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ARTICLE

Mathematical knowledge and overall practice in initial teacher education of early childhood teachers

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ABSTRACT

This paper presents an instrumental multiple-case study composed of two representative cases selected from a Mathematics Initial Teacher Education course. Data collected include concept maps, classroom observations, and interviews, which were analysed quantitatively and qualitatively. Differences in knowledge and practices were compared between pre-service early childhood teachers: one with greater content and pedagogical knowledge, and appropriate teaching practices; and another, with insufficient content and pedagogical knowledge, and repetitive mathematics activities. Findings from these two cases indicate a relationship among content, pedagogical knowledge, and good practice; and thus, support the existence of a dialogical and integrative relationship between knowledge and practice in teaching. Accordingly, the teacher practice dimension from the Mathematics Teaching Capability framework should play an important role in the initial teacher education of early childhood teachers.

KEYWORDS

mathematics; early childhood education; teaching practice; initial teacher education.

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CONHECIMENTO MATEMÁTICO E PRÁTICA EM GERAL NA FORMAÇÃO INICIAL DE PROFESSORAS DA EDUCAÇÃO INFANTIL

RESUMO

O artigo apresenta um estudo de caso múltiplo instrumental, composto por dois casos representativos selecionados de um curso de formação docente em Matemática. Os dados coletados incluem mapas conceituais, observação de aulas e entrevistas, submetidos às análises quantitativas e qualitativas. Diferenças de conhecimento e prática foram comparadas entre as professoras em formação em Educação Infantil: uma com alto conhecimento do conteúdo, pedagógico e práticas apropriadas; e a outra, com baixo conhecimento do conteúdo, pedagógico e atividades matemáticas repetitivas. Os resultados desses casos indicam uma relação entre conhecimentos do conteúdo, do pedagógico e de boas práticas, portanto, apoiam a existência de uma relação dialógica e integrada entre conhecimentos e práticas. Nesse sentido, a prática docente a partir da Capacidade de Ensinar Matemática deve desempenhar um papel importante na formação inicial de professoras da educação infantil.

PALAVRAS-CHAVE

matemática; educação infantil; prática de ensino; formação inicial de professores.

CONOCIMIENTO MATEMATICO Y PRACTICA COMO UN TODO EN LA FORMACION INICIAL DE LAS MAESTRAS DE EDUCACION INFANTIL

RESUMEN

El artículo presenta un caso múltiple instrumental, compuesto por dos casos representativos seleccionados desde un curso de formación inicial docente en matemática. Los datos recopilados incluyen mapas conceptuales, observación de clases y entrevistas, sometidos a análisis cuantitativos y cualitativos. Diferencias en conocimiento y práctica fueron comparadas entre las maestras de infantil en formación, una de alto conocimiento del contenido, pedagógico y prácticas apropiadas; y la otra, con insuficiente conocimiento del contenido, pedagógico y actividades matemáticas repetitivas. Los resultados de los casos indican una relación entre el conocimiento del contenido, pedagógico y buenas prácticas, por lo tanto, apoyan la existencia de una relación dialógica e integrada entre conocimiento y práctica. En consecuencia, la dimensión de práctica docente desde el marco de la Capacidad de Enseñanza de la Matemática debería jugar un rol importante en la formación inicial de las maestras de infantil.

PALABRAS CLAVE

matemática; educación infantil; práctica docente; formación inicial docente.

INTRODUCTION

The objectives of initial teacher education research seek to determine what knowledge is needed for prospective teachers to learn to teach, and how this knowledge can be turned into teaching practices that benefit student learning (Hammerness *et al.*, 2005; Leavy and Hourigan 2018). These points are especially relevant for Mathematics Initial Teacher Education (MITE), a topic garnering increasing global awareness (Parks and Wager, 2015).

Indeed, international MITE research has reported that prospective preschool teachers lack sufficient knowledge for teaching mathematics. For example, Esen, Özgeldi and Haser (2012) found that future preschool teachers in Turkey were insufficiently educated regarding mathematical concepts, which results in poorly implemented learning activities for children. Similarly, Samuel Sánchez, Vanegas Muñoz and Giménez Rodríguez (2015) reported that pre-service early childhood teachers in Spain presented difficulties in understanding the mathematical thinking of children approaching problem-solving activities.

Existing research in Chile — where the present study was performed — suggests that there are deficiencies in teacher education for future early childhood teachers, especially regarding mathematics (Goldrine Godoy *et al.*, 2015a; Goldrine Godoy *et al.*, 2015b). These MITE studies indicate that existing educational methods do not positively affect the education of prospective early childhood teachers, and indeed, that these teachers are likely certified without having the proper knowledge and practices needed to promote mathematics in early childhood education.

