Building a guide to analyse mathematics textbooks based on the didactical suitability theory*[1]

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Abstract

A textbook lesson can be considered as a potential or planned instructional process proposed by the author's book, which can help the teacher to design and implement an effective instructional process. This permits the application of didactical analysis tools of the onto-semiotic approach to mathematical knowledge and instruction in order to assess the didactical suitability of the instructional process, identify possible conflicts of meaning and potential improvements. In this article, we describe the process of developing a Mathematics Textbook Lesson Analysis Guide using the didactical suitability theory and its operational breakdown into components, subcomponents and indicators. The formulation of the suitability criteria, as rules that permit to guide an informed assessment of the suitability of a teaching and learning process, has led us to carry out a content analysis of the key research on textbook analysis and the consensus adopted in the research community regarding the suitability criteria. Assuming that a mathematics teacher has decided to use a textbook lesson as a resource to help the teaching and learning process of some mathematical content, the analysis guide is presented as a tool that allows the teacher to assess the didactical suitability and facilitate the making of informed decisions about the lesson use in the classroom.

Keywords

Didactical analysis – Mathematics textbook – Teacher education – Didactical suitability.

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Introduction

Textbook analysis has gained importance because this resource is a mirror of what is done in the classroom and its use is undeniable, becoming an object of study in itself (AAAS, 2000; BONAFÉ, 2008; FAN; ZHU; MIAO, 2013).

Teive (2015) emphasizes that the study of textbooks offers different research opportunities, which has involved the creation of several centres dedicated to specific research issues on these resources. One of these organizations is the Centro de Investigaciones en Manuales Escolares (MANES) in Spain, which focuses on two research perspectives: one related to the analysis of text content and the other focused on the context of book production and use. Specifically, in the Spanish context, Bel and Colomer (2018) point out that the textbook constitutes a particular component of the school context that requires the implementation of a specific research methodology.

In the case of mathematics textbooks, Schubring and Fan (2018) point out that this research area has received increasing international attention as it has opened up to new developments such as electronic resources, historical reflections, and international comparisons. There are proposals for the analysis of textbooks focusing on specific content and others that propose more general models of analysis (MONTERRUBIO; ORTEGA, 2012; SANTAOLALLA, 2014). Some lines of research focus on specific aspects such as task analysis (ORRANTIA; GONZÁLEZ; VICENTE, 2005), representations (MARTÍNEZ; PENALVA, 2006), reasoning (STYLIANIDES, 2009), error detection (FERNÁNDEZ; CABALLERO; FERNÁNDEZ, 2013) or on affective aspects (REY; PENALVA, 2002), all of them under different theoretical models.

In addition, works of research such as those of Balcaza, Contreras, and Font (2017), Burgos et al. (2020), Godino, Font and Wilhelmi (2006) or Vásquez and Alsina (2015) have focused on the specific analysis of a particular lesson or theme using tools of the onto-semiotic approach (OSA) of mathematical knowledge and instruction (GODINO; BATANERO; FONT, 2007). This research reflects the interest in investigating and establishing the reference meaning of a particular concept or theme through a historical-epistemological review and then analysing how that meaning is developed in the textbooks and what the potential difficulties that students may have, are.

Research such as that carried out by Braga and Belver (2016), Font and Godino (2007), Godino, Font and Wilhelmi (2006) highlight that textbooks are the immediate source that reflects the teachers' practical experience, and therefore, the analysis and evaluation of their relevance or appropriateness should be a component in the training programs for mathematics teachers. To make effective use of textbooks, teachers should have the mathematical and didactic knowledge to select the most appropriate sources and adapt them to the educational level (GODINO; FONT; WILHELMI, 2006), as well as to acquire a critical positioning towards them (MONTERRUBIO; ORTEGA, 2012).

Starting from the fact that the analysis of textbooks must be one of the competences contemplated in the training of teachers, and that there are few previous works that contemplate a global analysis of the textbook lessons for a specific subject, we consider it relevant to establish a guide that allows the prospective or in-service teacher to have guidelines to analyse the didactical suitability of a textbook lesson to address a

mathematical subject. Such an instrument should be useful in making informed decisions about the use of a textbook lesson in the classroom, as a resource to support the teaching and learning process of some mathematical content.

