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Dossier: History of Mathematical Education

MATHEMATICS IN CLASSROMS, NA ICONOGRAPHIC HISTORICAL STUDY[1]

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ABSTRACT

Historically studying representations of math-centered classrooms shows us arrangements revealing underlying teaching methodologies, learning theories, and technological innovations. This iconographic study draws on images in textbooks, magazines or collectors' collections from medieval times to the mid-twentieth century. Using an interpretative paradigm, involving historical and documentary research, we look for the perspectives of teaching and learning mathematics.

Keywords: mathematics classroom, iconography, classroom culture, history of mathematics education.

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A MATEMÁTICA NA AULA, UM ESTUDO HISTÓRICO ICONOGRÁFICO

RESUMO

Estudar historicamente representações de aulas centradas na matemática revela metodologias de ensino subjacentes, teorias de aprendizado e inovações tecnológicas. Este estudo iconográfico baseia-se em imagens de livros didáticos, revistas ou coleções de colecionadores desde os tempos medievais até meados do século XX. Utilizando um paradigma interpretativo, envolvendo pesquisa histórica e documental, buscamos as perspectivas de ensino e aprendizagem de matemática.

Palavras-chave: aula de matemática, iconografia, cultura da aula, história da educação matemática.

MATEMÁTICAS EN EL AULA, UN ESTUDIO HISTÓRICO ICONOGRÁFICO

RESUMEN

Estudiar históricamente representaciones de aulas de matemáticas revela metodologías de enseñanza subyacentes, teorías de aprendizaje e innovaciones tecnológicas. Este estudio iconográfico se basa en imágenes de manuales, revistas o colecciones desde la época medieval hasta mediados del siglo XX. Utilizando un paradigma interpretativo, que involucra investigación histórica y documental, buscamos las perspectivas de la enseñanza y el aprendizaje de las matemáticas.

Palabras clave: aula de matemática, iconografía, cultura de clase, historia de la educación matemática.

MATHÉMATIQUES EN CLASSE, UNE ÉTUDE HISTORIQUE ICONOGRAPHIQUE

RÉSUMÉ

L'étude historique des représentations de classes de mathématiques révéle méthodologies d'enseignement, théories d'apprentissage et innovations technologiques. Cette étude iconographique s'appuie sur des images de manuels, de magazines ou de collections depuis l'époque médiévale au milieu du XXe siècle. En utilisant un paradigme interprétatif, impliquant des recherches historiques et documentaires, nous recherchons les perspectives d'enseignement et d'apprentissage des mathématiques.

Mots-clés: classes de mathématiques, iconographie, culture de classe, histoire de l'enseignement des mathématiques.

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INTRODUCTION

The main thrust of this article is to have a glimpse of how did mathematics classes look in the past? In other words, we will seek to interpret the representations of mathematics classes using images (photographs, engravings, pictures) found in textbooks and newspapers and linking them with other studies of past mathematics teaching. The use of images as historical evidence extends the historian's field of work beyond traditional material sources: official statements, texts published in newspapers, books, documentation in archives, etc. Contesting what he calls visual invisibility, Peter Burke (2001) discusses the ways in which cultural history can incorporate the study of images.

Using an iconographic approach, we intend to identify the elements shown in pictures of classrooms and the relationships among them and thus understand change and stability in mathematics classrooms. It is a journey through modes of organization of the teaching process, the *aula* (that we translate here by the word *class*)[4], and not about the content of that teaching (the curriculum). It will necessarily be a preliminary trip, as a thorough study on the subject has not yet been carried out in Portugal, as opposed to more consistent work carried out in other countries[5].

METHODOLOGY

In this paper we intend to perform an iconography, that is, to examine, categorize and interpret images. The method, used in art history, is influenced by the reflections of Erwin Panofsky (1939) who distinguished among three levels of interpretation, corresponding to three levels of meaning: the *pre-*

[4] The Portuguese word *aula*, comprehensively designates the entire social and cultural process involved in the educational act.

[5] See, for example, Moreno and Viñao (2017).

iconographic description (component identification, corresponding to natural meaning), the *iconographic analysis* (study of these components eventually interconnecting them with other studies, corresponding to the conventional meaning), and, most importantly for Panofsky, the *iconological interpretation* (establishing relations with our knowledge about the historical period, looking for intrinsic meaning)[6].

Our work is based on images (photographs, engravings, and pictures) that refer to the Portuguese context systematically collected in bibliographic collections, archives and a magazine. In one case we use architectural vestiges. A prominent role is assumed by the documents collected in textbooks and by photographs found in the weekly journal *Ilustração Portugueza* electronically available. This magazine was published between 1903 and 1924 and it is an indispensable pictorial archive for researching the first quarter of the twentieth century in Portugal. Generously illustrated, its images are an important part of the journal that will "enjoy both men of today and generations to come" ... because they constitute "the most documented history of current Portuguese customs in their many features" (MARTINS, 1903, p. 2), as announced in the first paragraph of its first number. The images published in this magazine are not only cultural productions of an era, but they have also the potential of shape meanings within the culture of that time, as Peter Burke argues (2001).

Thus, we collected images in Portuguese mathematics textbooks between the 16th and mid 20th centuries that were completed with photographs from various archives and magazine articles. Occasionally, we found pictures of "generic" classrooms that could be used to teach several disciplines. We decided to include some of them considering that mathematics teaching could plausibly occur in those settings. Lastly, we also included an image of a class for professional teaching that, although vaguely related to mathematics, illustrates a very different approach to the teaching endeavour.

Following Panofsky´s approach, in the first phase, we proceeded to

[6] An in-depth look at the method and its application to mathematics can be found in Vaz (2013).

identify the components of the chosen images that, in a second phase, were confronted with each other, with other historical (namely textual) material, and with conclusions from other research studies. In the last phase, we established relationships with our knowledge about the social contexts of the times. Although Panofsky was somehow uninterested in a social view of art (BURKE, 2001), inclosing our study within the framework of cultural history would not be complete without incorporating this latter dimension into our analysis.

