### Society, culture, mathematics and its teaching

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#### Abstract

One of the effects of the globalized world is a strong tendency to eliminate differences, promoting a planetary culture. Education systems are particularly affected, undergoing strong pressure from international studies and evaluations, inevitably comparative, and sadly competitive. As a result, one observes the gradual elimination of cultural components in the definition of education systems. The constitution of new social imaginaries becomes clear; imaginaries empty of historical, geographical and temporal referents, characterized by a strong presence of the culture of the image. The criteria of classification establish an inappropriate reference that has as its consequence the definition of practices and even of education systems.

On the other hand, resistance mechanisms, often unconscious, are activated seeking to safeguard and recover the identifying features of a culture, such as its traditions, cuisine, languages, artistic manifestations in general, and, in doing so, to contribute to cultural diversity, an essential factor to encourage creativity. In this article, the sociocultural basis of mathematics and of its teaching are examined, and also the consequences of globalization and its effects on multicultural education. The concept of culture is discussed, as well as issues related to culture dynamics, resulting in the proposition of a theory of transdisciplinar and transcultural knowledge. Upon such basis the Ethnomathematics Program is presented.

A critique is also made of the curriculum presently used, which is in its conception and detailing, obsolete, uninteresting and of little use. A different concept of curriculum is proposed, based on the communicative (literacy), analytical (matheracy), and material (technoracy) instruments.

### Keywords

Ethnomathematics - Multiculturalism - Globalization - Curriculum.

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#### **Preliminaries**

Society is going through momentous transformations, with profound consequences for education. Today, we speak of bilingual education, of alternative medicine, of interreligious dialogue. Countless other forms of multiculturalism can be seen in education systems and in society at large. This seems to be contradictory when one observes that the world undergoes an intense process of mundialization, impacting on economic and financial aspects, and which manifests itself strongly in the new information and communication technologies, which socialize and spread new paradigms, thought systems, values and patterns of behavior. The word mundialization is therefore associated to the collective and alter-native construction of a new social world order1, whose transformations of the transport, commu-nication, informatization, production, and job systems result in an acceleration of this process and, consequently, give birth to globalization.

In globalized society there is a strong tendency to eliminate differences, promoting a planetary culture. Education systems are particularly affected, undergoing pressure from international studies and evaluations, inevitably comparative, and sadly competitive. As a result, one observes the gradual elimination of cultural components in the definition of education systems. The constitution of new social imaginaries becomes clear; imaginaries empty of historical, geographical and temporal referents, characterized by the heavy presence of the culture of the image. The criteria of classification establish an inappropriate reference that has as its consequence the definition of practices and even of education systems. On the other hand, resistance mechanisms, often unconscious, are activated seeking to safeguard and recover the identifying features of a culture, such as its traditions, cuisine, languages, artistic manifestations in general, and, in doing so, to

contribute to cultural diversity, an essential factor to encourage creativity.

Education systems have reacted to this contradictory situation of having to "align themselves" with international parameters while satisfying the demands of cultural contexts that seek identity, recognition and recovery. The results have been, as expected, misunderstood, provoking reaction and a tightening of punitive measures to bring systems "into line". Multiculturalism in education has been the main victim of this reaction. In the present work I shall focus on this situation.

### A multicultural proposal

One expected outcome of education systems is the acquisition and production of knowledge. This takes place fundamentally through the manner in which an individual perceives reality in its various manifestations: an individual reality in its sensorial, intuitive, emotional, and rational dimensions; a social reality, which is the recognition of the essentiality of the other; a planetary reality, which reveals the individual's dependence on a natural and cultural heritage, and his responsibility in its preservation; a cosmic reality, which makes him transcend time and space and existence itself in search of explanations and historicity.

The ad hoc practices to deal with problematic situations that emerge from reality are the result of the action of knowing. That is to say, knowledge is brought forth from reality. Knowledge is knowing and doing.

The production and accumulation of knowledge in a culture follows a kind of coherence. In the Harmonia Mundi (1618), Johannes Kepler suggests a commonality of action in which the Zeitgeist manifests itself, an idea that would prove itself fundamental for the historiographical proposal of Hegel (1770-

**<sup>1.</sup>** In relation to this point it is interesting to visit the site www.unesco.org/most and have a look at the Unesco MOST (Management Of Social Transformations) Program.

1831). Such commonality of actions characterizes a culture.

A culture is identified by its systems of explanations, philosophies, theories and actions, and by its daily behaviors. All of which are based on processes of communication, representations, classification, comparison, quantification, counting, measuring, inferences. These processes take place in different manners in the various cultures, and transform themselves through time. They always betray the influences from the environment and organize themselves in an internal logic, codify themselves, and formalize themselves. And so knowledge is born.

We try to understand human knowledge and behavior in the various regions of the planet throughout mankind's evolution, naturally recognizing that knowledge takes place in different manners in different cultures and at different times.

In the mid 1970s an educational program called the Ethnomathematics Program began to take shape. Although the name suggests an emphasis on mathematics, it is a study of the cultural evolution of mankind in its widest sense, starting from the cultural dynamics that can be seen in mathematical manifestations. But it should not be mistaken for mathematics in an academic sense, as a structured discipline. Such mathematics is undoubtedly important but, in the words of the eminent mathematician Roger Penrose, it represents only a very small area of the conscious activity, and is practiced by a reduced minority of sentient beings for a very limited portion of their conscious life. The same can be said about academic science in general.

In essence, the Ethnomathematics Program is a proposal for a theory of knowledge, whose name was chosen for reasons that shall be explained further ahead. In truth, it could equally have been called the Ethnoscience Program. Etymologically speaking, science comes from the Latin scio, meaning to know, and mathematics comes from

the Greek matema, meaning teaching, and therefore we can seen that the Ethnomathematics and Ethnoscience Programs complement each other. Actually, in the meaning 1 propose they cannot be distinguished. This is extensively discussed in D'Ambrosio (1990, 2001).

The idea for the Ethnomathematics Program originated from the analysis of mathematical practices in several cultural environments, and it was extended to analyze various forms of knowledge, and not just mathematical theories and practices. It is also a study of mankind's cultural evolution in its widest sense, starting from the cultural dynamics that can be seen in mathematical manifestations.

The point of departure is the examination of the history of the sciences, arts, and religions in various cultures. We adopt an externalist approach, which means searching for the relations between the development of scientific disciplines or artistic schools or religious doctrines and the sociocultural context in which such development took place. The program goes beyond this externalism, for it also deals with the close relationships between cognition and culture.