The aim of this paper is to describe the development of a Mathematics Textbook Lesson Analysis Guide (GALT-Mathematics) using the OSA didactic analysis tools (GODINO; BATANERO; FONT, 2007). The manuscript is organized in the following sections. The second section introduces the elements of the theoretical framework we use in this work and the specific research problem. The methodology used is described in the third section. The fourth section shows the GALT-Mathematics based on a review of research results or expert judgments assumed by the academic community, which leads to expanding and delimiting the indicators of suitability in Godino (2013). The article ends with some conclusions and future lines of research.

Theoretical framework and research problem

The notion of didactical suitability, its dimensions, criteria and operational breakdown into components and indicators (BREDA; FONT; PINO-FAN, 2018; GODINO, 2013; GODINO *et al.*, 2006) is introduced in the OSA framework due to the need to develop specific instructional theories to guide teachers in the design, implementation and evaluation phases of instructional processes. This theoretical tool guides the transition from a descriptive-explanatory Didactics to another oriented towards the effective intervention in the classroom (GODINO, 2013; GODINO; BATANERO; FONT, 2007).

The criterion of didactical suitability is understood as "a standard of correction that establishes how a teaching and learning process should be carried out" (BREDA; FONT; PINO-FAN, 2018, p. 264). These standards, or principles, must be agreed upon by the community interested in mathematics education or by a relevant sector of it. They are useful a priori since they provide guidance on how things should be carried out, and a posteriori since they allow for the assessment of the teaching and learning process implemented.

> In the OSA, the didactic suitability of a teaching-learning process is understood as the degree to which it (or a part of it) has certain characteristics that allow it to be qualified as optimal or adequate to achieve the adaptation between the personal meanings achieved by the students (learning) and the institutional meanings intended or implemented (teaching), taking into account the circumstances and available resources (environment). (BREDA; FONT; PINO-FAN, 2018, p. 268).

Didactical suitability is a scalable feature of teaching and learning processes that involves the coherent articulation of the following six facets (GODINO; BATANERO; FONT, 2007):

- Epistemic suitability: the degree of representativeness of the institutional meanings implemented with respect to a reference meaning.
- Cognitive suitability: the degree to which the implemented meanings are in the zone of the students' potential development; proximity of the personal meanings achieved to the implemented meanings.

- Interactional suitability: the degree to which the configurations and didactic trajectories allow the identification of potential semiotic conflicts and the resolution of the conflicts that occur in the instructional process.
- Mediational suitability: the degree of availability and adequacy of material and temporal resources.
- Affective suitability: the degree of student involvement in the instructional process.
- Ecological suitability: the degree to which the instructional process is adjusted to the educational project of the centre, the school and the society, and the setting in which it takes place.

It is important to clarify some terms involved in the above description that will appear in the GALT-Mathematics description. The meaning of a mathematical object refers to the systems of operative and discursive practices, carried out by a person (personal meaning), or that are shared within an institution (institutional meaning), to solve a type of problem-situation (GODINO; BATANERO; FONT, 2007). If there is a disparity between institutional meanings (for example, between the reference meaning and the one implemented in a textbook), we speak of an epistemic conflict, while a mismatch between the meaning expressed by a subject and the reference meaning is a cognitive conflict.

Six types of primary objects – situations-problems, languages, definitions, propositions, procedures and arguments – are considered in the OSA framework to analyse the mathematical activity. These entities are related to forming configurations, which are understood as networks of intervening and emergent objects of the systems of practices and the relations that occur between them. These configurations can be epistemic or cognitive (GODINO; BATANERO; FONT, 2007).

As Breda, Font and Pino-Fan (2018) point out, the notion of didactic suitability can be used as a tool to organize the teacher's reflection on his/her own practice. To carry out this reflection, each of the six facets of partial suitability is broken down into components and indicators, which become specific criteria. Therefore, it would be appropriate to "complement the list of indicators from the previous step of reconstructing the reference meaning of the specific topic to be taught" (BREDA; FONT; PINO-FAN, 2018, p. 272). In this sense. In this sense, the first works by Aroza, Godino and Beltrán-Pellicer (2016) and Posadas and Godino (2017) establish criteria for the subject of proportionality and for the case of the quadratic equation, respectively.

