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The contribution of Didactic Engineering as a methodological contribution to the teaching of Probability[1]

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ABSTRACT

This article is part of a research carried out in a private school in the city of Rio de Janeiro, with students in the sixth year of elementary school based on the theme: experimentation through frequentist probability to verify the classic probability. The objective of this research was to investigate the contributions of the qualitative methodological approach, subsidized by Didactic Engineering, for teaching probability. To this end, some experiments were carried out with the students, made possible by the use of a technological roulette wheel created in an application for this study and installed on their cell phones, which had equiprobable and non-equiprobable spaces. It is noteworthy that working with experimental activities that culminate in regularity can bring about a movement towards learning probabilities, in addition to keeping the focus on the specific skills of the BNCC, enhancing the understanding of the basic ideas of probability, thus leading to the understanding that the qualitative methodological approach can contribute to the teaching of this theme.

KEYWORDS: Mathematics Education. Statistical Education. Probability. Didactic Engineering.

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A contribuição da Engenharia Didática como aporte metodológico para o ensino de Probabilidade

RESUMO

Este artigo é parte de uma pesquisa realizada em uma escola da rede particular na cidade do Rio de Janeiro, com estudantes do sexto ano do ensino fundamental a partir da temática: experimentação por meio da probabilidade frequentista para verificação da probabilidade clássica. O objetivo desta pesquisa foi investigar as contribuições da abordagem metodológica qualitativa, subsidiada pela Engenharia Didática, para o ensino de probabilidade. Para tal, foi realizado com os estudantes, algumas experimentações, possibilitadas pelo uso de uma roleta tecnológica criada pelas autoras em um aplicativo que foi instalado nos celulares dos alunos previamente, que contava com espaços equiprováveis e não equiprováveis. Destaca-se que o trabalho com atividades experimentais que culminam a uma regularidade, pode trazer um movimento para a aprendizagem de probabilidades, além de manter o foco nas habilidades específicas da BNCC, potencializando o entendimento das ideias básicas de probabilidade, levando, assim, à compreensão que a abordagem metodológica qualitativa pode contribuir para o ensino desta temática.

PALAVRAS-CHAVE: Educação Matemática. Educação Estatística. Probabilidade. Engenharia Didática.

El aporte de la Ingeniería Didáctica como aporte metodológico a la enseñanza de la Probabilidad

RESUMEN

Este articulo hace parte de una investigación realizada en una escuela de la red particular en la ciudad de Rio de Janeiro, con estudiantes de sexto año de escuela primaria a partir de la temática: experimentación por medio de la probabilidad frecuentista para la verificación de la probabilidad clásica. El objetivo de esta investigación fue poner en claro las contribuciones del enfoque cualitativo usando la ingeniera didáctica, para la enseñanza de la probabilidad. En este sentido, fue realizado con los estudiantes, algunas experimentaciones, posibilitadas por el uso de una ruleta tecnológica creada con un aplicativo para este estudio e

instalada en sus celulares que contaba con espacios equiprobables y no equiprobables. Se destaca que el trabajo con actividades experimentales que culminan con regularidad, logrando traer un movimiento para el aprendizaje de la probabilidad, además de eso, mantener el foco en las habilidades especificas de la BNCC, potencializando el entendimiento de las ideas básicas de probabilidad, llevando así, a la comprensión que el enfoque cualitativo puede contribuir para la enseñanza de esta temática.
PALABRAS CLAVE: Educación Matemática. Educación Estadística. Probabilidad. Ingeniería Didáctica.

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Introduction

In the field of Mathematics Education, qualitative research has been standing out and outlining research possibilities, especially with regard to classroom research. With the aim of analyzing and praising the path, or the process and not just the results obtained, this research methodology has been unfolded in several directions such as, for example, ethnographic research, action research, case studies, discourse analysis , narratives, memory studies, life stories, among others (ANDRÉ, 2001). In qualitative research methodology, there is a dense analysis of the process and results, which aims to add discussions, results, possibilities and even limitations to existing knowledge.

In qualitative research, the idea of the subjective is present, capable of exposing sensations and opinions, taking into account the perspectives and interpretations of the participants. The meaning that is attributed to this method encompasses notions regarding perceptions of differences and similarities of comparable aspects of experiences (ARAÚJO; BORBA, 2006).

Thus, Didactic Engineering is a methodological approach that is inserted in the context of qualitative research as it was characterized by Artigue (1988) as an experimental scheme based on didactic achievements carried out in the classroom.