The Ethnomathematics Program emerges as a research program on the history and philosophy of mathematics, with important consequences to education, as showed in D'Ambrosio (1992).

At this point it is important to make clear that I understand mathematics as a strategy developed by mankind during its history to explain, to understand, to deal with, and live with the sensible, perceptible reality, and with its imaginary, all, of course, within a natural and cultural context. The same happens with the techniques, the arts, the religions, and the sciences in general. It is essentially the construction of bodies of knowledge in total symbiosis, within the same spatial and temporal context, which has obviously varied according to the geography and history of the

individuals and of the various cultural groups to which they belong — families, tribes, societies, civilizations. The greater purpose of those bodies of knowledge has been the will, which is actually a need of those cultural groups to survive in their environment, and of transcending, spatially and temporally, that environment.

I see education as a strategy to stimulate individual and collective development, engendered by those same cultural groups with the purpose of maintaining themselves as groups and of advancing towards the satisfaction of their needs of survival and transcendence.

Consequently, Mathematics and Education are contextualized and interdependent strategies. I try to understand the evolution of both, and to analyze the trends, as I seem them in the current stage of civilization. From that, I put forward some proposals. This is, within its own limits, the essence of this work.

By recognizing that the social moment is at the root of knowledge, the program, which is of a holistic nature, seeks to match cognition, history and sociology of education in a multicultural approach.

# On the issue of knowledge and the notion of culture

The holistic approach to the history of knowledge consists essentially of a critical analysis of the generation and production of knowledge, of its intellectual and social organization, and of its diffusion. In the disciplinary approach, these analyses are unrelated to each other, subordinated to often separate areas of knowledge: cognitive sciences, epistemology, sciences and arts, history, politics, education, communication theory.

Considering that the perception of facts is influenced by knowledge, when we speak of the history of knowledge, we are at the same time speaking of the history of Man himself, and of his habitat in the wider sense, that is, of

the Earth and even of the Cosmos. Modern science, when proposing "final theories", that is, explanations that intend to be definitive about the origin and evolution of the natural things, leans towards an arrogant stance that has as its inevitable consequence unquestionable behaviors. How do you question the behavior of someone who is convinced of knowing?

By contrast, transdisciplinarity is a holistic approach to knowledge based on the recognition of the impossibility of achieving total and final knowledge and, therefore, is constantly in search of new explanations and new knowledge, and thus modifying behaviors. It replaces the arrogance mentioned above with the modesty of relentless pursuit, whose consequences are respect, solidarity, and cooperation. Consequently, it must perforce draw support from the recovery of the various dimensions of the human being.

Disciplines give birth to specific methods of knowing well-defined objects of study. The methods and results thus obtained, which refer to clearly identified inquiries, constitute a named body of knowledge.

Since the beginning of the identification of bodies of knowledge as disciplines, variants of this organization have been proposed. Multidisciplinarity seeks to unite results obtained through the disciplinarity approach as practiced in the programs of a school course. Interdisciplinarity, much sought after and practiced nowadays, above all in schools, transfers methods from some disciplines to others, thereby identifying new objects of study.

Both extensions of the concept of discipline had already been anticipated in 1699 by the then secretary of the Paris Academy of Science de Fontenelle when he said: "Until now the Academy has considered nature only in parcels... Maybe the moment will come when all these scattered members (the disciplines) will come together in a regular body; and if they are as one wishes, they shall somehow come together by themselves" (1699, p. XIX).

Thus, transdisciplinarity goes beyond the limitations imposed by the methods and objects of study of the disciplines and interdisciplines. The psychoemotional process of knowledge generation, which is the essence of creativity, and can be considered a research program in itself, is characterized by questions such as: how can we move from ad hoc practices to ways of dealing with new situations and problems, and to methods? How can we move from methods to theories? How to proceed from theory to invention? These are the basic questions that give support to the research inherent to the Ethnomathematics Program.

To make it more explicit, the above questions involve processes such as generation and production of knowledge, intellectual organization, social organization, and diffusion, which are usually treated in isolation, as specific disciplines such as cognitive sciences (generation of knowledge), epistemology (intellectual organization of knowledge), history, politics and education (social organization, institutionalization and diffusion of knowledge).

The so-called modern method of knowing something, explaining a fact and a phenomenon, is based on the study of specific disciplines, which includes specific methods and objects of study. This method can be traced back to Descartes, and is characterized by its reductionism. But soon this method revealed itself insufficient, and already in the 17th century attempts were made to gather the knowledges and results from various disciplines to tackle a problem. The individual must try and know more things to know better. Schools practice this multidisciplinarity, which is today present in almost every school program.

Metaphorically speaking, disciplines are like television channels or computer programs. It is necessary to switch from a channel or close a computer application to open another. This is multidisciplinarity. But a huge innovation is to be able to work with several channels or applications at once, allowing new

creative possibilities and use of resources. That is what interdisciplinarity corresponds to. It not just juxtaposes results, but merges methods and, consequently, identifies new objects of study.

Interdisciplinarity experienced good development during last century, and spawned new fields of study. Neurophysiology, physicochemistry, and quantum mechanics appeared. Inevitably, those interdisciplinary areas created their own methods and defined their own objects of study. They became themselves disciplines, and then the specialists in interdisciplinary areas appeared. The interdisciplines quickly revealed limitations similar to those of the traditional disciplines.

At this point it is timely to say a few words about culture. There are many strongly ideological writings and theories about what culture is. We consider culture as the collection of myths, values, rules of behavior, and styles of knowledge shared by individuals living at a particular time and space.

Throughout history the perceptions of time and space have changed. Communication between generations and the meeting of groups with different cultures create a cultural dynamics, and we cannot think about a static culture, frozen in time and space. This dynamics is slow, and what we notice in the mutual exposure of cultures is either a cultural subordination, and sometimes even the destruction of one of the cultures, or a multicultural relationship. An interesting discussion on cultural dynamics can be found in Bateson (1972). Naturally, the multicultural relationship represents a progress in the behavior of societies, many times achieved after violent conflicts. Nowadays, and not without problems, multiculturalism gains space in education. The cultural dynamics intensifies thanks to the new information and communication technologies, and we contemplate the possibility of a planetary culture, the non-displaced union of various cultures localized in time and space.

While the instruments of observation

(devices - artifacts) and analysis (concepts and theories - mentifacts) were more limited, the interdisciplinary approach was satisfactory. But with the sophistication of the new instruments of observation and analysis, which intensified since the mid 20th century, it can be seen that the interdisciplinary approach has become insufficient. There is a flagrant contradiction between the yearning for a planetary culture, of a wider and deeper knowledge and the necessary ideal of respect, solidarity and cooperation among all individuals and nations, and the preservation of traditional cultures. This is the major difficulty we face when dealing with the mundialization, and I do not believe that the interdisciplinary practices are adequate to overcome such difficulty.