The consideration of a textbook lesson as a potential or planned instructional process by the book´s author can help the teacher design and implement an effective instructional process; it also allows the application of the didactic analysis tools of the OSA to assess the didactical suitability of such process, identify possible conflicts of meaning and consider potential improvements (GODINO, 2013).

Assuming that a teacher has made the decision to use a textbook lesson as a resource to help his/her teaching and student learning, the following questions arise:

- What aspects and criteria should be taken into account to optimize the teaching and learning processes that can be implemented based on the use of the lesson?
- How to manage the use of textbook lessons to increase the suitability of the planned instructional processes.

In this case, it is not a question of approaching the global analysis of a textbook, from a pedagogical perspective, but rather the didactic analysis of developing a specific topic. To achieve this end, it is necessary to build a guide that leads the analysis in a general way, and later adapt this guide to a specific mathematical theme. In this article we limit ourselves to describing the development of the GALT-Mathematics, making it clear that in order to carry out the analysis of a textbook lesson in a specific mathematical topic, the general indicators proposed here will have to be adapted to the didactic-mathematical knowledge of that content.

Methodology

A methodology based on content analysis is followed (COHEN; MANION; MORRISON, 2011) insofar as it allows processing and reviewing qualitative dimensions, describing trends and characteristics of the content, as well as formulating valid inferences from certain data. As suggested by Godino, Rivas, and Arteaga (2012), this method allows us to progressively improve the instruments for evaluating the didactical suitability of mathematical instruction processes using the technique of content analysis. The idea is to make

> Adjustments to existing instruments for their application in particular content areas, and also to "base" the indicators explicitly, either on the results of research or on expert judgments assumed by the academic community, often expressed in national and international curricular guidelines (GODINO; RIVAS; ARTEAGA, 2012, p. 333).

Research on the analysis of textbooks in education is a good reference for identifying and organizing standards of didactical suitability well established in the scientific community. The analysis of content requires delimiting units of analysis in the selected texts —in our case, the works of research which refer to the analysis of textbooks— which are classified according to the facets and components of the theory of didactical suitability. They are then compared with the criteria and indicators proposed in Godino (2013) and in Godino, Rivas and Arteaga (2012), to assess whether the information contained in these units reiterates, complements, or adds new elements with respect to the already pre-established indicators. It was necessary to adapt some suitability indicators and components in certain facets that were no longer relevant given the nature of the content (textbook lesson) to be analysed. As a result of this analysis, in the following sections, we will refer to the main ideas discussed in the papers found and reviewed, specifying when these "standards" were already included in the original indicators, when they complemented some indicator, or when they established new criteria to be incorporated. From this confrontation, an improved and expanded version of the original suitability components and indicators emerges.

Elaboration of a guide for the analysis of mathematics textbooks

The analysis of the textbook is based on the components and indicators proposed by Godino (2013) and Godino, Rivas and Arteaga (2012), these criteria were formulated

with the intention of "analysing the interaction between the functions of the teacher and the students with regard to a specific mathematical content" (GODINO, 2013, p.17), which implies considering issues of teacher-student and student-student interactions, as well as aspects of timing and classroom environment. Since our object of study is the lesson of a textbook understood as a planned instructional process, it is necessary to adapt or reformulate some of the criteria and components of Godino (2013) and Godino, Rivas and Arteaga (2012) to the new context.

For this work, the system of categories according to components and sub-components in each of the facets proposed in Godino et al. (2019) is adopted. The new organization implies special changes in the epistemic and cognitive facets, where the notion of partial meaning of a mathematical object is put in the foreground. In this way, up to four levels of categories are considered. Level 1 being each one of the facets, level 2 being the components of the facets, level 3 being the sub-components, and finally level 4 being the suitability indicators.

A review of the literature background has shown that several of the guidelines found for textbook analysis propose indicators, but do not go so far as to formulate an explicit criterion of suitability, as is the case of the Theory of Didactical Suitability. For this reason, those aspects from the literature background that establish a criterion and complement those already considered by Godino (2013) are taken into account.

To follow, we specify the indicators adapted to the interests of this research for each one of the dimensions of the theory of didactical suitability, as well as the changes or considerations made to elaborate the different tables that make up the GALT-Mathematics.