These issues are not limited to Chile only. Parks and Wager (2015) highlighted that courses do not necessarily include content, teaching techniques, or practices that would be appropriate for teaching mathematics in early childhood education. Unfortunately, minimal evidence is available for determining the skills and content that future educators need to learn for teaching mathematics.

As such, recent investigation has stressed the urgent need for research on suitable course contents and methods in MITE. Horm, Hyson and Winton (2013) reported a lack of consistency in MITE for future early childhood teachers, notably concerning research on contents and training methodologies. Platas (2008) proposed that future research should focus on describing the integration between knowledge and classroom practices. All of that indicates there is a need to study the influence of content and pedagogical knowledge on the practices of teachers. Thus, the goal of this study is to describe the relationship between knowledge and practice of pre-service teachers in mathematics initial teacher education.

KNOWLEDGE FOR MATHEMATICS TEACHING AMONG PRE-SERVICE EARLY CHILDHOOD TEACHERS

The bibliographic review consulted is based on Shulman's model (1987). Subsequent research followed this model regarding early mathematics teaching.

For Shulman (1987), the knowledge bases needed for teaching are held within the following categories: content knowledge; general pedagogical knowledge; curriculum knowledge; pedagogical content knowledge; knowledge of learners and their characteristics; knowledge of educational contexts; and knowledge of educational ends, purposes, and values. This model defines a dichotomy in teachers characterised by disciplinary subject matter knowledge and knowledge about teaching disciplinary domains of matter knowledge, and so some studies have applied Shulman's model (1987) to conceptualisations of knowledge and teaching regarding the abilities of early childhood teachers in teaching mathematics.

The model further presents a three-component construct of knowledge for teaching: content knowledge (CK), pedagogical knowledge (PK), and pedagogical content knowledge (PCK). This model has been heavily adapted to studies on early childhood teachers in mathematics instruction (Lee and Ginsburg, 2007; McCray, 2008; Platas, 2008; Lee, 2010; McCray and Chen, 2012; Oppermann, Anders and Hachfeld, 2016; Lee, 2017; Leavy and Hourigan, 2018).

According to McCray (2008), PCK of early childhood teachers consists of CK, PK, and knowledge of thought processes in children. This proposed understanding of PCK enables teachers to determine what mathematical contents are age appropriate and, consequently, which teaching strategies apply to each respective age group in terms of content and cognitive development. For Lee and Ginsburg (2007), PCK involves identifying mathematics in children's activities or play, interpreting such mathematical situations, and enhancing the mathematical thinking of children. McCray (2008) further maintains that PCK for early childhood teachers greatly impacts childhood learning.

For Lee (2010), PCK ensures quality in mathematics initial teacher education. Nevertheless, while Lee (2010) found that numeric sense is the most preponderant factor for PCK in early childhood teachers, Platas (2008) placed emphasis on numbers and operations given the relevance of these factors in early childhood education. Both studies agree that teachers with a wider range of experience and specific training on the mathematical development of children tend to have more advanced knowledge about teaching.

Next, the study of Leavy and Hourigan (2018) highlighted the importance of specialised content knowledge, describing the types of mathematical content knowledge held by prospective primary school teachers. That study indicated that teachers' existing knowledge of various number concepts was not sufficiently developed to support classroom teaching or offer learning opportunities to children. Also, their knowledge was inadequate to predict potential responses by children, which is only acquired through actual practice.

Since greater importance should be given to children's development, teachers should be equipped with the tools and confidence needed to guide children towards enriching mathematical experiences. Isikoglu (2008) reported that courses for future early childhood teachers should integrate content and teaching techniques linked to a constructivist method.

In this sense, Ormeño Hofer, Rodriguez Osiac and Bustos Barahon (2013) showed that Chilean early childhood teachers have insufficient knowledge of the teaching methodologies and skills needed for their respective grade levels. Previous work by our group using tests indicated that there is a relationship between choosing good practices and the knowledge of those practices (Goldrine Godoy *et al.*, 2015b). This paper then seeks to identify the influence of content and pedagogical knowledge on the extremely different practices of two early childhood teachers.

MATHEMATICS TEACHING CAPABILITY AMONG PRE-SERVICE EARLY CHILDHOOD TEACHERS

Considering the above, we propose a Mathematics Teaching Capability construct for pre-service early childhood teachers. Chart 1 shows each component in the first column, and the dimensions of teacher knowledge and teacher practice in creating learning opportunities for children in the second and third columns, respectively. This construct is comprised of the following three components of teacher knowledge:

1. content knowledge (CK);
2. pedagogical content knowledge (PCK), which includes PCK for teaching (PCK-T); and
3. pedagogical content knowledge on mathematics thinking during early childhood (PCK-MTEC).

Chart 1 – *Mathematics Teaching Capability* in pre-service early childhood teachers.