We do not deny that the disciplinary knowledge, and consequently the multidisciplinary and interdisciplinary, are useful and important, and shall continue to be extended and cultivated, but the only way they can lead to a full vision of reality is if they are subordinated to transdisciplinary knowledge. And, as showed in D'Ambrosio (1999), education is moving quickly towards a transdisciplinary education.

The most visible consequence of such tendency will be discussed in the present text, retrieving and detailing aspects of the relationship between society, culture, mathematics and its teaching.

# On education, peace and mathematical education

Education in general depends on variables that spread in very many directions: a) the student who is in an education process as an individual trying to fulfill his aspirations and find an answer to his concerns; b) his insertion in society and the expectations this society has about him; c) the strategies of this society to fulfill those aspirations; d) the agents and instruments to carry out those strategies; e) the content which is part of that strategy.

Sadly, in the organization of our teacher education courses and, equally, in graduate studies, there has been a reductionist emphasis to deal with those variables. Also, specialists have been established, along with their areas of competence. Psychologists should worry about "a", philosophers with "b", pedagogues with "c" and "d", and mathematicians with "e". As if it was possible to separate those areas.

We propose a holistic approach to education, and in particular to Mathematical Education. To talk about holistic approaches always makes the reader or listener shudder a bit. As does talking about transdisciplinarity, ethnomathematics, systemic approach, globalization, and multiculturalism. Apart from nuances, all those terms reflect the same wide effort to contextualize our actions as individuals and as society, in the concretization of the ideals of Peace and of a happy mankind. I admit that this is my utopia. And as an educator I try to guide my actions in that direction. How can one be an educator without a utopia?

When one speaks of Education for Peace most people ask: "But what has that got to do with Mathematical Education?" And I answer: "It has everything to do."

I could summarize my position by saying that it is only justified to insist on education for everyone if, through that, we achieve a better quality of life and greater dignity to mankind as a whole. The dignity of each individual manifests itself in the encounter with the others. Therefore, achieving a state of inner peace is a priority. But that is difficult, above all due to the countless problems we face daily, in particular in our relation with others. We should not stop making an effort to see if the other is also having difficulty achieving a state of inner peace. Many times we see that the other is facing problems resulting from material difficulties, lack of security, joblessness, shortage of money, often even lack of housing and food. The solidarity to others is the first sign of feeling part of a society. Social Peace will be a state in which such situations do not occur. And undoubtedly someone will ask the classical question: "But what does Mathematics have to do with it?" I cannot answer in any other way but suggesting thinking about and understanding the history of mankind to realize that it has "everything to do".

There are also not many people who understand how environmental peace is related to mathematics, when the latter is seen as applied to development and progress. I recall that modern science, which rests largely upon mathematics, gives us remarkable instruments for a healthy relationship with nature, but also supplies powerful instruments for the destruction of that same nature.

The multiple dimensions of peace, namely, inner peace, social peace, environmental peace, and military peace, which should be the first objectives of any education system, are the only justifications for any effort for scientific and technological advance, and should be the bedrock of any political discourse. Mathematics has a great responsibility in the efforts to achieve the ideal of an education for peace, in all its dimensions.

That should be the dream of the human being. That is the essence of being human. It is the human being trying to be human. I repeat the words of two distinguished mathematicians, Albert Einstein and Bertrand Russell, in the Pugwash Manifesto of 1955: "Remember your humanity, and forget the rest". I try in my proposals for Mathematical Education to follow the teachings of those two great masters, with whom I learned much mathematics, but above all much of humanity. My proposal is to make an Education for Peace and, specially, a Mathematical Education for Peace.

Many continued to be intrigued: "But how do I relate second degree trinomials with Peace?" It is likely that these same people use to teach second-degree trinomials giving the trajectory of a cannonball as an example. But 1 am almost sure that they do not say, or even suggest, that that same beautiful mathematical tool which is the second degree trinomial give to certain individuals - professional shooters who have probably been the best mathematics students in their class - the ability to fire a deadly cannon shot at a population of human beings, of flesh and blood, of emotions and desires, and kill them, destroy their houses and temples, trees and animals around, polluting any lagoon or river in the vicinity. And at coming back from their mission calmly receive praise and decorations. In the end, the implicit message is: learn well your second degree trinomials and you will have the opportunity to do all that, for only those who excel in Mathematics will have enough theoretical basis to point cannons at populations.

Obviously, my opponents will say, as they have already done: "But that is a demagogical discourse. That horrible destruction will only happen when necessary. And it is important that our youth be prepared for the necessary." Others will say: "It is necessary to know the instruments of the enemies to defeat them." Millions have been lulled by this talk throughout mankind's history and in particular during the Cold War, with material and moral losses to both sides of the conflict. We should note that those interested in such state of affairs say those things are necessary because the target of our devastating bomb is an individual that does not profess our religious credo, who does not belong to our political party, who does not follow our economic model of property and production, whose skin does not have the same color as ours or who does not speak the same language as we do, in short, the target of our bomb is an individual who is different from us.

The example of the second-degree trinomial was mentioned for the sake of argument. The ugly relevance of something as beautiful as the second-degree trinomial is interesting to be commented. We do not propose to eliminate the second-degree trinomial from the curriculum, but that time should be taken to show critically the ugly things made with it, and also the wonderful things that can be done with it.

There is effectively a morality associated to knowledge and, in particular, to mathematical knowledge. Why insist on education and Mathematical Education, and in the mathematical doing itself, if we do not realize how our practice can help building a society founded on respect, solidarity and cooperation?

Total peace depends essentially on each individual knowing himself and integrating into his society, into mankind, into nature and into the cosmos. Throughout our existence, each one of us can learn mathematics, but we cannot lose the knowledge of ourselves, and create barriers between the individuals and the others, between individuals and society, and promote habits of distrust of the other, of disbelief in society, of disrespect and ignorance about the single mankind, about a nature that is common to us all, and about the universe as a whole.

As a Mathematical Educator I see myself as an educator that has mathematics as his area of competence and his instrument for action, but not as a mathematician the uses education to advertise his abilities and competences, proselytizing for his discipline. My science and my knowledge serve under my humanism. As a Mathematical Educator I try to use what I have learned as a mathematician to fulfill my mission as an educator. To promote this message is my purpose as an educator of educators.