The choice of Didactic Engineering to develop this research is due to the understanding of its contribution to the construction of knowledge in the area of Mathematics Education. In other words, by writing this article, in which this research methodology was used, it is expected to contribute to the construction of knowledge in the area of Mathematics Education, providing evidence on pedagogical practices and teaching and learning strategies. Therefore, to elucidate how this methodology works, we will describe an investigation that focused on the topic of Probabilistic Education and used Didactic Engineering as a means, or method, to achieve the results.

With this, we will follow Almouloud (2007) and present the experimental process of Didactic Engineering, which is composed of four phases, namely: (i) Preliminary Analysis; (ii) A priori Construction and Analysis; (iii) Experimentation and, (iv) A posteriori Analysis and Validation of the Hypothesis.

In this context, with the aim of investigating the contributions of the qualitative methodological approach, supported by Didactic Engineering, to teaching probability, two activities were developed that sought to unite the frequentist and classical approaches to probability. In the first activity, a study was proposed in an equiprobable sample space, while in the second activity, a non-equiprobable sample space was explored.

In addition to the objective mentioned above, this research is guided by a question related to the topic: what are the possibilities arising from the integration of classical and frequentist approaches in teaching Probability?

To present this research, the writing of this article was divided into six sections, including this introductory text. Next, in the preliminary analyzes of Didactic Engineering, an explanation is presented about the teaching of probability, bringing the National Common Curricular Base (BRASIL, 2018), its indications of knowledge objectives for the sixth year of elementary school, the schooling in which research has been carried out and the skills expected in the official document. Important points are discussed about teaching probability in equiprobable and non-equiprobable spaces and about the classical and frequentist approaches to probability, pointing to the importance of a possible combination of these two approaches. Furthermore, qualitative research supported by Didactic Engineering in Mathematics Education is conceptualized.

In the following sections, it is described how the research was consolidated within each of the four phases of the methodology adopted, from the preliminary analyses, through the second phase of didactic engineering, the construction and a priori analysis, followed by experimentation with students in classroom, until the comparison between the predictive results and those actually obtained, that is, in the a posteriori analysis and validation of the hypothesis, in light of the theory discussed in the section on teaching probability in preliminary analyses.

Application of Didactic Engineering: Preliminary Analysis

The implementation of Probability and Statistics in basic schools began in 1980 with the creation of the International Association for Statistical Education (IASE) by the United States. The objective of IASE is the dissemination, implementation and consolidation of Statistical Education in basic schools. Acting as a document that guides teaching in Brazil, we have the National Common Curricular Base (BNCC) (BRASIL, 2018) which, in line with the IASE recommendation, points to the teaching of Probability and Statistics.

Early Childhood Education, which encompasses the period up to six years of age, according to the BNCC, does not indicate any work related to the fields of Probability. In Elementary School - Early Years, it organizes the school curriculum into five thematic units, namely: numbers, algebra, geometry, magnitudes and measurements and probability and statistics.

At this stage of Basic Education, Probability must be worked on in order to

> [...] promote the understanding that not all phenomena are deterministic. To this end, the beginning of the proposed work with probability is centered on developing the notion of randomness, so that students understand that there are certain events, impossible events and probable events. (BRASIL, 2018, p. 274).

This excerpt from BNCC highlights the importance of promoting compression on statistical phenomena, especially non-deterministic phenomena, so that work proposals on this topic can explore the notion of randomness, thus favoring the fundamental understanding that events are uncertain and vary according to different circumstances. By promoting differentiation between types of events, this approach develops students' critical thinking and ability to evaluate and estimate the probability of events occurring in different contexts.

For Elementary School - Final Years, the BNCC (2018) advises that the study of Probability

> [...] it must be expanded and deepened, through activities in which students carry out random experiments and simulations to compare the results obtained with theoretical probability – frequentist probability. (BRASIL, 2018, p. 274)

Thus, the BNCC (BRASIL, 2018) already presents the confrontation between classical probability and frequentist probability, which can be calculated by empiricism, or even, with the use of technologies so that a certain event can be carried out in a number big times. Coutinho (2002) already pointed to a focus on teaching Probability through the classical mode together with the frequentist one, since

> [...] this approach allows the comparison of the two main points of view when we define a probability: the classical or Laplacian point of view and the frequentist point of view. Under these conditions, the construction of the concept by the student is done in such a way that he has fewer possibilities of mobilizing them outside his domain of validity, that is, with fewer possibilities for this concept to become an obstacle to future learning in the domain. of Probability Calculation. (COUTINHO, 2002, p.9).