In writing this iconography we essentially adopted a chronological order. We started with traces of medieval classes and continue with Jesuit classes. We moved on to the nineteenth century, when a more frequent incorporation of engravings and photographs in books provides us with a larger array of evidences. In that century we found traces of classes of first letters that will gradually incorporate innovations such as the blackboard, student's desks, etc. As for secondary education, we found traces of high school classes and special classes for vocational education. On our journey, we encountered the influence of the New School movement, occurring both in the early twentieth century and in the 1970s, when the end of the Portuguese dictatorship allowed for larges degrees of freedom in education.

THE MEDIEVAL CLASS

The earliest engraving picturing a class we found is located coincidently at the end of the first edition of the *Tratado da pratica Darismetyca* by Gaspar Nicolas (1519/1963)[7] which is the first mathematical textbook written in Portuguese and printed in Lisbon (Figure 1). Following the tradition of the *Libri d'Abbaco*, and most likely intended for the merchant class, it explains the use of Arabic numbers in arithmetic algorithms among other contents. The image seems to represent a typical medieval class, in which the master sits on a higher

[7] This engraving was used in other books of the same publisher.

chair and points at the book while the learners presumably flip through other copies of the same book or write in personal notebooks.

Together with the first national grammars, the *Tratado* suggests a new "mentality" in Portuguese society, when writing, calligraphic, and arithmetic skills start to play a central role either in the economic activities or the daily life of a thriving merchant bourgeoisie and in a series of professions orbiting around it (ALMEIDA, 1994). Although the image represents a "generic" class, it is significant that it is included in an arithmetic book, suggesting that this pedagogical organization was appropriate to teach this subject.

Figure 1 - Engraving in the last folio of *Tratado da p*ratica Darismetyca.

Source: Nicolas (1519/1963).

To perform the analysis of the engraving we highlighted the following elements: the characters, the books and notebooks, the stools, the position of each character and the overall composition of the image. The master is at the centre and points to a book symbolizing the centrality of written knowledge and his mediating role between two forms of knowledge: the one contained in the book and the other learnt by heart or written down by the students. Note the posture of the leftmost student who transcribes the words of the master. The print seems to be "staged", that is, it may not pretend a factual representation of a class, but rather, through a global composition of characters and artefacts,

give a message, in this case, that school knowledge is essentially textual its authority lies in the book and the master — the *lente*, that is, the one who reads — is an intermediary between the book and the students. The iconological interpretation reveals to us a teaching done through the reading of the book and learning consists in its faithful written and oral reproduction.

THE JESUIT CLASS

The Jesuit Order grew after the Council of Trent (mid-16th century) highlighted the importance of teaching as a factor in opposing Protestantism. The kingdom of Portugal had a close connection to both the genesis of the Jesuit order in the 16th century and its banishment in several European countries in the 18th century (CARVALHO, 2008).

Gradually developing a set of rules, the *Ratio Studiorum*, which, especially from the seventeenth century on, governed and standardized teaching in all colleges, the Order will perfect a whole new pedagogical technique that was sought more effective than that of the medieval tradition. In colleges of higher rank, and just as in the medieval school, the Jesuit teacher takes a prominent place, speaking ("reading") from the pulpit. Innovation now lies in new teaching techniques: "explaining", "repeating" or "disputing" techniques developed in the colleges. A photo from one class at *Colégio do Espírito Santo* in Évora (Figure 2) shows the pulpit in its higher ground in a room decorated with *azulejos* (tiles) representative of school subjects.

Figure 2 - Pulpit in classroom of *Colégio do Espírito Santo*, Évora.

Source: authors' photo (2011).

Jesuit pedagogical innovations include the clarification of didactic procedures appropriate to the subjects to be taught. The teacher of mathematics, for example, was subject to a specific set of "Rules":

> 1. Authors, time, math students. — To physics students explain in class for 3/4 hour the elements of Euclid; after two months, when the students are a little familiar with these explanations, add something about Geography, the Sphere[8], or other subjects they like to hear, and this simultaneously with Euclid, on the same day or every other day.
>
> 2. Problem — Every month, or at least every two months, in the presence of an auditorium of philosophers and theologians, ensure that one of the students solves some famous mathematical problem; and then if appropriate, defend the solution.
>
> 3. Repetition — Once a month, usually on a Saturday, instead of the lecture, repeat the main points explained in the month (SILVA, n. d. , authors' translation).

Jesuit colleges also developed other innovative techniques, such as sabbatines, Saturday sessions in which the subjects given during the week were recapitulated through discussion or repetition.

[8] The study of the Sphere is what we today call Cosmography.

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In some Jesuit colleges in Portugal, the classroom walls were decorated with *azulejos* (tiles) representing allegories of key points of the doctrine. For example, at *Colégio do Espírito Santo*, Évora, the mathematics classroom is covered with panels depicting mathematical diagrams, scientific instruments, various machinery, mathematical applications, etc. At *Colégio de Santo Antão*, Lisbon, a similar allegory illustrates distinct mathematical topics: geometry, use of instruments, Archimedes' theorems, optics, ballistics, navigation, etc. (Figure 3, left). These allegories are not a teaching device *per se*. Instead, they create an atmosphere highlighting key features of mathematics.

Figure 3 - On the left, an allegory about geometry (Colégio de Santo Antão, Lisbon). On the right, a mathematical azulejo (unkown Colégio, Coimbra.

Source: Left imagem: (Rocha, 2017, p. 1). Right image: Simões and Duarte (2007, p. 40)

Another kind of azulejos was also found representing specific figures taken from the *Elements of Euclid* written by André Tacquet (1612-1660) (Figure 3, right). There are reasons to believe these azulejos came from a Jesuit college in Coimbra (SIMÕES; DUARTE, 2007). We do not know anything how these tiles fitted actual mathematics classes. It is apparent that they did not have a decorative and symbolic purpose as the previously mentioned allegories, but were reproductions of figures from an actual mathematical textbook meant as a means to improve the quality of mathematical and scientific learning by focussing in actual content (LEITÃO, 2007).