In very clear and direct terms: the student is more important than programs and contents. Education is the most important strategy to bring the individual into peace with himself and with his social, cultural and natural surroundings, and to position himself in a cosmic reality. If we do not accomplish that, education will have failed.

## The issue of knowledge revisited

The production, intellectual and social organization, and dissemination of knowledge form the general framework within which I try develop my specific proposals for Mathematical Education. My ideas often seem somewhat vague, inaccurate and exploratory. This reflects what could be called the state of the art in the theory of knowledge. We know very little about how we think. The recent contributions from cybernetics and artificial intelligence and, more recently, from neurologists make what is commonly studied in courses of psychology, theories of learning and similar disciplines at least obsolete. Hence the general presentation and the tone sometimes vague and inaccurate of this part in which I propose a model that seeks to embrace virtually every modern approach to knowledge. Mathe-matics is a critical area in these studies, as we learn from Butterworth (1999).

The efforts from individuals and from all societies throughout history can be seen in the search for explanations, ways of dealing and living with natural and sociocultural reality. They gave birth to modes of communication and to languages, to religions and arts, as well as to sciences and mathematics, in short, to everything we call knowledge. Individuals - and the same goes for the species as a whole stand out among their peers and reach their full creative potential because they know. All knowledge is the result of a long cumulative process in which stages can be identified, of course not dichotomic to each other, when the production, intellectual organization, social organization, and the dissemination of knowledge take place. These stages are normally the object of study of the theories of cognition, of epistemologies, of history and sociology, and of education and politics. The process as a whole is extremely dynamic and never complete, and it is obviously subjected to very specific conditions of encouragement and subordination to the natural, cultural and social context. Thus is the cycle of individual and social acquisition of knowledge.

My reflections on multicultural education have led me to see the act of creation as the most important element in this whole process, as a manifestation of the present in the transition from past to future. In other words, the acquisition and elaboration of the knowledge takes place in the present as a result of the whole past, individual as well as cultural, with a view to the strategies of action of the present and projecting into the future, from the most immediate future till the long term, thereby modifying reality and incorporating to it new facts, namely, "artifacts" and "mentifacts". Such behavior is intrinsic to the human being and result from natural impulses to survive and transcend. Although one can recognize there a process of construction of knowledge, my proposal is wider than constructivism, which has effectively become a pedagogical proposal, and which favors the rational. The holistic approach that I propose incorporates to the rational the sensorial, the intuitive, and the emotional, through the individual will to survive and transcend.

Survival and transcendence constitute the essence of being human. The human being, as every living species, seeks just its survival. The will to transcend is the most distinctive mark of our species.

Nobody knows where the will to survive as an individual and as species comes from, but it is no doubt imprinted into the genetic machinery since the origin of life. We simply observe that this force is the essence of all living species. No species, and therefore no individual, guides itself towards its extinction. Every moment is an exercise of survival for the individual and for the species.

Equally, we do not know how the human species acquires the will to transcend, which also seems to be written into our genetic code. This has been the major philosophical question in the whole of mankind's history and in all

cultures. In the shape of soul, of will, of free will, the impulse to transcend the moment of survival is recognized in various manifestations of the human being and every culture has a proposal to explain it.

The reflections about the present as realization of our will to survive and to transcend must necessarily be of a transdisciplinary and holistic nature. In this view, the present, presenting itself as the interface between past and future, is associated to action and to practice. The present is a philosophical question of the same nature as the irrational, which dominated philosophy since the Ancient Greece. In the 19th century, when Richard Dedekind put the concept of irrational in precise terms, meaning was given to the instant.

The focus of our study is the man as an integrated individual, immersed in a natural and social reality, which means being in permanent interaction with his environment, natural and sociocultural. The present is when the (inter)action of the individual with his environment, natural and sociocultural, takes place, something I call behavior. The behavior, which can also be called practice, doing, or action, is identified with the present, and motivates the search for organized explanations, that is, for theorizations, as the result of a reflection upon the doing. The theorization and elaboration of a system of explanations is what we generally call knowing, or simply knowledge. Actually, knowledge is the substratum of behavior. Life is action, and behavior and knowledge are the essence of being alive.

That idea of life cycle took shape in the 1970s and can already be seen in D'Ambrosio (1986). The life cycle is: "(...) the REALITY informs the INDIVIDUAL, who processes the information and defines strategies of ACTION, which insert new facts into REALITY, which informs the INDIVIDUAL, who processes (...)", and so on as long as the individual is alive.

This is the permanent cycle and in

evolution, which allows every human being to interact with his environment, that is, with reality considered in its totality as a complex of natural and artificial facts. That action takes place through the processing of information captured from reality by a processor that constitutes a true cybernetic complex, with a multitude of non-dichotomic sensors, variously identified as instinct, memory, reflexes, emotions, fantasy, intuition, and other elements that we can still barely imagine. As Oliver Sacks notes, referring specially to visual perception, but that applies equally well to all senses:

We achieve perceptual constancy — the correlation of all the different appearances, the transforms of objects — very early, in the first months of life. It constitutes a huge learning task, but is achieved so smoothly, so unconsciously, that its enormous complexity is scarcely realized (though it is an achievement that even the largest supercomputers cannot begin to match). (1995, p. 141)

### Going beyond survival

The processing of information (input) has as a result (output) strategies for action. There is evidence that those actions are intelligent products. In other words, man executes his life cycle not just for the animal motivation of survival, but subordinates survival to greater objectives through the consciousness of doing/knowing, that is, he does because he knows, and knows because he does. This argument is similar to that of Paulo Freire when he says that "the human being is the only (living being) that has consciousness of his inconclusion" (1997, p. 8). That is to say, who transcends the impulse to survive. The actions for transcendence, which always accompany the actions for survival, have their effect on reality, creating new interpretations and uses of natural and artificial reality, modifying it by the introduction of new facts, artifacts and mentifacts. I prefer the terminology artifact/

mentifact to concrete/abstract, for I see in the latter an incoherence due to the fact that it rests on a way of capturing those facts, whereas when we speak of artifact/mentifact we put the emphasis on the generation of the facts.

Knowledge is the producer of knowing, which will be decisive for action. Consequently, it is in the behavior, in the practice, in the doing that one evaluates, redefines and reconstructs knowledge. Consciousness is the driving force of man's action towards the know/doing and do/knowing, that is, towards survival and transcendence. The process of acquisition of knowledge is thus this dialectic relation knowing/doing, propelled by consciousness, and that takes place in many dimensions.