Component		Dimensions	
		Teacher Knowledge	Teacher Practice
CK		Appropriate mathematical concepts and semiotic representations for Early Childhood Education.	Teaching mathematical notions appropriate to Early Childhood Education. Mathematical objects and representations.
PCK	PCK-T	Focuses on mathematics teaching. Sequence of mathematical tasks. Manipulative and representative materials.	Organisation of teaching. Creation and implementation of learning opportunities. Analysis of manipulative and representative materials.
	PCK-MTEC	Teacher knowledge of mathematical thinking in children. Stages of learning mathematical notions. Knowledge of children's actions, strategies, and difficulties.	Teacher mediation of children's actions, strategies, and difficulties.

CK: content knowledge; PCK: pedagogical content knowledge; PCK-T: pedagogical content knowledge for teaching; PCK-MTEC: pedagogical content knowledge on mathematics thinking during early childhood.

These components can then be expressed through dimensions that entail conceptual knowledge (i.e., teacher knowledge) and teacher practice to create learning opportunities for children (i.e., teacher practice). These components and dimensions dialectically interact to form teacher knowledge and practices, which dialectically amalgamate into our proposed construct, Mathematics Teaching Capability.

MITE should have a positive impact on the Mathematics Teaching Capability of pre-service teachers, allowing them to create learning opportunities for children. These opportunities arise when teachers ensure a classroom is a place for enriched, active mathematics learning, providing children with access to:

- play-based mathematics activities;
- hands-on activities;

- interactive materials and representations;
- problem-solving; and
- an activation of mathematical thinking abilities, among others (Cohrssen and Tayler, 2016).

Mathematics education that provides these conditions gives children access to a high-quality mathematics curriculum (Samara and Clements, 2009). Accordingly, pre-service early childhood teachers must not only be knowledgeable, but also be aware of the most appropriate ways of creating mathematics learning opportunities for young children (Thornton, Crim and Hawkins, 2009).

THE PRESENT STUDY

This paper presents a case study of two pre-service teachers. The participants attended a course with contents that included logical concepts, number sense, play-based mathematics activities, ludic situations that align with problem-solving skills, use of manipulative materials and representations, and the role of teachers in stimulating mathematical thinking in early childhood.

Teaching techniques included lesson plans, case studies, and video analyses of early childhood mathematics teaching. The course consisted of weekly 90-minute sessions over a period of 15 weeks, as part of an Early Childhood Teaching Degree Program at a Chilean university. In parallel, the participants were active trainee teachers at a preschool establishment for children aged 5 to 6. Video recordings were taken of the trainee teachers teaching number sense to children.

At the start and end of the course, the participants were given a "*Knowledge Test on Number Teaching for Future Early Childhood Teachers*" (Goldrine Godoy *et al*., 2015a, see Appendix A). Test scores were used to select the two representative cases from among the total group of participants, with further analysis of these cases being the topic of the present research report (Flyvbjerg, 2006). The selected participants were Helen[1], a trainee teacher with a high final test score, and Lauren, a trainee teacher with a low final test score.

The research questions addressed by this study were as follows:

- Do Helen and Lauren differ in their Mathematics Teaching Capability?
- Are knowledge and practices related to each other?

RESEARCH DESIGN

The case studies reported here made use of different information collection tools, for example, videos, maps, a test, and interviews (Stake, 2005, p. 445). Two representative cases were selected (i.e., Helen and Lauren) for further in-depth inquiry into understanding learning in mathematics initial teacher education. Case comparisons can be a grand epistemological strategy, representing a powerful conceptual mechanism for analyses and comparisons (Stake, 2005; Creswell, 2013).

1 Names have been changed to maintain participant anonymity.

Data were collected from several information sources using quantitative and qualitative instruments. Data were triangulated for cross-case analysis (Creswell, 2013) to build patterns, facilitate interpretations, and provide tentative explanations about the learning of pre-service teachers in MITE.

CASE STUDY PARTICIPANTS

Helen and Lauren were selected from a class of 39 students who participated in a Mathematics Initial Teacher Education course given during the fourth year of studies in an Early Childhood Teaching Degree Program at a Chilean university. The case studies selected are consistently referred to herein as Helen (H), who represents the highest test score on the *Knowledge Test on Number Teaching for Future Early Childhood Teachers* (Goldrine Godoy *et al.*, 2015a); and Lauren (L), representing the lowest test score on the *Knowledge Test on Number Teaching for Future Early Childhood Teachers* (see Appendix A for examples). Each pre-service teacher was required to provide signed informed consent before participating in this study. Teaching practice centres and student tutors authorised some lessons to be video recorded. Data is used for research purposes only.