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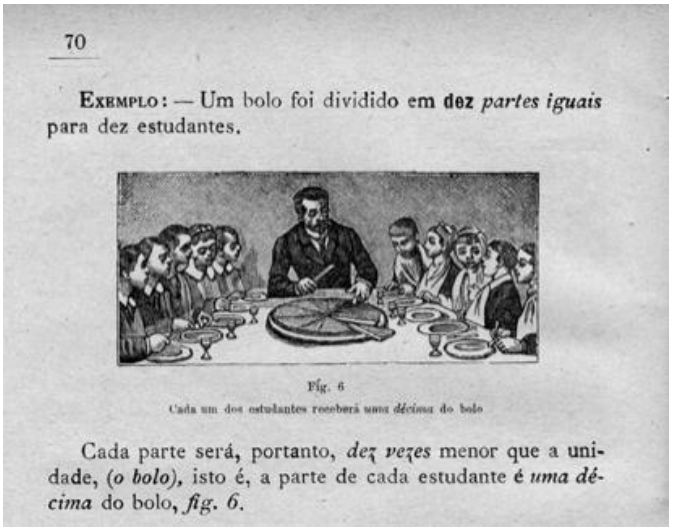
TRACES OF ELEMENTARY CLASSES IN THE XIX CENTURY

Jesuit schools, as well as those of other religious orders, were not intended to provide the elementary formation of reading, writing, counting and praying. For this there were the masters of first letters that will last well into the 20th century. The iconography of the late 19th century shows us some representations of this type of teaching.

For example, Ulysses Machado's book for primary education (1914) represents a teacher sharing a meal with his students in a scene that mimics a family dinner in an illustration accompanying an arithmetic problem (Figure 4).

Figure 4 - Family atmosphere between teacher and students.


70

EXEMPLO: — Um bolo foi dividido em **dez** *partes iguais* para dez estudantes.

Fig. 6

Cada um dos estudantes receberá uma *décima* do bolo

Cada parte será, portanto, *dez vezes* menor que a unidade, *(o bolo)*, isto é, a parte de cada estudante é *uma décima* do bolo, *fig. 6*.

Source: Machado (1914, p. 70).

The components of the image are clear: the characters, the table and its artefacts and the overall composition of the situation portrayed. Now it is not a staging, but rather a representation of a real situation that aims to illustrate the context of the problem. The iconological interpretation tells us that the situation echoes the hybrid school-house common during the 19th century (SILVA, 2005), although it was published in the following century and we can hardly

imagine how this situation could be similar to our current reality. But it should have some meaning at the time the book started to be published (beginning of the 20th century) and for the students for whom it was intended. In fact, many of these "schools" were also the teacher's house, and it would be natural that school practice was also be similar to a family practice with the teacher and the family who lived with him, a situation perfectly framed by legislation. According to a 1809 notice, "the masters, both regular and secular, could continue to give lessons in their homes or convents, having in them the 'necessary amenities'" (ALBUQUERQUE, 1960, p. 27).

The reality would, however, be much more complicated, as reported by Francisco Santos Morocco, who in 1799 is in charge of preparing a report on the state of primary and secondary schools. Referring to some primary school teachers, he writes, in an indignant tone:

> These teachers, like hawkers in a hardware store, sell paper, paint, rules and folders to their disciples; they impose monthly taxes, each contributing to the drinking water, having the most price for a glass, sweeping the school, and the most I omit (apud ALBUQUERQUE, 1960, p. 41, authors' translation).

CLASSES IN THE FIRST LICEUS

The Passos Manuel Reform instituted "liceus" as secondary schools in 1836. The first ones were housed in adapted buildings and there would naturally be some variation in their physical configurations for whom we did not find detailed descriptions. In Liceu of Aveiro, the first installed in a purpose-built building inaugurated in 1860, classrooms are sometimes referred as "casas de aula", literally, "houses of classes" (MARQUES, 2003, p. 42).

We could not find pictures of the classrooms of these first liceus. In this text we will try to fill this gap with descriptions collected in the literature. Take, for example, a depiction of the arrangement of furniture in the classroom "In

class, places for students form in front of the teacher's chair, a semicircle armchair amphitheater" (MARQUES, 2003, p. 43).

The class would thus have an armchair amphitheater format, with the teacher at the centre, thus solving the problems of student vigilance and good communication between them and the teacher.

About the organization of the class, we can get an idea through the Regulation for the National High School published in 1860 under the direction of Fontes Pereira de Melo. Chapter IV, "About Classes", stats: "Article 26 Classes of liceus are public. There will be places for visitors, entirely separate from the students' places" (PORTUGAL, 1861).

The possibility of "visitors" stops to be mentioned by the end of the 19th century. The norm would be tacitly valid for male classes (the only ones that existed in the public system), but when women's high schools started in 1890, their regulation stipulates in art. 14th that the classes are no longer public and can only attend "parents, guardians or persons entrusted with their instruction". The 1860 Regulation also states:

> Art. 27.º The places of the students in the classes will be arranged so that everyone can equally receive the lessons of the teachers and be supervised by them.
>
> Art. 28.º There will be three places of distinction in each class, which should be occupied by students who in the previous week were most distinguished in the performance of their school duties. [...]
>
> Artº 30.º Of the two hours of the class teachers shall employ at least one to hear as many students as possible about the previous lesson, and the rest of the time to give explanations which they deem fit for the full intelligence of the doctrines which are the object of the lesson given that day or that students have to study for the next day of class.
>
> Art. 31.º There will be written exercises or themes in all classes, which will be analysed and amended by the teacher, aloud and for the whole class. (PORTUGAL, 1861, authors' translation)

This formulation will be repeated in later legal documents and

disappears at the end of the century.

However, we can hardly speak of a secondary education system until 1895 (CARVALHO, 1996; VALENTE, 1973).The numerous and contradictory reforms, the low remuneration of the teachers, the precariousness of the facilities, are factors that will keep students away from liceus. A description of how students would acquire the education needed to gain access to the University cann be found can be found in a text by Agostinho Campos (1870-1944), writer, journalist, educator and native politician from Porto, who reports his school career.

> At that time [around 1880], the boys wondered and informed each other. Who is good for Algebra and Drawing? [Teacher] Teófilo de Faria, in Rua do Sol. For English and Introduction to Sciences? Carlos Chambers and Bento Carqueja, at Colégio da Glória in Cedofeita. For History and Geography? Muffler, in the course of Júlio Moreira, Rua de Passos Manuel. For Legislation? Alves da Veiga, in Santa Catarina. And so on (ADAMOPOULOS; VASCONCELOS, 2009, p. 24; authors' translation).