Among the various dimensions of the acquisition of knowledge we have highlighted four, which are the more recognized and interpreted in the theories of knowledge, namely, the sensorial, the intuitive, the emotional, and the rational. Granting a point to disciplinary classifications, we could say that religious knowledge is favored by the intuitive and emotional dimensions, whereas scientific knowledge is favored by the rational, and the emotional prevails in the arts. Naturally, these dimensions cannot be dichotomized or ranked, but they are complementary. So, there is no interruption, no dichotomy between knowing and doing, there is no priorization among them, nor is there any dominance among the several dimension of the process. Everything complements in a whole - the behavior - that has as its result the knowledge. Consequently, the body/mind, matter/spirit, manual/ intellectual, and so may other dichotomies ingrained in the modern world are merely artificial. Nobody expresses better this complementarity than the renowned Norwegian mathematician Sophus Lie, cited by Arild Stubhaug:

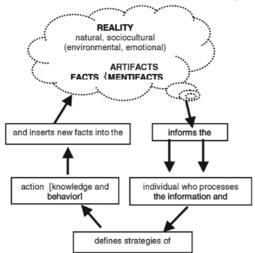
"(...) without fantasy nobody can become a mathematician, and what gave me a place

among the mathematicians of our day, despite my lack of knowledge and form, was the audacity of my thinking" (2002, p. 409).

# From the individual to the collective

The present as an interface between the past and the future manifests itself through action. The present is thus identified with behavior, it has the same dynamics of behavior, that is, it feeds on the past, it is the result of the history of the individual and of the collective, of prior knowledge, individual and collective, conditioned by the projection of the individual into the future. All from the information afforded by reality, therefore by the present. Inside reality all past facts are stored that inform the individual.

Information is processed by the individual and results in strategies for action, which will originate new facts (artifacts or mentifacts), which are incorporated into reality, obviously modifying it, getting stored in the collection of facts and events that constitute it. Reality is



therefore in relentless modification. The past thus projects itself, through the mediation of the individuals, into the future. Once more, the dichotomy between past and future is seen as artificiality, for the instant that comes from the past and projects itself into the future acquires in that way what could be called a transdimensionality, which we could see as a fold (a pli, in the sense of René Thom's catastrophes). Rethinking the dimen-sionality of the instant gives life, including here the "instants" of birth and death, a character of continuity, of fusion of past and future in each instant. Thence we recognize that there cannot be a frozen present, just as there cannot be a static action, just as there is no behavior without an instantaneous feedback (evaluation) that results from its effect. We can then see behavior as the link between the reality that informs and the action that modifies it.

Action generates knowledge, that is, the ability to explain, to deal with, to handle, to understand reality, it generates the mátema. This ability transmits itself, and accumulates horizontally in the relationship with others, contemporaries, through communications, and vertically, from each individual to himself (memory) and from each generation to the next (historical memory). Notice that what we call memory is of the same nature as the information mechanisms associated to the senses, to genetic information, and to emotional mechanisms, and retrieve the experiences lived by an individual in the past. Therefore, they all incorporate into reality and inform that individual in the same way that the other facts of reality do.

The individual is not sole. There are billions of other individuals of the same species Homo sapiens sapiens with the same life cycle, and billions of individuals of other species going through a life cycle specific to each species, but essentially similar to the one showed in the figure above. The process of producing knowledge as action is enriched by the exchange with others immersed in the same process, through what we call communication. The discovery of the other and of others, present or distant, contemporary or from the past, is essential for the phenomenon of life.

Everyone is incessantly contributing his

or her share to modify reality. Every individual is inserted into a cosmic reality, as a link between a whole history, from the beginning of time and things, that is, from a big bang or the like, and the present moment, the here and now. All experiences from the past, either recognized and identified or not, constitute reality in its totality and determine the behavior of each individual. His action results from the processing of retrieved information. Those include the experiences of each individual and those lived by others, in their totality. The retrieval of those experiences (individual memory, cultural memory, genetic memory) constitutes one of the challenges of psychoanalysis, of history and of many other sciences. It constitutes indeed the basis of certain modes of behavior (values) and knowledge (particularly the arts and religions).

In a temporal duality, these same aspects of behavior manifest themselves in the strategies of action that will result in new facts - artifacts and mentifacts - that shall take place in the future, and that, once performed, shall be incorporated into reality. The strategies of action are motivated by the individual's projection into the future (his wishes, ambitions, motivations, and so many other factors), both in the immediate future and in the remote future. This is the sense of transcendence I referred to above.

Although the mechanisms for capturing information and processing it, defining strategies for action, are absolutely individual, and keep themselves as such, they are enriched by the exchange and by communication, which is effectively a pact (contract) between individuals. The establishment of this pact is a phenomenon essential for life. In the human species, this pact allows the definition of strategies for common action. This does not presuppose eliminating the individual's capacity for his own actions, inherent to his own will (free will), but it can inhibit certain actions, that is, the common action resulting from communication can be interpreted as an in-

action resulting from the pact. So, through communication new actions can be produced desirable to both, and actions can also be inhibited, that is, in-actions can be produced undesirable to one of the parts or to both. I insist that these inhibiting mechanisms do not transform each individual's own mechanisms for capturing and processing information. Each individual has these mechanisms and that is what maintains the individuality and identity of each being, although they balance actions and in-actions, which make it possible what we identify as the living together. This has been well illustrated by Anthony Burgess in his classic A Clockwork Orange (1962), which inspired Stanley Kubrick's influential movie of 1971. Recently, a penal reform was proposed to the British Parliament, very similar to Anthony Burgess' fictional scenario.

These notions can be easily generalized to the group, community, and to a people through social communication and through a social pact, which, I insist, takes into account each individual's capacity, and does not eliminate each individual's own will, that is, his free will. The knowledge produced by the common interaction resulting from social communication will be a complex of codes and symbols, intellectually and socially organized to constitute what we call culture. Culture, therefore, is the substratum of knowledges, of knowing/doing, and of the resulting behavior shared by a group, community or people. Culture is what will allow life in society.

When societies, and therefore cultural systems, meet and are put in mutual contact they are subjected to a dynamics of interaction that produces an intercultural behavior manifested in groups of individuals, in communities, in tribes, and in societies as a whole. Interculturality has been intensifying throughout the history of mankind.

### The ethnomathematics program

The exposition above synthesizes the

theoretical fundamentals that serve as basis for a research program on the generation, intellectual organization, social organization, and diffusion of knowledge. In the academic jargon, we could call it an interdisciplinary program spanning what constitutes the domain of the so-called cognitive sciences, of epistemology, history, sociology, and diffusion.