DATA COLLECTION

The following were used to collect data on Helen and Lauren teaching outcomes:

- Concept Map: This instrument required participants to provide responses on what to teach (CK); how to teach (PCK-T) and at what age (PCK-MTEC); and which materials to use when teaching number sense to children (CK; PCK-T). Concept maps were scored based on an assessment rubric that considered the presence of the aforementioned components, with the maximum score being 18 points (i.e., 100%; see Appendix B). Two researchers reviewed the maps separately. Inter-assessor reliability was 75%, and final scores were assigned after discussion and until an agreement was reached;
- Classroom Observation Form: Two researchers separately reviewed videos of each trainee teacher implementing activities in a preschool classroom and identified indicators. These indicators were related to the use of mathematical notions and representations of number concepts (CK); the problem-solving steps and materials used (PCK-T); and teacher reactions to children's actions (PCK-MTEC). The maximum obtainable score was ten points (i.e., 100%; see Appendix C). Two researchers separately reported observations on trainee teachers. Inter-assessor reliability was 75%, and final scores were assigned after discussion until an agreement was reached;
- Interviews on Teaching Practices: Interview protocols included questions regarding the experience of planning and implementing the teaching of numbers (CK; PCK-T and PCK-MTEC). Interview transcripts were interpretatively analysed to establish categories related to the knowledge reported by trainee teachers.

FINDINGS

Chart 2 shows the achievement rates of Helen and Lauren on the Knowledge Test, for the Concept Map, and regarding Classroom Observation.

Chart 2 – Achievement rates of trainee teachers with a high score (Helen) and a low score (Lauren) established using the three research instruments.

Trainee teacher	Knowledge Test (%)	Concept Map (%)	Classroom Observation Form (%)
Helen	93	77	92
Lauren	43	30	36

CONCEPT MAP

Using rubric-based scoring and an 18-point scale, Helen's concept map scored 15 points (Figure 1).

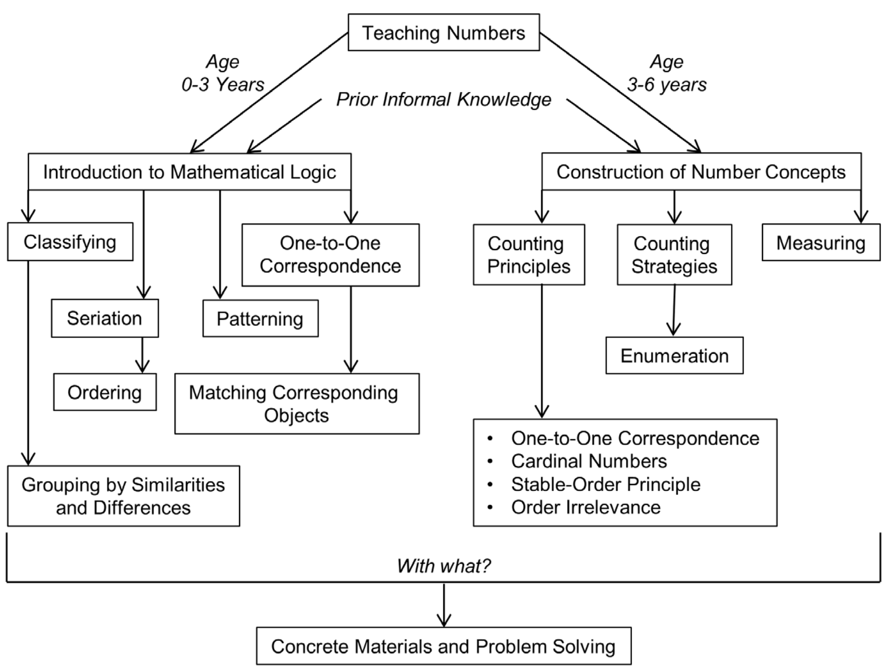

Figure 1 – The concept Map constructed by Helen.

Helen structured a map that groups ideas and shows relevant connections between main concepts. These main themes are characterised by the concepts of logic and numbers for early childhood teaching. Below these main themes are some subordinate mathematical and didactic concepts that evidenced CK and PCK.

The Concept Map constructed by Helen (Figure 1) distinguished between the construction of number concepts and an introduction to mathematical logic in direct relation to a child's stage of development, which demonstrates PCK-MTEC. In each mind map division, she mentioned the mathematical notions associated with problem-solving skills and the corresponding teaching methodology. She also showed strongly coherent CK and PCK-T throughout the duration of the course. According to the rubric and evaluation criteria, she used the words "number" and "logic" as major concepts.

Helen coherently used classifying, patterning, seriation, and one-to-one correspondence in association with the concept of initiation to mathematical logic. Subordinated to number concepts were notions associated with counting principles, such as cardinality, the stable order principle, and order irrelevance, as well as with counting strategies, which mentioned enumeration. Helen grouped concepts and showed relationships with arrows. Overall, the constructed Concept Map was fully organised and easy to interpret.