According to the excerpt, by exploring both points of view, the construction of the concept of probability seeks to ensure that students are less likely to generalize it inappropriately or apply it beyond its domain of validity. This means that approaching activities with this focus can favor the understanding of the conditions in which these probabilistic concepts are applicable.

The BNCC on the topic Probability in Elementary Education – 6th year points out two objects of knowledge (content and concepts), they are:

> Probability calculation as the ratio between the number of favorable outcomes and the total possible outcomes in an equiprobable sample space.
> Probability calculation through many repetitions of an experiment (frequencies of occurrences and frequentist probability) (BRASIL, 2018, p. 304).

In addition, BNCC also presents a skill (knowledge necessary for the full development of skills) for probability in the sixth year, which is:

> Calculate the probability of a random event, expressing it as a rational number (fractional, decimal and percentage) and

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> compare this number with the probability obtained through successive experiments (BRASIL, 2018, p. 305).

Following in this direction, it was decided to address both classical probability and frequentist probability with students. This decision was motivated by the relevance of allowing students to recognize that sample spaces do not always present equal probability for each event.

To define probability, the students worked with Classical Probability, which can be calculated in the classical way, defined by Laplace:

> Suppose randomized experiments have the following characteristics:
>
> a) There is a finite number (say n) of elementary events (possible cases). The union of all elementary events is the sample space Ω.
>
> b) Elementary events are equally probable.
>
> c) Every event A is a union of m elementary events where m ≤ n.
>
> We then define:
>
> Probability of A = P(A) = $\frac{\text{number of favorable cases number}}{\text{possible cases}}$.
>
> (MORGADO et al, 1991, p. 64)

This definition of probability assumes that all events are equiprobable, that is, they all have the same chance of occurring. As some events cannot be repeated under the same conditions, we see that classical probability alone cannot quantify all probabilistic situations, in addition to the fact that there are several experiments inserted in the cultural context of students that do not have the same chance of occurrence and that can be brought to the classroom as a form of encouragement by the teacher so that the students themselves can exemplify probability in their daily lives.

Coutinho (1996) is concerned about students being exposed only to classical probability, since they may construct a false idea that probability is always calculated in equiprobable spaces. Given this, the hypothesis was raised for this work that the combination of classical and frequentist approaches can enhance the learning of fundamental probability ideas.

In this way, performing events repeatedly can lead students to verify variability, randomness, uncertainty and also estimate a probability of occurrence of a certain event, using the frequentist mode. The probability calculation can be defined using the frequentist mode as:

> The frequentist definition is based on the relative frequency of a large number of experiment realizations. More specifically, we define the probability P(A) of an event A using the limit of the relative frequency of the occurrence of A in n independent repetitions of the experiment, with n tending to infinity, that is, P(A) = $\lim_{n \to \infty} \frac{1}{n}$ x (number of occurrences of A in n independent realizations of the experiment) (NETO, 2016, p. 23).

Thus, the probability of an event is verified through significant repetitions of that event and, with the results obtained, a value can be estimated that describes its possibility of occurrence, with this approach not only using the equiprobable space for calculations. . As the use of this approach requires a large number of experiments, research indicates the importance of using information technology as a tool for simulating experiments, thus being able to favorably achieve an estimate of the probability sought (BITTAR; ABE, 2013).

In addition to the relevance of probability that was highlighted by BNCC, it is important to highlight the ideas published by Gal (2005). According to the author, randomness, predictability and the calculation of probabilities are essential elements of the knowledge of a probabilistically literate person. Furthermore, the author also highlights the importance of the language used to communicate about chance, the context in which the questions are inserted and the importance of reflecting on the results obtained.

To carry out the research proposed in this work and described so far, a qualitative analysis of the information will be carried out, since “the direct source of data was the natural environment, constituting the researcher as the main instrument” (BOGDAN; BIKLEN, 1994, p. 47), and is interested in the context in which the research problem is being investigated. In this case, the sixth year classroom of a private school in the city of Rio de Janeiro is the environment in which the research was carried out and also where one of the authors works as a leading teacher in Mathematics classes.