Methodologically, this program recognizes that in its adventure as a planetary species man (the Homo sapiens sapiens species), as well as the other species that came before it, the various hominids that lived more than 5 million years ago, has its behavior fostered by the acquisition of knowledge, of doing(s) and knowing(s) that allowed it to survive and transcend, through ways, modes, techniques, arts (techné or "tics"), to explain, know, understand, deal with and live with (mátema) the natural and sociocultural reality (ethno) in which he, man, is inserted. By using, in a true etymological license, the roots "tics", "matema" and "ethno", I originated my conceptualization of Ethnomathematics.

Naturally, in all cultures and in all times, knowledge that is generated by the need for an answer to distinct problems and situations is subordinated to a natural, social, and cultural context.

Individuals and peoples have throughout their existences and throughout history created and developed instruments of reflection, of observation, theoretical instruments and, associated to them, techniques, abilities (arts, techniques, techné, tics) to explain, understand, know, learn, to know and to do in response to needs of survival and transcendence (matema) in various natural, social, and cultural environments (ethno). Hence we call what we have described above the Ethnomathematics Program. The name suggests the corpus of knowledge recognized academically as Mathematics.

In all cultures we can find manifestations related to and even identified as what we call today mathematics (processes of organization, classification, counting, measuring, inference), generally merged or hardly distinguishable from other forms, today identified as art, religion, music, techniques, sciences. In all times and in all cultures mathematics, arts, religion, music, techniques, sciences were developed with the purpose of explaining, knowing, learning, of knowing/doing and of predicting the future (divinatory arts). They all appear at a first stage of the history of mankind and of the life of each one of us, indistinguishable as forms of knowledge.

We live in a period in which the means of capturing information, and the processing of information by each individual are found in the communications and information technology, auxiliary instruments previously unimaginable. The interaction between individuals also finds in teleinformatics a great potential, still difficult to gauge, for generating actions in common. It can be seen in some cases the predominance of one form over another, sometimes the substitution of one form by another, and even the suppression and total elimination of some form, but in most cases the result is the production of new cultural forms, identified with modernity. Still, dominated by emotional tensions, the relations between individuals from a same culture (intracultural) and above all between indi-viduals from different cultures (intercultural) represent the creative potential of the species. Just as biodiversity represents the way to the appearance of new species, in cultural diversity resides the creative potential of mankind.

The importance of intercultural relations has been recognized. But unfortunately there is still reluctance to recognize intracultural relations in education. Children are still placed in series according to age, the same curriculum is still offered for a given series, and one even hears absurd proposals for a national curriculum. And the even greater absurd of evaluating groups of individuals with standardized tests. It is effectively an attempt to pasteurize the new generations!

The plurality of mass communication media, aided by improved transportation, has taken intercultural relations to truly planetary dimensions.

A new era thus begins, opening huge possibilities of behavior and planetary knowledge, with unprecedented results for the understanding and harmony of all mankind. We should say no to the biological or cultural homogenization of the species, but yes to the harmonious living together of the different, through an ethics of mutual respect, of solidarity and cooperation.

Of course, there have always been, and now will be more easily noticed, different manners of explanations, of understanding, of dealing with and living with reality, thanks to the new means of communication and transport, which create the need for a behavior that transcends even the new cultural forms.

Occasionally, the cherished free will, intrinsic to being human, will manifest itself in a model of transculturality that will allow each human being to reach his plenitude. An adequate model to facilitate this new stage in the evolution of our species is the so-called Multicultural Education, which has been growing in education systems around the world.

We know that today there are more than two hundred states and more than six thousand indigenous nations with a population of around 10% to 15% of the world population. Although I do not intend to discuss Indigenous Education here, the contributions from experts in the area have been very important to understand how education can be an instrument to reinforce the mechanisms of social exclusion.

The concept of knowledge and the practices associated with it in a culture are decisive to the national identity and, therefore, the encounter with other cultures can lead a nation to question its own identity. Perhaps the most important thing to underline here is the perception of a dichotomy between knowing

and doing that prevails in the so-called "civilized" world, and which is typical of the paradigms of modern science, such as created by Descartes, Newton and others.

Appearing nearly concomitantly with the age of navigation, with the conquests and colonization, modern science established itself as a form of rational knowledge originated from Mediterranean cultures, and substratum to the efficient and fascinating modern technology. From the central nations came the definition of conceptualizations structured and dichotomic of knowing (knowledge) and doing (abilities).

It is important to remember that practically all countries subscribed to the Declaration of New Delhi (16 December 1993), which is explicit in recognizing that education is the prime instrument for the promotion of universal human values, of the quality of human resources and of the respect for cultural diversity, and that the contents and methods of education need to be developed to serve the basic learning needs of the individuals and societies, giving them the power to tackle their more urgent problems - the struggle against poverty, increase in productivity, improvement of living standards, and protection of the environment - and allowing them to take up their rightful role in the construction of democratic societies and in the enrichment of their cultural heritage.

Nothing is more explicit in this declaration than the appeal to subordinate programmatic contents to cultural diversity. Equally, the recognition of a variety of learning styles is implicit in the appeal to the development of new methodologies.

In essence, these considerations establish a great flexibility both in the selection of contents and in methodology.

## Ethnomathematics and mathematics

The approach to distinct forms of

knowing is the essence of the Ethnomathematics Program. In fact, differently to what the name suggests, Ethnomathematics is not just the study of the "mathematics of various ethnic groups". I have created this word to signify that there are various ways, techniques, abilities (tics) to explain, understand, deal with and live with (matema) distinct natural and socioeconomic contexts of reality (ethnos).

The discipline called mathematics is, in fact, an Ethnomathematics that originated and developed in Mediterranean Europe, having received some contributions from Indian and Islamic civilizations, and which reached its present form in the 16th and 17th centuries, being then taken and imposed in the rest of the world. Today, this mathematics acquires a character of universality, above all due to the dominance of modern science and technologies, which were developed in Europe from the 17th century.

This universalization is an example of the process of globalization that we have witnessed in all activities and areas of knowledge. There was much talk about multinationals. Today, the multinationals are global enterprises, for which it is impossible to identify a nation or dominant national group.

The idea of globalization begins to appear already in the foundation of Christianism and Islamism. Differently from Judaism, from which those two religions originated, as well as from several other religions in which there is a chosen people, Christianism and Islamism are, essentially, religions for the conversion of the whole mankind to one faith, of the whole planet subordinated to the same Church. This can be clearly seen in the process of expansion of the christianized Roman Empire and of the Islam.

The process of globalization of the Christian faith comes close to its perfection with the age of navigations. The catechism, fundamental element of the conversion, is taken to the whole world. Just like Christianism

is a product of the Roman Empire raised to the character of universality with colonialism, so are mathematics, science and technology.