Epistemic suitability indicators

Meanings. From the OSA pragmatic point of view, meanings are defined in terms of the configurations of practices, objects, and processes that a person mobilizes to solve a certain kind of problem-situation.

• In relation to the problem-situations proposed in the textbook, it is analysed whether the tasks are heterogeneous with respect to their typology, that is, if they involve realistic, routine, application, deduction mathematical activity (BRAGA; BELVER, 2016) and if they deal with diverse issues (numerical, theoretical, graphical) (MONTERRUBIO; ORTEGA, 2012). It is also assessed whether the situations have different levels of difficulty (BRAGA; BELVER, 2016), the duration of their resolution (BONAFÉ, 1992) and that there is an adequate number of different exercises (MONTERRUBIO; ORTEGA, 2012). These considerations have been taken into account when analysing whether there is a representative sample of contextualization, exercising, and application problem-situations.

• Language. The inclusion of manipulative resources that are alternatives to pencil and paper is incorporated (BRAGA; BELVER, 2016). The construction, improvement, and use of representations to organize, interpret, and record ideas (GODINO; RIVAS; ARTEAGA, 2012) is also contemplated as an extension of the previous indicator.

• Rules (concepts, propositions, and procedures). It is necessary for the rules to be presented in a clear and correct way and adapted to the educational level to which they are addressed. This involves issues such as: that the rules are introduced with intuitive clarifications at the beginning and then more formal or general (MARTÍNEZ; PENALVA, 2006) and that they are presented with precision and follow an adequate didactical and mathematical logic (AAAS, 2000). The changes in these sub-components, according to Godino (2013), have been merely organizational.

• Arguments. Based on the considerations of the AAAS (2000) and Godino, Rivas and Arteaga (2012), it has been added as an indicator that the propositions and procedures are explained and argued. In addition, the indicator which allows the analysis of whether the justification of mathematical statements and propositions is favoured through different types of reasoning and testing methods, is incorporated and adapted.

Relationships. It is considered relevant that the textbook expresses the basic concepts of the content and the relations between them, through maps or conceptual networks, for example (BONAFÉ, 1992).

Processes

• It is relevant to organize and consolidate mathematical thinking through communication (NCTM, 2000) and to verbally describe reasoning and personal strategies (MEC, 2006). It is necessary that students work with mathematical tasks which have enriching discussion, given that "procedural tasks for which students are expected to have well-developed algorithmic approaches, are generally not good candidates for such discourse" (NCTM, 2000, p. 60). Interesting problems and tasks, in which the student writes about mathematics, and reflects on his/her work, should be promoted to help the student consolidate his/her thinking.

• Taking into account the contributions of AAAS (2000) and Martínez and Penalva (2006), it was decided to contemplate the modelling in a more explicit way. A mathematical model refers to "a mathematical representation of elements and relationships in an idealized version of a complex phenomenon. Mathematical models can be used to clarify and interpret the phenomenon and to solve problems" (MARTÍNEZ; PENALVA, 2006, p.70).

• In the sub-component generalization, it is analysed whether tasks are proposed that allow analysing and describing patterns, making mathematical generalizations, working on abstraction and generalization, reasoning, hypothesis building, and testing (NCTM, 2000; ONRUBIA; ROCHERA; BARBERÁ, 2001; STYLIANIDES, 2009).

Epistemic conflicts. We have considered it important to include a specific component on epistemic conflicts. This refers to whether the textbook contains errors of concept, ambiguities, problems with absurd statements where data are missing, statements of problems with error in the data or that contain contradictory orders, or errors in the answers to problems (FERNÁNDEZ; CABALLERO; FERNÁNDEZ, 2013).

Taking into account these considerations, Table 1 presents the final indicators contemplated in the epistemic facet according to the components and sub-components cited.