Most of the students did not attend liceus, where education was deemed of inferior quality and, after obtaining the necessary training in one-off teacher classes, as we see in the previous citation, they simply went to the exams in liceus that were almost deserted for the rest of the school year (VALENTE, 1973). It could hardly be construed a secondary school tradition on these premisses.

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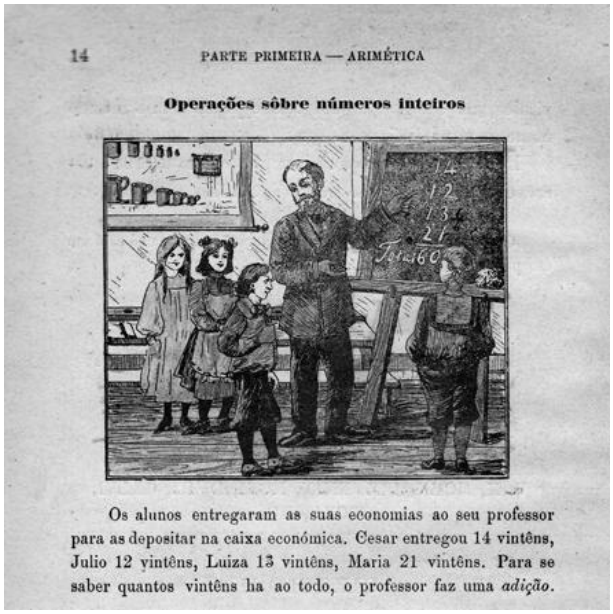
INOVATIONS IN THE END OF THE 19TH CENTURY

By the end of the 19th century and the beginning of the 20th, textbooks start to incorporate engravings as a way to enhance the way didactical topics were addressed. This new technique allows us to appreciate visual changes to the structure of classes that occur from the end of the 19th century. We find some of these innovations in a textbook by Ricardo Diniz de Carvalho which supported the introduction of the metric system in primary schools and has editions from the early 1880s until 1912. The book includes some engravings and one incorporates two references to new teaching methodologies and technologies[9] (Figure 5).

Figure 5 - Modernity is shown by the blackboard and the synoptic picture of the metric system on the left.


14 PARTE PRIMEIRA — ARIMÉTICA

Operações sôbre números inteiros

Os alunos entregaram as suas economias ao seu professor para as depositar na caixa económica. Cesar entregou 14 vintêns, Julio 12 vintêns, Luiza 13 vintêns, Maria 21 vintêns. Para se saber quantos vintêns ha ao todo, o professor faz uma *adição*.

Source: Carvalho (1912, p. 14).

In a engraving packed with components, a first sign of modernity is given by the blackboard that allowed the contents to became available to all

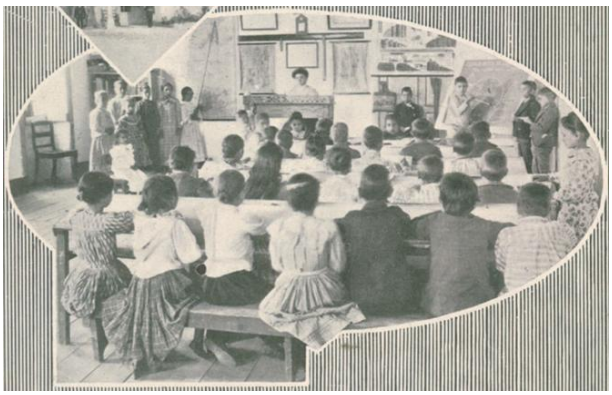
[9] We consulted the 17.ª edition from 1912.

students simultaneously and in large format. The second, in background, is the government-developed synoptic picture of the metric system that has been mandatory in classrooms since 1860 and which Ricardo Carvalho intended to highlight.

Note that the blackboard is still trying to find its place in the spatial arrangement of classroom. The class structure is not yet frontal to the board and students' work area is a running desk with built-in ink cartridges and a bench to sit on. To discuss on the board the algorithm of addition, teacher and students need to leave their usual places and put themselves around the board. The board begins to emerge as a place where mathematical knowledge materializes and where mediations between teacher, students, and content occur.

We found on the pages of *Ilustração Portugueza* a similar classroom in 1912 (Figure 6). The staged photography, intended to enhance the good quality of the school associated with *Quinta de Carvalhaes*, which specializes in wine production, shows us the willingness of students sitting on benches positioned along tables. The teacher sits at a desk on a higher ground. One student on the left uses the pointer to mark something on a map and on the right another student with a pointer draws attention to several geometric figures where an elaborate drawing of a cone of revolution can be distinguished. The room is literally lined with paintings, with the synoptic picture of the metric system standing out in the center.

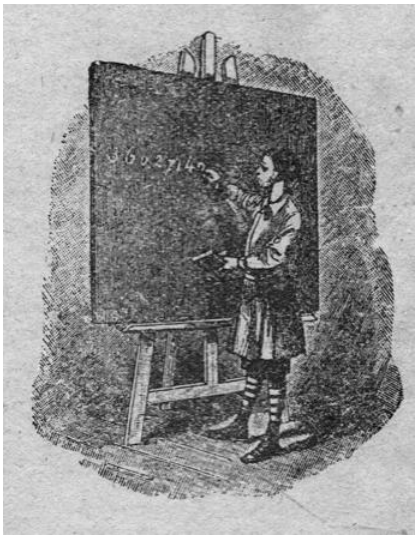
Figure 6 - Class in Quinta de Carvalhaes, Trás-os-Montes.



Source: *Ilustração Portugueza*, 1.ª Série, n.º 327, 27/5/1912, p. 28.

A book by Ulysses Machado (1865-1939) includes on the cover an engraving with a blackboard similar to the one in Ricardo Carvalho's book (Figure 7) showing a student performing an arithmetic algorithm (MACHADO, 1914). Here the student enacts the mathematical knowledge and the board is the place of sharing his actions with the teacher and the other students.

Figure 7 - Detail of the cover of Ulysses Machado's book (1914).



Source: Machado (2014).

At the end of the nineteenth century, classrooms began to be organized in rows of individual desks for students, as we can see in Figure 8, which represents the design of a primary school in Lisbon.

Figure 8 - Project of a primary school.