The option for qualitative methodology for the development of this research is reiterated when the researchers analyze the performance of each participant, as the main focus of this work is on the processes that will be carried out by the students to achieve the results (BOGDAN; BIKLEN, 1994) and not just the result itself.

Therefore, it is worth highlighting that analyzing the process of each activity is very important, considering that this characteristic distinguishes qualitative research from others. Thus, it is understood that:

> [...] qualitative researchers establish strategies and procedures that allow them to take into account experiences from the informant's point of view. The process of conducting qualitative research reflects a type of dialogue between researchers and their respective subjects, which are approached by them in a neutral way. (BOGDAN; BIKLEN, 1994, p. 51)

The aspects observed by the authors (BOGDAN; BIKLEN, 1994) and which were highlighted in this research, show that the use of Didactic

Engineering makes it possible to understand the students' perspectives and experiences, seeking a contextualized understanding of the subject. To this end, activities that involved active dialogue and meaningful interactions were used.

It is also worth highlighting that the theoretical assumptions of Didactic Engineering, according to Artigue (1988), support an experimental scheme based on “didactic achievements" in the classroom, that is, on the conception (or construction), realization, observation and analysis of sequences of teaching.

According to Pais (2002), Didactic Engineering's object of study is the development of concepts and theories that are compatible with the educational specificity of mathematical school knowledge, seeking to maintain strong links with the formation of mathematical concepts. This trend aims to understand the conditions of production, recording and communication of school mathematics content and its didactic consequences, in addition to establishing connections between theory and practice.

To implement the Didactic Engineering methodology, it is necessary for the researcher to follow the four phases that were presented previously and which will be detailed below.

For Machado (2002), preliminary analyzes are carried out mainly to support the conception of the Didactic Engineering research methodology, they are revisited and deepened throughout the course of the work. In these terms, Almouloud (2007) states that one of the objectives of preliminary analyzes is to identify the teaching and learning problems of the object of study, in addition to outlining in a well-founded way the questions, hypotheses, theoretical and methodological foundations of the research.

Still for Almouloud (2007), the second phase of Didactic Engineering, of a priori construction and analysis, includes a description part and a prediction part. The purpose of this phase is to develop and analyze in a predictive sense a didactic sequence to answer the questions and validate the hypotheses raised in the previous phase.

In the next phase, experimentation, Machado (2002) states that it is the phase of carrying out Didactic Engineering with a certain population of students. This is the time to put into operation everything that was built during the other phases.

This phase is followed by a posteriori analysis and validation that is based on the data collected in the experimentation phase. Almouloud (2007) states that this phase is the set of results that can be obtained from the exploration of the data collected and contributes to the improvement of teaching knowledge on the topic in question.

The comparison of a priori and a posteriori analyzes that validate or refute each other is the last phase of the research, as well as the validation of the hypothesis initially raised. The objective is to relate the observations with the objectives defined a priori and estimate the reproducibility and regularity of the didactic phenomena found (AMOULOUD, 2007).

Application of Didactic Engineering: Construction and a priori Analysis

Seeking to achieve the objective of this research, two activities composed of five tasks each were applied, which sought to relate the classical probability calculation with the estimate given in the frequentist approach, analyzing two sample spaces, one being equiprobable and the other non-equiprobable.

When developing the proposed activities, we took into account the understanding that children and many adults often have difficulty thinking rationally and quantifying probability, even though they understand the importance of randomness and probability in our lives (BRYANT; NUNES, 2012). Furthermore, taking into account the students' context (GAL, 2005), teams from the city of Rio de Janeiro were used, about which the students could have prior knowledge.

To do so, it is important to base yourself on four different aspects about events and the sequence in which they occur, they are: understanding randomness, working the sample space, comparing and

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quantifying probabilities and understanding the relationships between events (BRYANT; NUNES, 2012).

It is understood that activity and task are notions that mathematics educators consider to constitute basic didactic categories. An activity can include performing numerous tasks. Most importantly, the activity concerns the student and refers to what he does in a given context. The task, on the other hand, represents only the objective of each of the actions in which the activity unfolds (PONTE, 2014).

In the activity that proposed work in an equiprobable sample space, we sought to develop the calculation through Laplacian probability and estimate through successive attempts using a roulette wheel previously constructed by the authors and available via a link, to be accessed by cell phone of a representative of the student group.

To use roulette with alternatives, “the use of technologies deserves to be highlighted” (BRASIL, 2018, p. 270), as it makes it applicable a sufficiently large number of times that the experience is carried out, without being boring for the student.