Table 1– Components, sub-components and indicators of epistemic suitability

Components	Sub-components	Indicators
Meanings	Problems	A representative and an articulated sample of problem-situations that allows the contextualization, exercising, amplification, and application of mathematical knowledge, which comes from mathematics itself and from other contexts, is presented. Problem generation situations are proposed (problematization).
	Languages	A wide repertoire of representations (verbal, graphic, material, iconic, symbolic...) is used to model mathematical problems and ideas, analysing the relevance and potential of each type of representation and carrying out translation processes between them. The language level is made appropriate for the students to which it is addressed. The construction, improvement, and use of representations to organize, interpret, and record ideas are promoted.
	Concepts	The fundamental concepts of the subject are presented in a clear and correct way and are adapted to the educational level to which they are addressed. Situations are proposed where students have to generate or negotiate definitions.
	Propositions	The fundamental propositions of the subject are presented in a clear and correct way and are adapted to the educational level to which they are addressed. Situations are proposed where students have to generate or negotiate propositions.
	Procedures	The fundamental procedures of the subject are presented in a clear and correct way and are adapted to the educational level to which they are addressed. Situations are proposed where students have to generate or negotiate procedures.
	Arguments	Propositions and procedures are explained and argued (justified and demonstrated) in a manner appropriate to the educational level at which they are addressed. The justification of mathematical statements and propositions is favoured through various types of reasoning and proving methods.
Relations		Mathematical objects (problems, definitions, propositions, etc.) are related and connected to each other. The diverse meanings of the objects that intervene in the mathematical practices are identified and articulated.
Processes	Communication, argumentation	Situations are promoted where the student has to argue (describe, explain, verify) and formulate conjectures about mathematical relations, investigate and justify them. Situations are proposed that allow the student to communicate using mathematical language to express his/her ideas with precision. Situations are proposed where the student can analyse and evaluate the mathematical thinking and strategies of others.
	Modelling	Situations are presented that allow the student to use mathematical models to represent and understand quantitative relationships (identify, select characteristics of a situation, represent them symbolically, analyse and reason the model, recognize the characteristics of the situation, the precision, and limitations of the model). The use of technology and the use of functions to model patterns of quantitative change are promoted.
	Generalisation	Situations, where students have the opportunity to describe, explain, and make generalizations and conjectures about geometric and numerical patterns, are promoted.
Epistemic conflicts		The contents, problem-situations, and their solutions, concepts, propositions, language, etc. are presented correctly without errors, contradictions, ambiguities.

Source: Elaborated by the authors from Godino (2013, p.119).

Cognitive suitability indicators

Relationships. Several authors agree that mathematical concepts must be presented in such a way that the necessary interconnections can be created for their learning to be meaningful (SANTAOLALLA, 2014). This implies offering experiences that allow the understanding of the concepts and that students use them in problem-situations (BALLESTA, 1995).

In this component we analyse whether the proposed experiences permit the assessment of whether the student establishes connections between the different mathematical objects and between their corresponding meanings.

Previous knowledge. It will be evaluated whether the concepts and competences that are prerequisites are specified (or retaken and consolidated) so that students establish the appropriate connections between the different contents and, in this way, face the learning with guarantees of success. When analysing whether the level of difficulty of the intended contents is manageable in its various components, it is studied whether the concepts, procedures, propositions are presented in an increasing degree of complexity (MONTERRUBIO; ORTEGA, 2012).

Cognitive conflicts. It is considered whether the error is valued as a source of learning or whether the author foresees the possible difficulties that students may have in approaching the subject (BRAGA; BELVER, 2016). This implies considering whether the textbook includes proposals of activities to detect the ideas, errors or previous difficulties that students have in relation to such content.

Evaluation. This component will assess whether the textbook includes a variety of evaluation, co-evaluation, and self-evaluation instruments that are applied in a variety of contexts, such as in problem solving or project development, and not only in pencil and paper tests. The previous indicator of Godino (2013) is expanded, giving relevance to the consideration of generalization and modelling processes, analysing whether the students' progress towards higher levels of abstraction and generalization is contemplated (ONRUBIA; ROCHERA; BARBERÁ, 2001).

Since NCTM (2000), the importance of developing metacognition skills, understood as reflection skills that imply that students are aware of what they are doing and that they monitor and self-evaluate their own progress, as well as adjust their strategies when solving problems, is highlighted. This leads us to include the existence of evaluation questions or situations that promote such reflections as indicators, that is, that allow students to think about what they have done, about the solution they have given, about what they have learned. It is considered whether the textbook provides spaces that allow the student to become aware of the nature and purposes of the tasks, to control his/her own learning, to be aware of his/her own learning styles and to self-evaluate by becoming aware of his/her mistakes (CÓRDOVA, 2006; ESPAÑA, 2006; MONTERRUBIO; ORTEGA, 2012). The cognitive suitability indicators are shown in Table 2.