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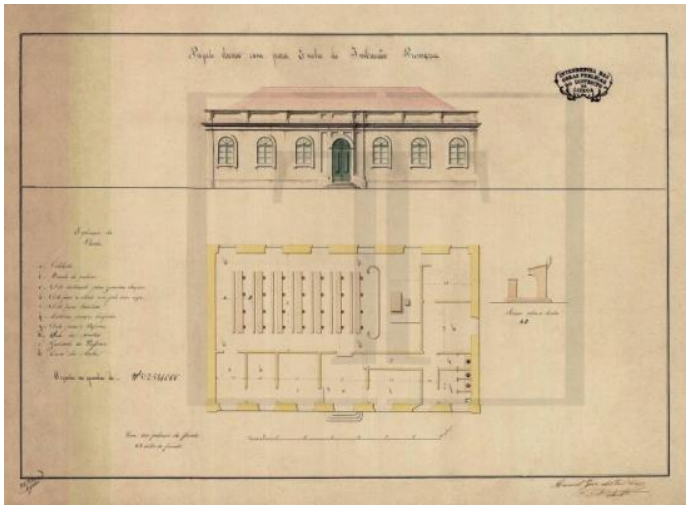
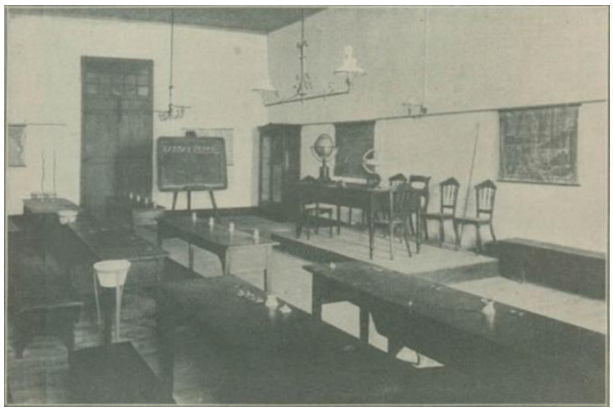


Source: LEITE, Restos de colecção.

In 1905 the magazine *Ilustração Portugueza* published a photograph of a classroom with this topology (Figure 9). Performing an iconographic analysis, we find the usual elements of previous images. The centre is occupied by the teacher's desk placed on a raised platform and on the walls hung various maps and paintings. A small blackboard appears in a lateral position. In addition to terrestrial globes, a pointer appears against the wall behind the teacher's desk. The room does not yet have individual student desks, but only stools and tables with a place for ink cartridges.

Figure 9 - Regimental school.



Source: *Ilustração Portugueza,* 1.ª Série, n.º 78, 16/1/1905, p. 11.

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Some photographs also show another widely used teaching device, the slate ("lousa") on which students wrote with a chalk stick. It is a cheap and reusable material. At the time paper was more expensive and could not be reused. An example of a 1905 photograph (Figure 10) illustrates the use of the slate. On the board some arithmetic calculations are written and the image is intended to show how students record the procedures on their slates. Some books or notebooks are perched on the left side of each desk, but do not appear to contribute to the event. On the left side of the teacher rests the pointer. Note also how the edge of the desk delimits (and underlines) the space of the teacher's authority. Here, the blackboard is already next to the teacher and the photography reveals its importance as a mediator of knowledge between the teacher and the student.

Figure 10 - Class in the school *Asilo da Ajuda* in Lisbon.

Source: *Ilustração Portugueza*, 1.ª Série, n.º 92, 7/8/1905, p. 9.

The photo was taken at *Asilo da Ajuda*, a school intended for deprived girls and the journal aims to show the excellence of teaching at this establishment.

This picture also shows the use of specialized students' desks. These desks, together with other educational materials (Fröebel's dons, metric boxes, etc.) were firstly manufactured in Portugal in the early years of the twentieth century (CARVALHO, 2004) and its use in the *Asilo* confirms it as an innovative school.

As other photos we include in this text, it is a "staged" photograph, that is, the participants compose a situation the photographer wish to illustrate. It was intended to show the relationship between the teacher, questioning (?) the student at the blackboard where an addition of fractions is presented. The other students, with their hands on the desk, appear to write something. Only one student undoes the scene by looking straight into the camera revealing the hidden character, the photographer.

We can also find the slates in conjunction with school notebooks in a photograph probably from the 1930s or 1940s on a wall in the Cartaxo County School Museum (Figure 11). Here the board has already found its present

position in the center of the wall behind the teacher and in front of the students.

Figure 11 - Slates and notebooks lying on the individual desks.

Source: Museu Escolar do Concelho do Cartaxo.

The individual slates were used for quite some time. In 1938 the primary teacher Dionísio das Dores Gonçalves describes in his diaries the constant use of the blackboard in parallel with that of notebooks, although he defends the use of paper for exams.

> I then sent them to the place and told them to draw on the slates the map of Portugal with the respective systems of mountains. At first glance, this lesson may seem too challenging for children, but it is not. Very good results are obtained by down playing the use of maps as much as possible and replacing them with drawing on the slate, on the slates and then on paper. Students memorize better the subjects studied and localize better any river, mountain, district or province (GONÇALVES, 2005, p. 197, authors' translation).

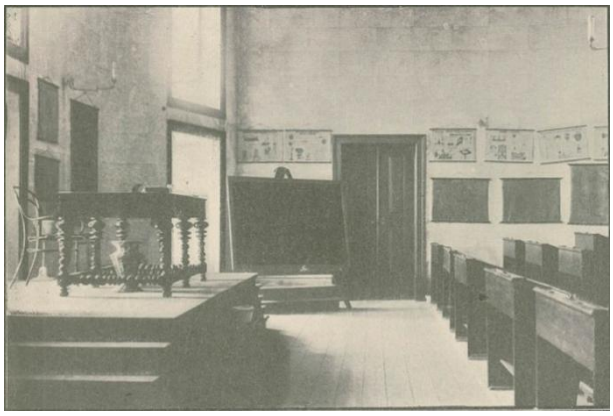
The long change from the use of slates to paper will gradually deepen the technology of the school notebook, incorporating more specialized notebooks: those with squared sheets and the special ones for calligraphy, music, etc. Slates will no longer be able to compete with these innovations.