By contrast, Lauren's concept map (Figure 2) shows a linear structure and incorporates few elements related to specific number sense teaching. The map

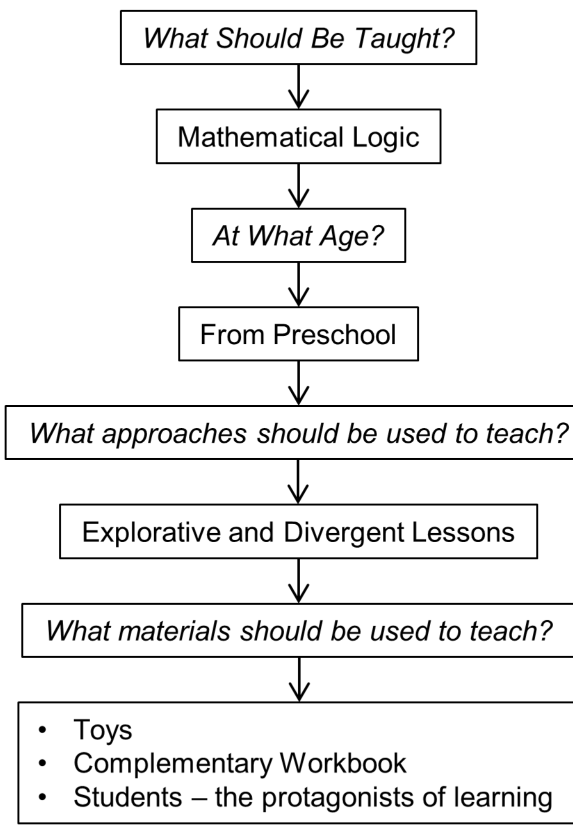

Figure 2 – The concept Map constructed by Lauren.

does not address the specific number concepts to be taught, showing a low degree of PCK-MTEC. Additionally, Lauren shows pedagogical alignments that apply generally to early childhood teaching, such as explorative and divergent activities, without making any connections to mathematical notions or teaching strategies characteristic of mathematical CK and PCK. On the concept of PCK-T, she mentioned using a complementary workbook, thus showing a preference for school methodology rather than for developing ludic situations that would nurture problem-solving skills.

CLASSROOM OBSERVATION FORM

In the mathematics teaching course, trainee teachers were given the opportunity to promote activities on number sense to children aged 5-6 in a preschool classroom. This activity was video recorded for subsequent analysis. The transcripts for activities conducted in-classroom, particularly those of main events, are shown in Chart 3 (Helen) and Chart 4 (Lauren).

During her teaching, Helen presented a problem in which the central question was about the amount of chickpeas needed to prepare lunch. This problem was connected to a previous lesson, thus showing the application of PCK-T in practice.

Chart 3 – Activity implemented by Helen (H).

Activity: How many chickpeas do we need for lunch?
Event: Presentation of the problem
H: Children, do you remember the activity we did yesterday? What did we have to do? Student 1 (S1): Count dots and get the same number of chickpeas. H: Let me tell you, my mom wants to cook a delicious lunch […] made with chickpeas, but she needs an exact quantity [of chickpeas]. […]. There are bags on each chair, so you will have to look to see how many chickpeas are missing until my mom has the exact number she wants – some bags already have some chickpeas in them. How many chickpeas do we need to get the desired quantity? [Children worked in pairs. The chickpeas were on another table, and the cards presented the numbers 13, 14, and 15.]
Event: Chickpea counting and calculation of chickpeas needed to reach the required quantity
H: How many chickpeas are there? [To a couple of children]. S2: Seven. H: How many do we need in the bag in total? N2: Thirteen. H: If there are seven, how many more do you need? S2: Thirteen. H: Are you sure you need thirteen? […] Let us count again. How many do we have? S3: One, two, three, four, five, six, seven. H: If we have seven, how many more do we need to get up to thirteen chickpeas? Let us keep on counting from seven. S3: Eight, nine, ten, eleven, twelve, thirteen. H: So, how many more do we need to get there? [She raised a finger for each number said by S3]. S3: [No reply.] H: How many do I have in this hand? Count them for me. S3: One, two, three, four, five, six. H: Good. Go and get six chickpeas, and then we will see if this gets us there. We will wait for the others to finish, and then you can share your strategy with the rest. How did you do it? [i.e., solve the problem]?

Chart 4 – Activity implemented by Lauren.