Table 2 – Components and indicators of cognitive suitability

Components	Indicators
Relations	The experiences (situations, examples, explanations...) proposed allow the assessment of whether the student establishes relationships or connections between mathematical objects and between their corresponding meanings.
Previous knowledge	The textbook contemplates the previous knowledge necessary for the study of the topic. The intended contents are attainable (have a manageable difficulty) in its various components.
Individual differences	Expansion and reinforcement activities are included. Access, achievement and support of all students is provided.
Cognitive Conflicts	Error is valued as a source of learning. Potential cognitive conflicts of the students are foreseen.
Evaluation	Evaluation and self-evaluation instruments are proposed. It is promoted that the results of the evaluations are disseminated and used to take decisions. The various modes of assessment included in the textbook are suitable for evaluating that the students achieve the appropriation of the knowledge, comprehension, and competencies intended (conceptual and propositional understanding; communicative and argumentative competence; procedural fluency; situational understanding; modelling and generalization competence, metacognitive competence). The evaluation takes into account different levels of comprehension and competence.

Source: Elaborated by the authors from Godino (2013, p.121).

Affective suitability indicators

Instructional models and teaching resources, including textbooks, have a strong influence on the configuration of attitudes and beliefs (SANTAOLALLA, 2014). Although there is no consensus on what aspects should be considered when defining the affective domain, most authors include emotions, attitudes, beliefs, and values (BELTRÁN-PELLICER; GODINO, 2020). For this reason, these components have been considered in this facet.

Attitudes describe orientations or predispositions towards certain emotional sensations (positive or negative) that are moderately stable (BELTRÁN-PELLICER; GODINO, 2020). In this component of the GALT-Mathematics, it will be taken into account whether there are situations that motivate the student to argue under equal conditions and value their explanations and ideas (AAAS, 2000), as well as tasks that promote collaboration and active participation and respect for strategies and solutions different from their own (SANTAOLALLA, 2014). Consideration will also be given to whether the textbook encourages students to develop an attitude of flexibility in exploring mathematical ideas and alternative methods of problem-solving (REY; PENALVA, 2002; SANTAOLALLA, 2014).

Emotions are understood as rapid changes in feelings that occur consciously, pre-consciously, or unconsciously during mathematical activity, and can range from weak to intense (BELTRÁN-PELLICER; GODINO, 2020). In this component, it is valued that the contents are related to social problems or the student's social life (BRAGA; BELVER,

2016), include motivational aspects through humour, play or connections with the history of mathematics and other disciplines, pose real-life situations through problem-solving, appropriate language, etc. (MONTERRUBIO; ORTEGA, 2012; REY; PENALVA, 2002).

Whether the tasks and content are of interest to the student, if original ideas and useful work are valued, is considered (SANTAOLALLA, 2014). Furthermore it is also considered in this component whether opportunities of encouraging the self-esteem, avoiding rejection, phobia or fear of mathematics are promoted (BELTRÁN-PELLICER; GODINO, 2020; SANTAOLALLA, 2014).

Beliefs imply assigning some kind of external validity to the system of cognitive configurations; they are highly stable and largely cognitive and structured (BELTRÁN-PELLICER; GODINO, 2020). It will be important to assess whether the beliefs about mathematics, its teaching, the students' metacognition, and in general about the social context in which they are developed, are considered (SANTAOLALLA, 2014; REY; PENALVA, 2002).

Linked to the values component, understood as personal truths and aspects appreciated by individuals, it is considered whether the textbook promotes the student's appreciation of the aesthetic and precision qualities of mathematics.

Finally, following Monterrrubio and Ortega (2012) who propose the need to analyse whether social and civic aspects and attitudes, in general, are valued, in the guide we have considered introducing the affective assessment as a new component, with respect to those considered in Godino, Rivas and Arteaga (2012), Godino (2013) or Beltrán-Pellicer and Godino (2020). Table 3 shows the indicators of affective suitability.