In the process of expansion, Christianism modified, absorbing elements of the subordinated cultures, and producing remarkable variants of the original Christianism of the colonizer. It should be expected that, likewise, the forms of explaining, knowing, dealing with, living with sociocultural and natural reality, obviously distinct from region to region, and which are the reasons for the existence of mathematics, sciences and technologies, would also go through this process of "acclimatization", a result of a cultural dynamics. However, that did not happen, and it does not happen, and those fields of knowledge have acquired a character of universal absolute. They do not admit of variations or any kind of relativism. This fact has been incorporated to the level of popular dictums such as "as sure as two and two are four". We not dispute the fact, but its contextualization in the form of a symbolic construction anchored in a whole cultural past.

Mathematics has been defined as the science of numbers and forms, of the relations and measures, of the inferences, and its features point to precision, rigor, and exactness. Mathematics big heroes, that is those individuals historically pointed out as responsible for the advancement and consolidation of this science, are identified in Ancient Greece and, later, in the Modern Age, in Central Europe, above all in England, France, Italy, Germany. Names most remembered are Thales, Pythagoras, Euclid, Descartes, Galileo, Newton, Leibniz, Hilbert, Einstein, Hawking. They are ideas and men from the Mediterranean northwards.

Therefore, to speak of this mathematics in diversified cultural environments, above all when dealing with indigenous peoples or Afro-Americans or other non-Europeans, with workers oppressed and from marginalized classes, in addition to bringing the image of the

conqueror, of the pro-slavery, in short, of the dominator, also refers to a form of knowledge that was built by the dominator, and of which he served, and still serves, himself to exercise his domination.

Many people say that the same thing happens with blue jeans, which are replacing all traditional clothes, or with Coca-Cola, which is displacing Guaraná, or with rap, which is becoming as popular as the samba. But none of them has, like mathematics, the aura of infallibility, rigor, precision, and of being an essential and powerful instrument in the modern world. This makes it a presence that excludes other forms of thinking. Actually, being rational is identified with mastering mathematics. Mathematics presents itself as the language of a god wiser, more miraculous and more powerful than the deities of other cultural traditions.

If that could be identified just as part of a perverse process of acculturation, through which the creativity essential to being human is eliminated, I would say that such schooling is a farce. But it is worse than that, for in the farce, once the spectacle is finished, everything returns to what is was before. Whereas in education, the real is replaced by a situation devised to satisfy the objectives of the dominator. Nothing returns to the real after finishing the educational experience. The student has his cultural roots, part of his identity, eliminated in the process. This elimination produces the excluded.

This is evidenced in a tragic way in Indigenous Education. The Indian goes through the education process and is no longer an Indian... neither is he a white man. It is probable that the high incidence of suicide among some indigenous populations is associated with that.

A natural question can occur after these observations: would it not be better not to teach mathematics to natives and to the marginalized?

This question applies to all categories of

knowing/doing characteristic of the culture of the dominator, with respect to all peoples that show a cultural identity. The question could be rephrased: would it be better to discourage or even stop the popular classes wearing blue jeans or drinking Cola-Cola or practicing the rap? Naturally, these are false questions, and it would be false and demagogical to answer with a simple yes or no. These questions can only be formulated and answered within a historical context, trying to understand the irreversible e(in?)volution of the cultural systems in the history of mankind. The contextualization is essential to any education program for native and marginalized populations, but no less necessary for the populations of the dominant sectors, if we want to achieve a society with equity and social justice.

Contextualizing mathematics is essential to everyone. After all, how can we fail to associate Euclid's Elements with the cultural scene of Ancient Greece? Or the acquisition of Indo-Arabic numbering system with the flourishing of European mercantilism in the 14th and 15th centuries? And we cannot understand Newton outside his context. I recall the fundamental work of Boris Hessen (1995). Surely, it is possible to repeat a few theorems, memorize multiplication tables, and automate operations, and even calculate some integrals and derivatives, which do not have any relation with anything in the cities, fields or forests. Some will say that they are worth it as the noblest manifestation of the human thinking and intelligence.

We persist with the false assumption that intelligence and rationality are synonyms with mathematics. It is believed that this construct of the Mediterranean thinking, taken to its purest form, is the essence of being rational. And thus the fact is justified that individuals, rational because they master mathematics, have treated, and continue to treat, nature as an inexhaustible resource for the satisfaction of their wishes and ambitions.

Naturally, there is an important political

component in these reflections. Despite many people saying that this is an outdated slogan of the left, it is obvious that dominant and subordinate classes still exist, in the central countries as well as in the peripheral ones.

It makes sense, therefore, to speak of a "dominant mathematics", which is an instrument developed in the central countries, and many times used as an instrument of domination. This mathematics and those that master it present themselves with a position of superiority, with the power to dislodge, and even eliminate, the "everyday mathematics". The same happens to other forms of culture, particularly with language, as very well discussed by Bernstein (1971). And the situations associated to behavior, medicine, art, and religion are well known. All these manifestations are referred to as popular culture. Naturally, although alive and practiced, popular culture is often ignored, disdained, rejected, repressed, and certainly belittled. This has the effect of discouraging and even eliminating the people as the producer and consumer of culture, and even as a cultural entity.

That is no less true of mathematics. In geometry and in arithmetic, particularly, violent contradictions can be seen. For instance, the geometry of people, of the balloons and kites, is colorful. Theoretical geometry, since its Greek origin, has eliminated color. Many readers will be confused at this point. They will be saying: but has this all have to do with Mathematics? Kites and balloons? Colors?

They have everything to do, for they are exactly the first and most notable geometrical experiences. The reunion of art and geometry cannot be accomplished without the mediator color. In arithmetic, the attribute of number in quantification is essential. Two oranges and two horses are distinct "twos". To reach the "two" without qualification, abstract, like reaching geometry without colors, may be the crucial point in the passage to a theoretical mathematics. Being careful about this passage

and about working adequately this moment may synthesize all that is important in the Elementary Mathematics programs. The rest of the components that make up the programs are a collection of techniques that become less and less interesting and necessary, more efficiently carried out by electronic machines.

One cannot define criteria of superiority between cultural manifestations. Appropriately contextualized, no form can be said superior to another. This is well illustrated by Ferreira (2002, p. 25-36). For example, we learn that the Xavante binary system was replaced, as if by magic, by a "more efficient" base 10 system. Why more efficient? Because it relates to the Xavante context? No, because it relates to the numbering system of the dominator. What happens to the native language is not substantially different.