Activity: Build a collection of objects according to a given number series
Event: Activity setup
L: We are going to engage in a math activity on numbers […]. In this bag, there are many numbers. For instance, we can make the number ten. Put your hand into the bag and grab a number […]. We are going to put [the number] on the table […]. This bag contains stuffed animals, and this other bag contains Legos and other figures […]. How many items [i.e., stuffed animals, Legos, etc.] do you have to select according to this number? [Children are sitting at their desk and one child stands up to perform the activity.]
Event: Activity implementation: take a number and build a collection
L: Alexis, come forward […] and take a number. S1: (Selects the number 1 and shows it to Lauren) L: […] but this number needs company, so take another number. S1: (Selects the number 6.) L: (Inverts the number.) This needs to be turned around so that it shows the number 6, because 9 has already been selected. How many items do we need to put on the table, Alexis?
Event: Ending the activity
L: To end the activity, we are going to review all of the numbers we saw […]. I will show you the numbers, and you will have to tell me which number it is that I am showing. L: […] And after 9, which number comes next? Class: Ten. L: Ten […]. Which numbers make up ten? Class: One and zero. L: One and zero, very good. […]. Let us count, but I want it to be really loud (points to a number sequence hanging from the ceiling.) Class: One, two, three, […] ten.

When presenting the problem, Helen used phrases such as the "exact number" and "the total number of chickpeas," demonstrating CK in practice. This teacher also asked how many chickpeas the children still needed to arrive at the total quantity. This strategy established a comparative connection between quantities and concrete counting to complete a problem-solving activity, thereby relating PCK-T to CK in the practice dimension of Mathematics Teaching Capability.

As seen in the corresponding transcript (see Chart 3), Helen encouraged the children to reason by asking questions, including "How many [chickpeas] do you need?" or, more specifically, "If we have seven [chickpeas], how many more do we need to get up to thirteen?" This encouragement through progressive questioning demonstrates PCK-MTEC. Moreover, she motivated the children to solve the problem by using counting and comparison strategies, which evidences PCK-T and PCK-MTEC in practice.

Helen also used materials with iconic (i.e., dots) and symbolic (i.e., numbers) representation, in addition to regulating the number intervals between 13 and 15. Both instances once again show the use of CK and PCK-T in practice. The children were further invited by Helen to share their problem-solving strategies so as to consolidate their learning effect, evidencing PCK-MTEC. Considering the various implemented practices, Helen was given a 92% approval on the Classroom

Observation Form, showing a highly positive development of her Mathematics Teaching Capability.

Lauren carried out an activity regarding number recognition and associating numbers with quantities (see Chart 4), showpresenting flashcards with the digits ranging from 0 to 9. Children were asked to arrange these flashcards to make two-digit numbers ranging from 0 to 98. This activity showcases CK, privileging symbolic representations unrelated to counting. In practice, Lauren preferred symbolic representations and did not use iconic representations, which would involve greater cognitive abstraction. This preference towards symbolic representation did not consider PCK-MTEC and its respective relationship with CK. Since the activity included numbers beyond those required for the educational level, Lauren showed weakness in relation to CK, PCK-T, and PCK-MTEC. While the activity was not based on problem-solving, it did support number quantity associations.

Additionally, the activity proposed by Lauren did not involve any counting strategies that would allow children to assign meaning to the number concept. As seen in the corresponding transcript (Chart 4), Lauren addressed the unexpected confusion between the numbers 6 and 9 by directly resolving the issue, which is indicative of weak CK. Her approach was to identify numbers without connection to the represented quantity, thereby evidencing insufficient CK and PCK-T in practice. In the final event during the practice period, Lauren committed a conceptual error associated with poor CK. Specifically, she taught that the number 10 was composed of two numbers (i.e., 1 and 0), which ignores suggestions for early childhood education that 10 should be taught as a collection of ten elements or as a decomposition (e.g., 6 + 4). This content was covered in the Mathematics Initial Teacher Education course. The knowledge demonstrate practiced by Lauren resulted in the 36% approval assigned on the Classroom Observation Form.

INTERVIEWS

Interviews were conducted with each participant to deepen their knowledge with regard to teaching mathematics in Early Childhood Education. The interviews were performed with the participants looking at a video recorder, and questions were related to their mathematics teaching experiences in a preschool classroom (see Chart 3 and Chart 4 for the activities implemented to promote number learning opportunities by pre-service teachers Helen and Lauren, respectively).

The interview questions are the same in both cases, and both interviews are analysed under the Mathematics Teaching Capability construct. See Chart 5 below which shows extracts from Helen's interview.

In responding to question 1) in Chart 5, Helen stated problem-solving, increasing the number range, representations of quantity, and problem-solving strategies, thereby evidencing CK and PCK-T. In the second question, Helen says that the proposed activity is challenging, and that it allowed the children to do something different; in this case, Helen means that the challenging aspects were counting and adding as central elements of the activity, thereby showing evidence of content knowledge (CK).

Chart 5 – Extracts from Helen interview.