As pointed out by BNCC (BRASIL, 2018), it is understood that:

> The study must be expanded and deepened, through activities in which students carry out random experiments and simulations to compare the results obtained with theoretical probability and frequentist probability (BRASIL, 2018, p. 270).

In these terms, the predictive nature of this phase focuses on students' ability to calculate the probability of a random event, expressing it as a rational number (fractional, decimal and percentage) and comparing this number with the probability obtained through successive experiments. .

Participating in the research were eight students in the sixth year of elementary school, from a private school in the state of Rio de Janeiro. In this school, Mathematics is responsible, in the students' school routine, for five weekly periods of fifty minutes each. The contents of the classes are established by the teaching material adopted by the school, in accordance with the BNCC.

At the time of implementing this activity, we were going through the process of returning to face-to-face teaching, due to the stoppage of face-to-face classes in 2020, due to the COVID-19 pandemic. In this context, we emphasize that in 2020 "we were left with little teaching, little learning, little content, little workload, little dialogue. On the other hand, we have many tasks" (SAVIANI; GALVÃO, 2021, p. 42).

In this context, although the majority of students knew how to calculate classical probability, as the ratio between favorable events and possible events, there was a weakness in the conceptualization of the random nature of probability, as well as a lack of distinction between equiprobable and non-equiprobable spaces. .

The first activity, presented below, sought to work with the equiprobable space and, in addition to the questions listed, showed the student a table with the four teams from Rio de Janeiro (Vasco, Flamengo, Fluminense and Botafogo) – all with the same area in roulette, that is, representing its equiprobable space.

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Table 1: activity in the equiprobable space

Activity 1
Consider the random experiment: spin the wheel and see which team the pointer stops on.
a) What is the sample space?
b) Do all teams have the same chance of leaving? Why?
c) Is the sample space equiprobable or not equiprobable? Why?
d) What is the probability of the pointer stopping at Flamengo's team? (Remember how we calculated probability)
e) Spin the roulette wheel 100 times. If you calculate the ratio “Flamengo team left” divided by the “total number of times the roulette wheel was spun”, what will be the result? Is the value found the same (or similar) to what you calculated in the previous item? How can we explain these results?

Source: created by the authors

The second activity sought to work with non-equiprobable space. Now, the roulette wheel represented the percentages of the number of fans of the Rio teams and, as a result, the area for the four teams from Rio de Janeiro is different, since the Flamengo fans are the largest among the fans of the Rio de Janeiro teams.

Table 1: activity in the non-equiprobable space

Activity 2
a) What is the sample space?
b) Do all fans have the same chance of leaving? Why?
c) Is the sample space equiprobable or not equiprobable? Why?
d) What is the probability of the pointer stopping at Flamengo fans?
e) Spin the roulette wheel 100 times. If you calculate the ratio “Flamengo fans came out” divided by the “total number of times the roulette wheel was spun”, what will be the result? Is the value found the same (or similar) to what you calculated in the previous item? How can we explain these results?

Source: created by the authors

Below, all intentions and possible responses to the activities previously discussed by the authors are presented. This prior description of the answers is foreseen in the a priori analysis stage of Didactic Engineering. Later, we will compare the expectations described here with the results obtained during the experimentation phase.

In task 1 of activity 1, it was expected that students could realize that the sample space was the four teams: Flamengo, Botafogo, Vasco and Fluminense. I also expected them to be able to answer 100% on this task. In task 2, students should realize that the chances of winning any of the teams were equal, since each of them occupied ¼ of the sample space, or even, that they had the same "size" on the roulette wheel.

In task 3, when the question was asked about the equiprobability of the sample space, the answer considered correct would be yes, although we did not expect the students to remember this concept, as it was included in the task precisely with the aim of bringing it to the discussion.

Task 4 was designed so that students could use probability calculation as the ratio between the favorable event (the pointer stops over the Flamengo team) and possible events. And it was expected that they would do so, as this concept had already been worked on in the classroom and, for this reason, we placed a reminder in the task statement.

Finally, in task 5 of the first activity, we asked students to spin the roulette wheel on their cell phone, with just one touch, 100 times, so that they could realize that the estimate could vary in relation to the probability calculation carried out in the previous task, and that the idea came to light that the more we increase the frequency of the experiment, the closer the estimate to the classical probability would be.