But there is undoubtedly a criterion of efficiency that applies to intercultural relations. Without learning the "arithmetic of the white man", the native will be swindled in his commercial transactions with the white man. This is dramatically illustrated in Céline's classic. Just as without command of the colonizer's language the native will find it difficult to have access to the dominant society. But that happens with all cultures. I have to master the English language if I want to participate in the international academic world. But nobody has ever said, or even suggested, that I should forget Portuguese, and that I should be embarrassed or even ashamed of speaking that language. But that is what is done to peoples, specially the indigenous populations, be it in language, be it in knowledge systems in general, and particularly in mathematics. Their language is labeled as useless, their religion becomes "fairy tales", their art and rituals are "folklore", their science and medicine are "superstitions", and their mathematics is "imprecise" and "inefficient", when not "nonexistent".

Now, that goes on in precisely the same way with the popular classes. But that is exactly

what happens to a child or a teenager or even an adult when they approach a school. Whereas Indians commit suicide, something permitted by their intracultural relations, the form of suicide practiced in other segments of the population is an attitude of disbelief, of alienation, so well depicted in the movie Kids.

There is no question about the convenience and even necessity of teaching to the dominated, either Indians or whites, poor or rich, children and adults, the language, mathematics, medicine, and laws of the dominator. We have reached a structure of society and such perverse concepts of culture, nation, and sovereignty that this need imposes itself upon us. What is questioned here is the aggression to the dignity and to the cultural identity of those subordinated to that structure.

The main responsibility of the education theorists is to call attention to the irreversible damages that can be caused to a culture, to a people and to an individual if the process is carried out unconscientiously, many times even with good intention, and make proposals to minimize those damages. Many educators are not aware of that. The consequences of naiveté and of perversity can be essentially the same.

Still referring to indigenous education, the conceptual conflicts that result from the introduction of the "mathematics of the white man", which manifest themselves above all in the formulation and solution of simple arithmetic problems, are very well illustrated in the cultural context of the Xavantes, Suyás, Kayabis, and Jurunas (Ferreira, 2002). Various examples, such as the transport in boats, balancing a bank account, and others show that the Indians master what is essential to their practices and to the elaborate arguments with the white man about the things that interest them, usually dealing with transports, commerce, and use of the land. So, mathematics is contextualized as one more resource to solve new problems that, having originated in another culture, have arrived demanding the intellectual tools of that culture.

The ethnomathematics of the Indian can do the job, it is efficient and adequate to many - really important - things, and there is no reason to replace it. The ethnomathematics of the white man is good for other things, equally important, and it cannot be ignored. To say that one is more efficient, more rigorous, in short, better than the other is, if removed from a context, a false and falsifying issue.

The mastery of two ethnomathematics, and possibly of others, obviously offers greater possibilities of explanations, understandings, of handling new situations and solving problems. But that is exactly what is done in mathematical research - and in fact in any other field of knowledge. The access to a greater number of intellectual instruments and techniques gives, when these are appropriately contextualized, a much larger capacity to deal with situations and to solve new problems, of modeling adequately a real situation to reach, with the use of those instruments, a possible solution or course of action.

This is learning par excellence, that is, the ability to explain, learn and understand, to critically face new situations. Learning is not the mere command of techniques, abilities, nor is it the memorizing of a few explanations and theories.

Formal education is based on the mere transmission of explanations and theories (theory-based teaching and expository classes), on the training in techniques and abilities (practice-based teaching with repetitive exercises). From the viewpoint of the most recent advances in our understanding of the cognitive processes, both methods are completely flawed. Cognitive abilities cannot be assessed outside their cultural contexts. Obviously, each individual has his/her own cognitive capacity. There are cognitive styles that must be recognized in different cultures, in an intercultural context, and also within the same culture, in an intracultural context.

Naturally, each individual organizes his intellectual process throughout his life history,

collecting and processing information, as discussed above. Metacognition offers a good theoretical apparatus to understand this process. The risk of the more common education practices is, when trying to match the intellectual organizations of different individuals, and thereby create a highly acceptable social scheme, to threaten the authenticity and individuality of each participant in the process. The frailty of this pedagogical structuralism, anchored in what we call the myths of current education, is evident when we reflect on the dizzying fall of the results of the education grounded on these myths around the world. The big challenge faced in education is precisely that of being capable of interpreting the abilities and the cognitive action itself in the non-linear, stable, and continuous way that characterizes the more current educational practices.

The alternative is to recognize that the individual is a whole, integral and integrated, and that his cognitive and organizing practices are not unrelated with the historical context in which the process takes place, a context that remains in permanent evolution. That is clear in the dynamics that prevails in the education for everyone and in multicultural education.

The adoption of a new educational stance is, in fact, the search for a new paradigm of education that will replace the already worn out teaching/learning paradigm, which is based on an obsolete cause/effect relationship. We are in search of an education that encourages the development of an open creativity, leading to new forms of intercultural relationships. These relationships characterize mass education and afford adequate space for the preservation of diversity and elimination of inequalities, bringing forth a new organization of society.

# A curriculum proposal: literacy, matheracy, and technoracy

1 employ a very wide definition of

curriculum: the strategy of educative action. Throughout history, the curriculum has reflected a conception of education and of its importance in society, which is very different from the academic importance of each discipline. We are talking about education systems as a whole and of curriculum as a strategy of education (D'Ambrosio, 1983).

The Romans have bequeathed us an institutional model that persists to this day, in particular in education. What would correspond to fundamental education was organized in the Roman world as the trivium (grammar, rhetoric, and dialectics), and the main motivator of this curriculum was the consolidation of the Roman Empire. With the expansion of Christianism in the Middle Ages, other educational needs were created, which reflected in what would be upper schooling, organized as the quadrivium (arithmetic, music, geometry, and astronomy). In both cases it is clear that the curriculum organization finds its raison d'être in the sociocultural and economic moment of the each epoch.

The extensive advances in the styles of explanation of the natural facts and in economy that characterized the European thought since the 16th century created a demand for new goals for education. The main goal was to create a school accessible to everyone, and following a new social and economic order. Already in 1656 Comenius said:

If therefore we want well ordered and flourishing Churches and States and good administrations, first of all let us order schools and let us make them flourish, so that they be true and live workshops of men, and ecclesiastical, political and economic nurseries. (1996, p. 71)

**2.**In Portuguese literacia is used. In English literacy is frequent, and matheracy seems to have been used before by the illustrious Japanese mathematics educator Tadasu Kawaguchi in a sense more restricted than the one I propose. I have never seen technoracy, although technological literacy is employed.

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