Question	Answer
1) Why did you select this activity for the children?	I had already done this activity [with this group of students] when we were asked to conduct a problem-solving activity, but [I had previously] set the numeric sequence too low […]. So, I said […], what would happen if I did it again but with a higher numeric sequence? […] [The students] did not pay much attention to the numbers [they did not understand] and paid much more attention to the configuration of the dots. […]. [To make the students] establish the relationship, I tried to use a strategy to get to the numbers […], it was challenging.
2) Do you think this was a problem-solving situation?	It was challenging […] because their background knowledge did not seem to allow them to add and count the way they have always been doing it.
3) Did you include elements not incorporated in your initial lesson plan?	Problem-solving […] is meaningful if [the students] ask appropriate questions and the teacher does not give away the answer. [With] learning [through] logical mathematics, [teachers] can start working [with students] from a very young age, maybe not with numbers or quantities, but by classifying, ordering, […]. Later you can introduce the concept of numbers little by little.
4) What is the relationship between the implemented activity and number learning?	[…] if the children just repeat a number, it means nothing […], I said seventeen and of course the number seventeen was the one that came after sixteen, which itself followed fifteen, so they repeated the numbers as they had done previously in the oral series. However, with the proposed dots configuration, I think they [the children] were starting to build an association between the quantities and the number-words.
5) What advantage does the work proposed in your activity have for children?	I think that it helps reciprocally, to use a different strategy, […] or the children think of a similar strategy, they go on trying until they achieve it […]. It develops children's logical thinking and language, since they explain what they are doing.
6) How do you think children learn math?	[…] There are many stages children have to go through. These stages must be developed little by little so that they can reach mathematics as we know it […]. Since we already know the principles of counting, it is obvious to us, but for children, it is not so obvious. These stages are very marked [mathematically], so you have to go on proposing challenging activities so that they can develop their learning little by little.

For the third question (Chart 5), Helen reported that problem-solving is something new, that is, not previously included in her lesson plan. Then she adds that this problem-solving is significant when there is minimal teacher intervention. The above shows deep reasoning in PCK-T, since, in Chile, it is often not believed during teacher education that teachers should mediate less to generate a significant learning effect. In the second part of the third question, we observed that Helen introduced a structure, of which she notes that children must first be able to classify and order to engage in initial mathematics. With this, we can say that Helen displays adequate CK. In her last sentence, we can assume that she applies her PCK-MTEC when she indicates that this process should be done slowly in consideration of the children.

Regarding question four (Chart 5), Helen comments that children remember the number as part of a sequence, but not focusing on the number as a cardinal.

This observation involves CK and PCK-MTEC. Helen believes that the number as a cardinal should be treated at this level and that apparently children do not have the concept of quantity.

For the fifth question (Chart 5), Helen talks about her activity with the children and maintains that the children are involved in this activity since they are asked to apply logical thinking and justify their argumentation. She refers to the questions that she asked during the activity, but not to the answers that the children actually gave.

Regarding question six (Chart 5), we observe that Helen draws on the theory learned in her education, which is related to PCK-MTEC. When she says that children learn little by little, in stages, with challenges according to a development scheme, Helen lists the points that should be addressed at the time of offering initial mathematics learning opportunities.

From the answers provided by Helen, we can infer that she applies a constructivist approach to teaching. This approach is demonstrated when she emphasises that children should solve problems with their own strategies. This is indicative of an evolution in Mathematics Teaching Capability.

Next, Chart 6 shows extracts from Lauren's interview. The questions asked Lauren were the same as the ones asked Helen.

Regarding question one in Chart 6, Lauren mentioned dynamic activities to get the children's attention, showing a rather minimal level of PCK-T. This answer is

Chart 6 – Extracts from Lauren's interview.

Question	Answer
1) Why did you select this activity for the children?	In the school system, mathematics are not applied dynamically; lessons are way too structured […]. I felt like I had to do something more dynamic, something that would get the children's attention easily […]. I feel they liked [the activity] and that they had a good time […].
2) Do you think this was a problem-solving situation?	A problematic situation was presented, and they faced it without difficulty.
3) Did you include elements not incorporated in your initial lesson plan?	The children were always attentive, and I liked that because I felt that it was authentic learning for them and that it was not something they were accustomed to, as they did not use their workbooks.
4) What is the relationship between the implemented activity and the number learning?	I believe that everything is less structured and that they can use mathematics in a more didactic way, and this is how we should work in math.
5) What advantage does the work proposed in your activity have for children?	To understand that a number is not just a number, but that it has a purpose. I can draw the number symbol, but at the same time, I can identify the quantity to which that number corresponds.
6) How do you think children learn math?	I believe that in mathematics, we need an illustrative system for children. For us, we did not like mathematics in school. Math was something so structured and always the same, there was a formula to solve the exercise. On the other hand, with a more illustrative activity that provides real interest, children may like math in the future.

not directly related to mathematics. Compared to Helen in the same question, Lauren does not mention key mathematical elements of her activity, for example, the support materials for counting (see Chart 6). In the second question, although Lauren stated that a problem-solving activity was used, the activity was actually repetitive in nature.