In activity 2, the tasks were similar to the tasks in activity 1, both in elaboration and in objective, the difference is that we expected the students to be able to realize that it was not an equiprobable space, where all events had the same chance .

Next, it is narrated how the experiment took place, with the eight students present.

Application of Didactic Engineering: Experimentation

As the Probability theme had already been worked on theoretically, in the classroom, with the teaching material used by the teacher and recommended by the school, the tasks were defined as using technology to calculate probability and then compare this calculation with the result obtained through successive experiments.

The class of eight students was divided into two groups of four students each. Each group received two printed sheets: one with the activity in which the sample space was equiprobable (activity 1) and another with the activity in which the sample space was not equiprobable (activity 2).

In order to study the results, we will call group 1 the group formed by the students: Maria, Ana, Joana and André. And group 2 made up of Edmundo, Sarah, Jaqueline and Denis.

Task 1 of both activities asked students what the sample space was. In the Times of Rio roulette, both groups answered “a circle”. In the Rio Times Fans roulette, group 1 answered “a circle” and group 2 answered “No, as they are about the percentage of fans”.

The second task of activity 1 on the Rio team roulette asked if, by spinning the roulette wheel, all teams would have the same chance of winning and why. Group 1 responded: “because they are different teams, but with equal chances”; and group 2 answered “Yes, because they all have the same shape and the same area”.

In the second task of activity 2, in the Rio Times Fans roulette, the question was regarding the possibility of all fans having the same chance of leaving. Group 1 responded: “if it were randomly played at roulette everyone would have the same amount, but if it were based on the percentage of fans they would not have the same chance". Group 2 responded: "No, because the areas are different.

The third task of both activities, 1 and 2, asked whether the sample space was equiprobable or non-equiprobable and why. For both roulette wheels, group 1 left this activity blank. Group 2 responded to the Rio Times roulette, in activity 1, that "the space is equiprobable because it has the same chance of leaving" and, in the Rio Times Torcidas roulette, in activity 2, group 2 responded: " Yes, because it has the same space."

The fourth task asked students to calculate the probability of the pointer stopping on the Flamengo fans, for the Rio Times Torcidas roulette and the Rio Times roulette, the request was that they calculate the probability of the pointer stopping on the Flamengo team. Flamengo. For this task, group 2 responded 25% in the two roulette wheels, in activities 1 and 2. Group 1 responded to the Rio Times roulette wheel: "25% because no team repeats itself and because equal amounts of each one, having 4 teams we could do 25 + 25 + 25 + 25 = 100 or 25 divided by 100 is equal to 4" and in the roulette wheel for the Rio Teams Fans, group 1 responded: "based randomly on the roulette wheel there would be a 20% chance" .

The last task proposed that students use each of the roulette wheels that were presented by the teacher through two respective links and accessed by a member of each group. This member was responsible for spinning each of the roulette wheels, one for each activity by touching the cell phone screen.

Group 1 spun the wheel 100 times and the needle stopped 23 times on the Flamengo team. For the answer that was given, the group wrote: "a similar result, since we rotated randomly and it fell 23 times, so there were only two left".

Group 2 also spun the wheel 100 times and the needle stopped 19 times on the Flamengo team. The group responded "Similar. It's a probability, it won't always be the same."

Group 1 spun the roulette wheel 100 times and of these, 23 times they ended up with Flamengo fans. They responded: “23%. Similar, as there was a 3% difference.”

Group 2 responded: “Similar (23)/100 is a possibility and will not always be the same”.

At the end of the class, the teacher collected the printed sheets on which the groups recorded the answers that were transcribed throughout this section.

Application of Didactic Engineering: A posteriori analysis and validation

In this section, with the data from the experimentation, a comparison will be made with the a priori conceptions and analyses.

Group 1, formed by students Maria, Ana, Joana and André, responded in task 1 of activities 1 and 2, that the sample space was a circle. In fact, the roulette wheel presented to the students is a circle, especially when we think that the sample space brings together all the possibilities of where the pointer can stop and, therefore, the area of the circle represents the sample space, which was immediately discussed with the students by the teacher. The predictive response would be 100%, or they would even list the names of the teams or fans, but their response is in any case surprising and directly confronts the a priori discussions held, showing how the classroom is dynamic, it is a movement constant and, most importantly, it is a place of knowledge production (GIRALDO, 2019).