CLASSES FROM THE FIRST HALF OF THE 20TH CENTURY

From 1895, with the reform of Jaime Moniz, the public and secular secondary structure idealized since Passos Manuel will become reality establishing a subsystem of intermediate (secondary) education between primary school and the university (MAGALHÃES, 2010; VALENTE, 1973). Adopting the class system in liceus will allow for classes of students of the same age that, as a greater or lesser variation in their composition, will go through the various years of schooling. Secondary school education thus gains stability and predictability.

Ilustration Portugueza publishes in 1903 a photograph of a classroom for secondary education at *Asilo Maria Pia*, intended for deprived students (Figure 12).

Figure 12 - Class of secondary education in *Asilo Maria Pia*.



Source: *Ilustração Portugueza*, 1.ª Série, n.º 9, 4/1/1903, p. 10.

As in the case of the reportage at *Asilo da Ajuda*, the journal intends to show the good quality of this school. We find here the same elements from previous photos. It is an amphitheater room, with the teacher's desk set high on a platform. In this classroom, the blackboard has a larger dimension than in previous images, but it is not yet placed on the wall in front of the students.

Sometimes the themes published by *Ilustration Portugueza* are related to the school space and we couldn't help but notice that when a picture of a class with its teachers and students is published, a mathematical theme is often placed on the board, as we previously showed. Mathematics seems to play an important role legitimizing the quality of teaching taking place in those schools. Figure 13 illustrates precisely this fact, as it registers King Manuel II's visit to a primary school in Santos, Lisbon in 1910. The photograph by Joshua Benoliel (1873-1932), one of the best-known Portuguese photographers of the early twentieth century, does not seem to be staged. The central action unfolds on the board, where a student develops a long multiplication (probably 6 or 7 digits by 4), closely observed by the King while the teacher seems to keep an eye on the children's behavior. Excellence in mathematics was chosen to illustrate excellence of the school. Note that in this primary school the wall behind the teacher is just an empty space with no maps or pictures.

Figure 13 - A visit of D. Manuel II to a primary school in Santos, Lisbon.

Source: *Ilustração Portugueza*, 1.ª Série, n.º 213, 21/3/1910, p. 10.

In the same year, after monarchy was replaced by a republican regime, the magazine reports a visit of the new Minister of Justice to an asylum in the outskirts of Lisbon (*Ilustração Portugueza*, 1.ª Série, n.º 244, 24/10/1910, p. 12). Again in the photographed class we find a multiplication represented on the

board.

THE NEW CLASSROOMS IN LICEUS IN THE BEGINNING OF THE 20TH CENTURY

The structure of the class will undergo some changes with the inauguration of new secondary school buildings in the beginning of the twentieth century that consolidate a new reference in architecture (MARQUES, 2003). It starts with the Liceu Camões in Lisbon in 1909, continues during the second decade of the twentieth century and supports the construction of a cultural imaginary of what was "the liceu" that until then was very diffuse (VALENTE, 1973).

In what pertains to the purpose of our text, the structure of the room integrates the blackboard that is affixed to the wall next to the teacher and facing the students. The two photographs in Figure 14 illustrate this new centrality in the walls of the new secondary schools: the teacher sitting at his desk, placed on higher ground, and the board, the privileged place for the mediation of knowledge. In front are rows of individual specialized desks. The photo on the left shows a classroom at *Liceu Camões* in Lisbon in 1909/10, the year of its inauguration. The one on the right dates probably from the 1950s shows the same arrangement in *Liceu Pedro Nunes* also in Lisbon.

Figure 14 - Classrooms in the new liceus. On the left, *Liceu Camões* (1910/11) and on the right, *Liceu Pedro Nunes*.

Source: Left - Adamopoulos; Vasconcelos (2009, p. 75). Right - Monteiro (2018).

The amphitheater class of the first liceus of the mid-nineteenth century is gone and it is no longer necessary to legislate on the importance of all pupils "being able to receive the lessons equally" as they appeared in official documents throughout the nineteenth century. Now the class naturally assumes the central, and usually higher, place of the teacher with his desk on a platform, the individual student desks and the blackboard on a wall in front of them.

A realization of this modernization project can also be observed in primary school classes. In a class at the *Colégio Nacional de Lisboa*, prior to 1911 (Figure 15) we can identify the components: specialized furniture (desks, dais, teacher's desk, blackboard, maps, metric box hanging on the right). The organization of the room is centered on the teacher and the board. The photograph is included in an advertising postcard, and thus is produced with propagandist intentions (again a staged photo) shows the teacher dictating a text that one student writes on the board and the others copy in notebooks.

Figure 15 - Class of the first degree of the primary school.

Source: LEITE, Restos de colecção.

This type of organization is already dominant and we will also find it in teacher training as we can see in a photograph of the *Escola do Magistério*

Primário de Lisboa[10]. In the classroom, arranged in rows of desks, we see a group of students consisting mainly of women - men are at the back of the room — wearing a uniform consisting of a white coat (Figure 16).

Figure 16 - A class with future teachers for primary schools (no date).

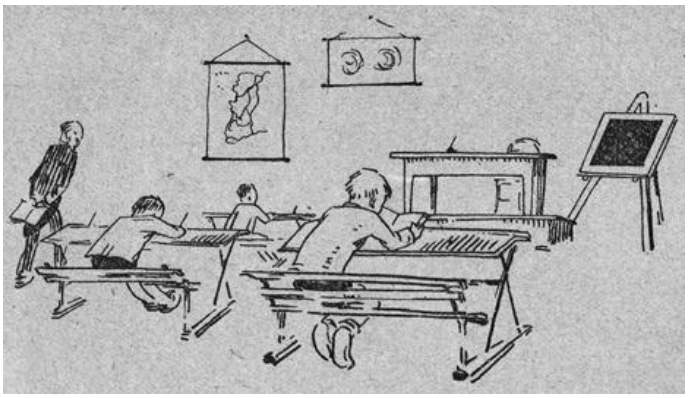
Source: LEITE, Restos de colecção.

This picture underlines the prevalence of women as primary teachers that occurs from the beginning of the twentieth century (NÓVOA, 1987) and that we can observe in previous images, in contrast with the male dominance in secondary schools that is maintained until the 1960s.

This structure of the class will be dominant until the 60's and prevails in contemporary schools. We also found a picture that constitutes perhaps the most schematic representation of this kind of class (Figure 17) from the late 1930s.

