673 students.

Questions about where to start teaching and how to measure learning, according to Faria de Vasconcelos, were initially raised by the *Committee of Seven*[9] that at the Illinois Conference sought to answer these questions "[...] through a series of investigations that extended over five years and required the cooperation of 148 cities and many thousands of children" (VASCONCELOS, 1933, p. 47).

Based on Washburne, Faria de Vasconcelos discusses the importance of knowing the right moments to teach operations, according to him the "[...] failures in arithmetic are due, in large part, to the fact that the teaching of operations and processes is done in inappropriate time" (VASCONCELOS, 1933, p. 50). This argument reinforces his defence that scientific pedagogy should guide the teacher's actions.

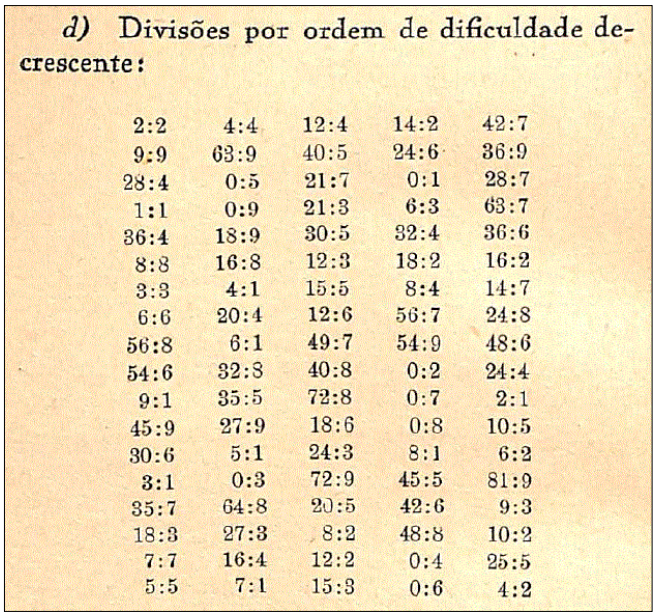
Regarding the operations, the author argues that Clapp, when testing 7,000 children, studied the relative difficulties of the combinations and elaborated a sequence for each of them according to the increasing order of difficulty. Take here as an example, the case of the division[10] shown in Figure 1.

textbooks and published articles disseminating the results of research developed within the scope of the Instituto de Educação, *locus* of circulation and production of scientific knowledge" (ALMEIDA; LEME DA SILVA, 2014, p. 52).

[9]A group of experts gathered by Washburne "[...] under well-defined conditions to elaborate scientifically a minimum programme for the teaching of arithmetic in primary school" (PINHEIRO, 2017, p. 56).

[10] Aguayo appropriates those results of Clapp's research in his handbook on scientific pedagogy, thus, discussions on the sequence of the addition operation can be read in authors who to some extent analysed Aguayo's handbook, as did Almeida and Leme da Silva (2014), and Ferreira and Valente (2020).

FIGURE 1: Sequence of Clapp combinations of division in ascending order.



d) Divisões por ordem de dificuldade decrescente:

2:2	4:4	12:4	14:2	42:7
9:9	63:9	40:5	24:6	36:9
28:4	0:5	21:7	0:1	28:7
1:1	0:9	21:3	6:3	63:7
36:4	18:9	30:5	32:4	36:6
8:8	16:8	12:3	18:2	16:2
3:3	4:1	15:5	8:4	14:7
6:6	20:4	12:6	56:7	24:8
56:8	6:1	49:7	54:9	48:6
54:6	32:8	40:8	0:2	24:4
9:1	35:5	72:8	0:7	2:1
45:9	27:9	18:6	0:8	10:5
30:6	5:1	24:3	8:1	6:2
3:1	0:3	72:9	45:5	81:9
35:7	64:8	20:5	42:6	9:3
18:3	27:3	8:2	48:8	10:2
7:7	16:4	12:2	0:4	25:5
5:5	7:1	15:3	0:6	4:2

Source: (VASCONCELOS, 1933, p. 64).

With those scientific pedagogy characteristic data, the teacher could teach division following graded difficulties pre-established by the psychological tests. It is an example of how this pedagogy affects elements of the professional knowledge of the teacher who teaches mathematics, an *arithmetic for teaching*, given that it is a specific one of the teaching profession. By such references, we understand that the teacher's pedagogical work should not be performed according to the personal preferences of each teacher. It cannot be up to the teacher to elaborate a set of exercises or arithmetic problems to work with their students. He/she should use what has already been the subject of mental and pedagogical tests. In doing so, he will teach scientifically.

Conclusion

This article aimed to analyse aspects of how the grading of the arithmetic teaching was proposed by Faria de Vasconcelos's handbook *Como se ensina aritmética*.

Supported by tests carried out by other researchers, the author addressed some elements of this grading, such as when to start teaching numbers, based on the children's level of mental development, which should be determined statistically. Also, how to organise exercises and problems, based on scientific investigations - i.e., statistics-based experimental psychology studies –to follow the course of what was believed to agree with the development of children's learning.

In the handbook analysed, the tests were also responsible for determining the sequence of teaching operations and the most appropriate moments for this. Thus, they produced a transformation in elements of the professional knowledge of the teacher who taught mathematics, as they affected the very logic of knowledge organisation, since the teaching sequence that the teacher should follow was no longer that produced by the disciplinary field of mathematics, but that which had been scientifically elaborated and tested, a logic proper to scientific pedagogy.

Farias de Vasconcelos's handbook addresses the teacher to guide him/her to perform the pedagogical work in a way that is considered scientific. Specifically with regard to arithmetic, there is a new organisation of this knowledge ongoing in school, where elements of a new tool that the teacher should use are evident, a knowledge about arithmetic that the teacher did not have, and that is provided by the tests, by the statistical studies that indicate to the teacher how he/she can make his/her craft more efficient and, thus, promote the learning of tailored arithmetic, an arithmetic forged by mental and pedagogical tests (PINHEIRO, 2017). In this movement, the teaching of such arithmetic needs an *arithmetic for teaching*. Vasconcelos (1933) reveals elements of this new knowledge.

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