Group 2, formed by students Edmundo, Sarah, Jaqueline and Denis, used the same reasoning as group 1 in task 1 of the first activity. But in the second, when dealing with the fans of the Rio teams, as we left the percentage of each fan, the group responded that it was not a sample space, since it was a roulette wheel on the percentages of the fans.

It can be seen, from the response given by the group, that the students in this group do not have a formed conceptualization about the definition of the sample space, which causes us concern, since the students already know how to use classical probability for calculations, that is, they act in an equiprobable sample space, but they do not know what the sample space will be.

At this point it is crucial to emphasize the importance of conceptualization. And, in this group's response, it is clear that knowing how to calculate a probability through the ratio between favorable events and possible events does not provide a student with an understanding of what these events actually are and the implications of the calculation carried out, such as states Gal (2005), when discussing the teaching of probability.

In task 2 of activity 1, group 1 responded in line with the predictive answer, responding that yes, all teams have the same chance of leaving. Group 2, in addition to highlighting that the teams have the same chance of leaving, when spinning the roulette wheel, the group justified this statement with the use of the area, pointing to the equality of the areas occupied by the four teams in the roulette wheel. This answer once again points to the combination of geometric probability with the teaching of probability, as it becomes clear to the group, visually through the roulette wheel, the equality of chances and the equality of areas.

In task 2 of the second activity, group 1, which previously used equality of chances through percentages, now justifies its answer by calculating the area, as the areas are not equal, with Flamengo fans being the ones with the highest probability chosen precisely because it occupies a larger area. This observation could also be made by the percentage that was placed together with the name of the fans, all of which are different from each other.

Group 2, in this same task, brings context knowledge to this activity, pointing out that the answer would be one in roulette, but if it were to consider the real crowd, it could be another. This response is important, as it shows how students have knowledge to be presented and dialogued with the knowledge seen at school, pointing to probability as a possibility of reading the world (FREIRE, 1996), and not just another content of a curriculum or of a crowded booklet.

Although the question in task 3 in both activities was the same, about the equiprobability of the space, group 1 left this task blank in the first activity, while in the second they responded that it was an equiprobable space (conversing with their previous task, where they responded that they had equal chances) and group 2 did the opposite, answering correctly in activity 1, about it being an equiprobable space and in the second activity, leaving it blank. The teacher left this possibility for the groups, to hand in a blank task, if the group decided together that they would not know how to answer, for the purposes of analysis in the research. In this way, it is clear and still current, as mentioned in this text, when Coutinho (1996) points to the non-conceptualization of equiprobability, leading students to always think they are acting in equiprobability spaces.

In task 4, of both activities, students were expected to calculate the probability using classical probability, which was directly answered by them, without much difficulty. In this way, it appears that one of the objectives of the activities had been achieved.

In task 5, of both activities, we sought to work with the estimation of the frequentist probability. The importance of this work is highlighted, as students were able to discuss while using cell phone technology, to repeat the experiment in each activity 100 times. Along this path, as described in the previous section, when making the comparison, students were able to realize that it is an approximation to Laplacian calculation, which is in line with the second objective of carrying out the activities and with the reference presented in the second section of this work.

Conclusion

This article aimed to investigate the contributions of a qualitative methodological approach to teaching probability in the light of Didactic Engineering, verifying which potentialities can be extracted from the combination of classical and frequentist approaches in a sixth-year elementary school class. .

Following the theoretical assumptions of Didactic Engineering in line with its phases, the research was consolidated on the teaching of probability in equiprobable and non-equiprobable spaces and on the classical and frequentist approaches to probability, under the recommendations of the National Common Curricular Base (BRASIL, 2018).

The qualitative nature of the research was essential so that there was an analysis of the written responses obtained by the students and not just the numbers that are found after using the formulas.

Furthermore, Didactic Engineering played a fundamental role in this research, allowing the recording of studies carried out on the didactic sequence and its validation, as proposed by Machado (2002).

This validation is an important point in research using Didactic Engineering and what makes this aspect of qualitative methodology different from others. Validation is internal, that is, the comparison takes place in the same group, with the expected results and the results obtained, different from what happens in research that uses external validation. It is important to note that this validation process was implemented in our work from the a priori conception and analysis phase.

With this, the objective of investigating the contributions of a qualitative methodological approach to teaching probability in the light of Didactic Engineering was achieved, since it is possible to describe the circumstances involved in the production, recording and communication of school mathematics content, as well as as well as its didactic implications, by establishing connections between theory and practice.

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