[10] The correct name of the school can be *Escola Normal Primária* (1919-1930) or *Escola do Magisterio Primário de Lisboa* (1930-1988), depending on the date the photograph was taken.

Figure 17 - Cover of *Caderno de problemas de aritmética*, *4ª classe*, late 1930s.



Source: Caderno de problemas de aritmética, 4ª classe.

In this picture the artist's trait reduces the elements to the essentials: students desks, teacher's platform and desk, books and notebooks, maps and a blackboard. The teacher is not in his central position and walks around the room. It is a book of exercises for the primary school and the editors chose to represent a specific didactic technology, students performing a written exercise under the teacher's observation. The author of the picture adopts a style representing with an energetic synthetic trace all the relevant elements in interaction with the actors: the students with the worksheet and the teacher watching over the class. Although the cover dates from the late 1930s, the image represents an older board, as we found at the beginning of the 20th century.

CLASSES IN PROFESSIONAL SCHOOLS

It will be in commercial and industrial education that training needs will dictate the introduction of very different classroom structures in the late nineteenth century, which we include as a means to present other kinds of learning environments. In business education, for example, which is designed to train small and large businessmen, as well as bookkeepers and senior accounting employees, their training included bookkeeping, general business accounting, and financial accounting. In private schools presenting innovative

training methods, which we would today call in-context training (RODRIGUES, 2014; RODRIGUES; MATOS, 2017) new classrooms will emerge.

We found images of some of these schools, for example the *Escola Prática de Comércio de Lisboa* founded in 1903 or the *Instituto Comercial Pereira de Sousa* founded in 1899[11]. For this article we chose an image of the *Escola Prática Comercial Raul Dória* located in Porto with photos published in *Ilustração Portugueza* I.st Series, no. 563 of 4/12/1916 (Figure 18).

Figure 18 - Commercial offices in *Escola Prática Comercial Raul Dória*.

Source: *Ilustração Portugueza*, 1.ª Série, n.º 563, 4/12/1916, p. 459.

In these schools, more theoretical teaching spaces coexist with others for the practical learning of the technical contents necessary for the exercise of the profession. In photography we observe the very realistic simulation of a working commercial space exposing students to various types of business practices.

[11] Images of these schools can be found at: https://restosdecoleccao.blogspot.com/search?q=escola+prática+de+comércio+de+lisboa. Accessed on: 1/10/2018.

THE INFLUENCE OF THE NEW SCHOOL MOVEMENT IN PRIMARY SCHOOL

Not all classrooms had the grid structure of students desks. Since the beginning of the twentieth century, what is commonly referred to as the New School movement has gained strength in Portugal, especially after the establishment of the Republic. Putting the student at the canter of education, the movement proposed the adoption of intuitive and discovery methodologies, close to the reality of children, the manipulation of materials and respect for their psychological development. This movement will have a special focus on teacher training schools (PINTASSILGO; MOGARRO; HENRIQUES, 2010) and some private primary schools, for example in *Jardins de Infância João de Deus*.

The *Jardins de Infância*, founded in Coimbra in 1911 by João de Deus Ramos (1878-1953), intended to use the method created by his father, the Portuguese poet and educator João de Deus (1830-1896). With regard to mathematics, early use was made of manipulative materials proposed by Maria Montessori and Friedrich W. Fröebel, which required the organization of the class in working groups with children sitting around a large table (Figure 19).

Figure 19 - On the left, children in a large room playing with Fröebel's gifts, 1938. On the right, children working with manipulatives, 1930s.

Source: Iconographic Archive of *Museu João de Deus* - material collected by Joseane Arruda.

These photographs were not collected in advertising materials, so this

time they are unlikely to be staged and represent actual lessons. Components include the desks, benches, and materials students work on. Schoolwork is performed around a table with the children exploring (individually?) the material. Teachers are not the center and their role is to support students' play.

THE INFLUENCE OF THE NEW SCHOOL MOVEMENT IN LICEUS

Innovative didactic approaches influenced by the New School were not limited to primary education and have had great expression, at least at the level of discourse, in secondary teacher education institutions since the start of the *Escolas Normais Superiores* in 1915 (MATOS, 2014). We will find them again in the mandatory reports that temporary teachers sent annually to the Ministry of Education (MATOS and FISCHER, 2010) which contain constant references to the "heuristic method". For example, Joaquim Manuel Preguiça, who writes the report on his work at the Liceu Passos Manuel in 1960, describes his first year of the 1st cycle:

> The teaching method used was, whenever possible, an experimental active method, in which children were sought to learn through classroom experiences.
>
> The class often took on the look of a laboratory in which children performed drawings, cut-outs, constructions, measurements and weighings and thus learned the first mathematical notions through the use of intuition and material objects (mathematical models).
>
> In the study of (experimental) geometry, mathematical models were used - devices or material objects capable of translating or suggesting mathematical ideas constructed by the students themselves. According to the most up-to-date theories of psychologists such as Piaget, it is intended to provide mental representations of models for use in the abstraction of mathematical ideas and to avoid verbalism without any concrete reality, while at the same time providing for the action of the students. and develop initiative by giving them freedom of choice in material and construction (PREGUIÇA, in MATOS and FISCHER, 2010, p. 89, authors' translation).

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And Maria Eduarda Sousa describes her work at Liceu of Faro in 1959/60:

> As a rule, I always followed each definition, each rule, each theory, of numerous examples exercises, so that the knowledge that the students were acquiring could be better defined and focused.
>
> In high school classes, I tried to use the heuristic method, where possible, preferably, as it can be applied to forty-something students (SOUSA, in MATOS and FISCHER, 2010, p. 89, author's translation).