Table 3 – Components and indicators of affective suability

Components	Indicators
Attitudes	Active participation in activities, perseverance, responsibility, etc., is promoted to encourage a mathematical attitude. The argument is favoured in situations of equality; the value of an argument does not depend on who says it. Flexibility is encouraged to explore mathematical ideas and alternative methods for problem solving.
Emotions	Tasks and the contents involved are of interest to students. There are motivating elements: illustrations, humour, poetry, riddles, etc. Logical reasoning, original ideas or useful, practical or realistic work are encouraged and promoted. Specific moments are programmed throughout the sessions so that students can express their emotions towards the proposed situations. Self-esteem is promoted, avoiding rejection, phobia, and fear of mathematics.
Beliefs	Beliefs about mathematics, its teaching, the student's metacognition, and about the social context in which learning takes place are analysed and considered.
Values	The student is encouraged to value the qualities of aesthetics, accuracy, and usefulness of mathematics in daily and professional life.
Affectivity evaluation	Assessment activities are proposed to evaluate the affective aspects of teaching and learning.

Source: Elaborated by the authors from Godino (2013, p.122).

Instructional suitability indicators

In this facet, questions that refer to the author-student interaction are addressed, understanding that, at least in the traditional textbook, the interaction is not bidirectional. In this sense, it is valued whether a clear and well-organized presentation of the contents is made, and if the key concepts of the subject are emphasized through some strategy. It is also observed whether the vocabulary used is understandable, the illustrations are adequate in terms of graphic quality and purpose, and whether varied and clear examples are presented throughout the theory (MONTERRUBIO; ORTEGA, 2012).

The interaction between students is contemplated through the presence of tasks or situations that promote dialogue and communication among students, considering flexible groupings (BRAGA; BELVER, 2016).

The existence of situations that allow students to be spontaneous, dynamic, participative, and unsettled (SANTAOLALLA, 2014), developing intellectual autonomy to face real problems and new situations (REY; PENALVA, 2002) is included in the autonomy component.

The formative evaluation component considers the presence of forms of continuous assessment throughout the lesson that serve as feedback to the students and not only at the end (AAAS, 2000; MONTERRUBIO; ORTEGA, 2012). It is also discussed whether the textbook contemplates the use of various assessment techniques consistent with the learning goals (GODINO; RIVAS; ARTEAGA, 2012).

Table 4 includes the indicators finally considered for each of these components of interactional suitability.

Table 4 – Components and indicators of interactional suitability

Components	Indicators
Author-student interaction	The author makes an adequate presentation of the topic (clear and well-organized presentation, emphasizes the key concepts, etc.) Situations where consensus is sought based on the best argument are promoted. Diverse rhetorical and argumentative resources are used to involve and capture the students' attention. The inclusion of students in the dynamics of the lesson is promoted or facilitated.
Students interactions	Tasks are proposed that encourage dialogue, communication and debate among students in which different points of view are explained, justified and questioned using mathematical arguments. Situations in which students have to convince themselves and others of the validity of their statements, conjectures and answers, supported by mathematical arguments.
Autonomy	Sometimes students take responsibility for the study (raise questions and present solutions; explore examples and counter-examples to investigate and conjecture; use a variety of tools to reason, make connections, solve problems, and communicate).
Formative evaluation	Forms of evaluation are included to allow for the systematic and continuous observation of students' cognitive progress. Assessment is seen as a process in the service of teaching and learning (serves as feedback to students). The use of diverse evaluation techniques is contemplated (problem solving, practical tasks, ...). Self-evaluation, co-evaluation and hetero-evaluation activities are included. Assessment is consistent with the learning goals (tasks similar to learning situations are included).

Source: Elaborated by the authors from Godino (2013, p.123).

Media suitability indicators

Two components are considered in this facet: material resources and time. With regards to the use of material resources, an assessment is made of the extent to which the textbook promotes the incorporation of complementary materials into the book itself (BRAGA; BELVER, 2016); whether appropriate tasks requiring the use of a calculator, computer, Internet, and whether the sources used are made explicitly and diversely (BONAFÉ, 1992). In relation to the time component, it is considered whether the temporal space is adequate to cover the most difficult contents and if, in general, the timing of activities and contents is feasible. Table 5 includes the indicators finally considered.