As for the 3rd cycle, currently 10th and 11th years, her perspective is different:

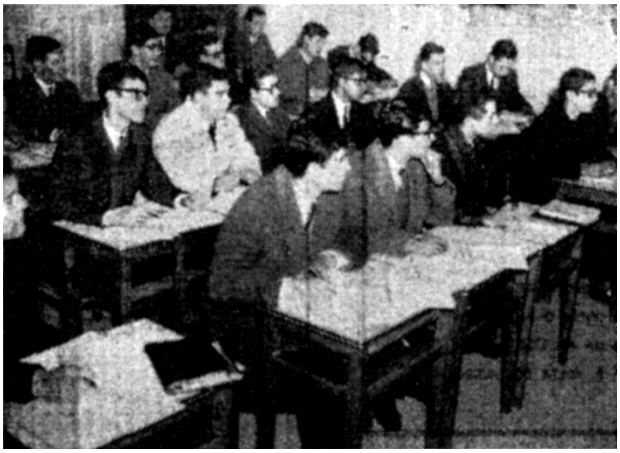
> In the 3rd cycle classes, where I taught Math in the 6th grade and Drawing in the 7th, I adopted different methods depending on the subject in question. Thus, in Mathematics, the classes were based on the expository method, which is compatible not only with the number of students in the class [we know they were over 40] but especially with the extension of the program. Moreover, I believe that the logical rigor to which deductive reasoning requires is highly fruitful when it is intended to lead the student to a higher view of known problems, which was largely the case in this subject's syllabus (SOUSA, in MATOS and FISCHER, 2010, p. 90, authors' translation).

This view contrasts with those of the official high school programs of 1931, which will continue until 1947 and which value memorization and repetition:

> The solving of numerous exercises of mental and written calculus is therefore the basis of this teaching, - exercises solved in class under the direction of the teacher, serving as preparation and presentation of the subject to study and home-made exercises recalling the work of the class and The student will present them in his notebook, submitting them to the teacher's corrections and notes (PORTUGAL, 1931, p. 2187).

We must wait for the experience of Modern Mathematics in the early 1960s to see the emergence in official secondary education of experiences with different kinds of classes influenced by the early-twentieth New School movement. This reform is reported with pomp and circumstance in a series of four articles in the *Diário Popular* and in the March 8, 1963, and it is announced, as a factor of modernity, that the desks have been replaced by working tables, now allowing for team work and laboratory methodologies. The news is accompanied by a photograph that presumably was taken in one of the experimental classes of *Liceu Pedro Nunes* (Figure 20).

Figure 20 - Experimental class of Modern Mathematics in *Liceu Pedro Nunes*.



Source: FONSECA, 1963.

From reports written by future teachers that participated in these experiences we also know that the platform for the teacher's desk was removed (SERROTE, 1966).

Other forms of organization were adopted at this time. For example, in the Mathematics Room of the *Liceu D. Manuel II* in Oporto small group organization facilitated the use of the various teaching materials (Figure 21).

Figure 21 - Students working at the Mathematics Room of *Liceu D. Manuel II*, Porto.

Source: Personal archive of António Augusto Lopes.

In the following years, especially from the pedagogical innovations introduced from 1968 with the Preparatory Cycle of Secondary Education, this "deconstruction" of the class will deepen. Figure 22 shows Natália Vaz, then a in-service training teacher, supporting a group of first year students (currently fifth grade) engaged in a task related to number systems a class at *Escola Preparatória Eugénio dos Santos* in Lisbon.

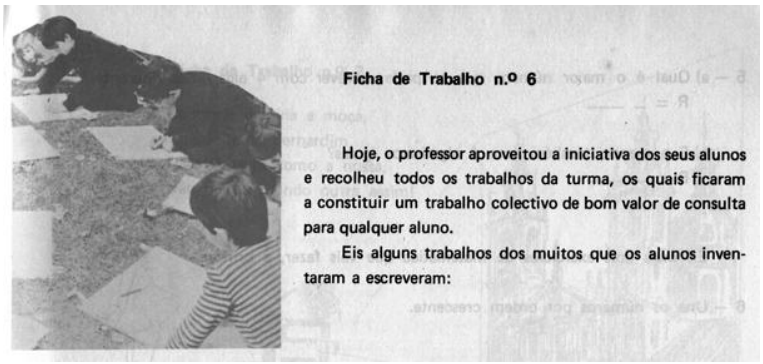
Figure 22 - Class with students working in groups in 1972.

Source: Vaz (1972, p. 100).

Lastly we found another image of a class of a different type (Figure 23)

which appears, among many others, in the book Mathematics and Me, 2nd Phase, 1st Year by Belarmina Lopes and Maria Jorge Costa, apparently committed to encouraging the use of methodologies. which was published in 1976 when, after the 1974 revolution, everything seemed possible.

Figure 23 - Other kinds of in-class work.



Source: Lopes e Costa (1976, p. 11).

CONCLUDING

Similar to the German hermeneutic method of textual analysis, the iconographic method has been criticized for being too intuitive and speculative (BURKE, 2001). However, the iconographic analysis of the representations found in textbooks, photographs or pictures intertwined with the social, economic and cultural context of the time found in teacher, student and legislative documents allowed us to identify mathematical practices in class (in the sense of JULIA, 1995) framed between the medieval period and the mid-twentieth century. Through this iconographic portrait it was possible to recognize an evolution of the relationships established between teachers and students, the materials used and the organization of the classroom space, which, over the centuries, has transformed the formal rigidity of spatial organization. New educational equipment, especially the blackboard, will gradually transform the structure of the class, not only from an architectural point of view, but more

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importantly from a symbolic point of view. The framework assumes special importance for mathematics teaching and learning as it gains centrality and becomes a space for sharing knowledge. New learning dynamics come with new ways to explore materials and organize space, such as hands-on classes for vocational training, the use of manipulative materials or the organization of students in small groups. Many of the teaching practices we have found throughout the ages have coexisted with each other and continue to the present.

The iconographic analysis linking the components of each image with other studies and their iconological interpretation that allowed the establishment of relationships with our knowledge of the historical period, allowed to give a more global meaning to each artefact we commented here.

We hope to have brought the reader to a view of history (particularly that of mathematics education) as a search for stability and changes that allow us to reflect on the present. The student, the teacher and the content as abstract categories are permanent, but everything else changes: the relationships established in the classroom, the valued and the repressed, the artefacts and their meaning. But it changes, beyond all, the concrete social and cultural identity of the students, the teachers (note again the images of students and teachers) and mathematics itself (which has incorporated different views of what is legitimate and desirable mathematical knowledge by eliminating simultaneously other dimensions)[12].

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[12] Aknowledgement. This work would not have been possible without the collaboration of Mária and Tó Zé Almeida who provided access to some books in their archive. Teresa Monteiro and Joseane Arruda collected some of the photographs used. A final mention must be made to Helena Martinho who in 2011 suggested the exploration of the topic.

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