Tabla 5 – Components and indicators of Mediational suitability

Components	Indicators
Material Resources	The use of manipulative, audio-visual, and computerized materials is promoted, allowing the introduction of good situations, languages, procedures, arguments adapted to the intended content. Definitions and properties are contextualized and motivated using specific situations, models and visualizations. The bibliographic sources used are made explicit and diverse.
Time	The contents which are more difficult to understand are given sufficient space in time. The timing of the sequencing of activities and content is appropriate.

Source: Elaborated by the authors from Godino (2013, p.125).

Indicators of ecological suitability

The ecological dimension considers first of all the degree to which the contents, objectives, situations are adapted to the curricular guidelines (AAAS, 2000; MONTERRUBIO; ORTEGA, 2012). For education in values, it is valued that the educational resource avoids the transmission of stereotypes, racist, sexist, homophobic, discriminatory elements, etc., both in the text and in the images (BRAGA; BELVER, 2016). It is also taken into account whether diversity is addressed and whether different points of view are offered on the same cultural phenomenon (BONAFÉ, 1992).

Finally, in the component of intra/interdisciplinary connections, it is analysed to what extent the textbook indicates links with previous and subsequent learning (BONAFÉ, 1992), connections with the history of mathematics (MONTERRUBIO; ORTEGA, 2012), and the presence of transversal themes in the contents and situations (BRAGA; BELVER, 2016). All these suitability criteria are shown in Table 6.

Tabla 6 – Components and indicators of ecological suitability

Components	Indicators
Adaptation to the curriculum	The objectives, contents, their development and evaluation correspond to the curricular guidelines.
Didactical innovation	Innovation based on research and reflective practice.
Socio-professional adaptation	The contents contribute to the students' socio-professional education.
Education in values	The training of democratic values (respect for diversity, tolerance, integration, cooperation, environmental awareness, pacifism, other values and prejudices) and opportunities for students to question the apparently obvious or given as natural (critical thinking) are contemplated and promoted.
Intra/Interdisciplinary connections	The contents are related to other intra- and interdisciplinary contents (cross-cutting themes, history of mathematics, others).

Source: Elaborated by the authors from Godino (2013, p.126).

Conclusions

In this article, we have described the development of a guide for the analysis of mathematics textbooks based on the theory of didactical suitability (BREDA; FONT; PINO-FAN, 2018; GODINO, 2013; GODINO; RIVAS; ARTEAGA, 2012) whose indicators in each of the facets have been re-analysed and enriched with the review of literature background on the analysis of textbooks. The GALT-Mathematics incorporates the most relevant aspects as specific criteria to assess the overall didactical suitability of a textbook lesson previously chosen to implement the teaching and learning process of specific mathematical content. The guide should be understood as a tool that facilitates the teacher's decision making on how to use a textbook lesson in the classroom to optimize the instructional process.

The GALT-Mathematics should be understood as an analysis tool of a textbook lesson that, when applied and discussed by prospective or in-service teachers, and researchers themselves, should be progressively improved and enriched.

In future research, we propose to contemplate indicators that refer to the analysis of textbooks addressed to teachers, to evaluate the use of this guide to apply it in the comparison of textbooks, and of course in the adaptation to specific mathematical contents. Another open question corresponds to the design and evaluation of lessons based on the use of hypertexts and other technological resources.

The school textbook presents some peculiar characteristics: it is a curricular material of preferential use by the teachers and a mediator of the student's learning. Therefore, the analysis of the textbook offers enormous possibilities in the initial training of educational professionals (BRAGA; BELVER, 2016). In this sense, the guide elaborated in this research provides a tool for systematic and reflexive analysis that prospective and in-service teachers can use in the detection of characteristics of textbooks. It is also useful for

identifying possible ways to use the textbooks in combination with other complementary resources to optimize the teaching and learning of a specific subject.

We consider that performing didactic analysis of textbook lessons should be a competence of the mathematics teacher as part of a more general lesson planning competence (BURGOS et al., 2020). Therefore, the design and experimentation of specific training interventions on lessons analysis, which allow the use of textbook analysis guides by teachers themselves, is an open research topic. A pilot experience of this research topic is described in Burgos, Castillo and Godino (2020).

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