When responding to the question of whether additional elements not included in original lesson plans had been incorporated (question three), Lauren insisted on distractive motivational elements when assessing the chosen activity. Lauren described a teaching approach that should be attractive and fun, but without integrating ludic teaching strategies referencing mathematical notions applicable to Early Childhood Education. The activity led by Lauren was repetitive, which is associated with more empirical methods of teaching mathematics and, consequently, with a behaviourism approach.

In question four, Lauren states that initial mathematics follow a school-defined structure, while implying at the same time children should be entertained. Note that Lauren bases her answer on the assumption that her activity for number learning is entertaining. We observe that Lauren does not answer adequately that the assumptions are outside the contexts of the mathematical activity performed. This indicates that Lauren's practice is not related to certain CK or PCK-T dimensions.

In answering question five, Lauren indicated that the key purpose of her lesson is to distinguish between symbolic representation and meaning in numbers, in this case, quantity. According to the construct, Lauren's observation corresponds to CK, although it is observed that the final portion of the activity she performs (see Chart 6) does not correspond to the number as a cardinal.

In question six, Lauren first refers to teaching and argues that it can be taught playfully. Then she refers to an experience in which mathematics consists of structured, monotonous, and meaningless techniques. In the previous answers, Lauren places emphasis on entertainment, which seems to be a response to her bad experiences in the past. Lauren's bad experiences are her motivation for changing the way she teaches mathematics.

In summary, the analysis revealed a low degree of evolution regarding Mathematics Teaching Capability in Lauren. In contrast, Helen demonstrated CK by mentioning mathematical notions applicable to early childhood education, PCK-T by referring to problem-solving, and, lastly, PCK-MTEC by mentioning the concept of working in accordance with the children's stage of development. Helen shows prowess of mathematical learning, manifesting knowledge about the role of mathematics in childhood development. In contrast, the answers provided by Lauren allow us to infer that the CK, PCK-T and PCK-MTEC categories are not of interest to her, and it seems that she does not even take any interest in mastering them. Lauren reflected non-specific knowledge, based only on her experiences. In this context, although Lauren highlights concepts such as attractive, meaningful mathematics, it is not based on what she learned during her teacher education.

DISCUSSION

The results demonstrate the relationship between knowledge and practice in teaching mathematics education. The study describes two cases of Mathematics Teaching Capability: in Helen, with a high score in the knowledge test; and in Lauren, with a low score in the knowledge test. While both Helen and Lauren were exposed to the same contents and teaching techniques, quantitative and qualitative differences were detected regarding knowledge and practices for mathematics initial teacher education.

Specifically, Helen showed high Mathematics Teaching Capability manifested through sufficient knowledge and the creation of problem-solving mathematics activities with materials that could be manipulated by children. In contrast, Lauren's Mathematics Teaching Capability was poor, showing insufficient knowledge and the creation of a repetitive mathematics activity that had little meaning to children.

These two case studies — along with the results obtained from teacher practice and map analyses — reveal a potential relationship between the organisation of learned content, the pertinence of content, and the interrelation of concepts and the practice developed. Consequently, we agree with Lee's conclusion (2010), who proposes that content knowledge and pedagogical content knowledge guarantee high-quality mathematics teaching in initial education.

Thus, in furthering the work by Olfos Ayarza, Goldrine Godoy and Morales Candia (2019), Lee (2010), Leavy and Hourigan (2018), and Platas (2008), this study reveals that teachers with broader experience and specific training related to children's mathematical development tend to show advanced knowledge with regard to number teaching. The case study with Helen shows a complex map, adequate practice, and reflections related to mathematical knowledge. On the other hand, that of Lauren shows a linear, simplistic vision of teaching, weak practice, and knowledge based on experience.

The main differences between these two case studies are the levels of knowledge and the clarity and quality of their teaching practices. While the case of Helen is based on content and pedagogical knowledge, the case of Lauren is based on her experience in mathematics classes. The case of Lauren allows us to highlight the weak areas in initial education courses, such as the apparently weak relationship between CK and PCK-MTEC.

CONCLUSIONS

This study contributes with a construct specific to number teaching, which helps to describe learning in pre-service early childhood teachers in regard to a mathematics teaching education course. This construct, which we named Mathematics Teaching Capability, integrates components of CK, PCK-T, and PCK-MTEC, all of which dialectically interact to form teacher knowledge and teacher practice. This study also sheds light on the dimensions that approach what is expected in early childhood mathematics teaching capabilities.

We have shown that good teaching practices require these specific teaching domains, that early childhood learning demands special considerations related to number teaching, and that highly integrated content aids in teaching understanding. Knowledge and practice are critical for teachers to promote rich and appropriate experiences to develop mathematics initial teacher education.

Additionally, the construct we have presented may be used by programs as a conceptual reference to integrate knowledge and practice within MITE. Accordingly, the "teacher practice" dimension must play an important role in mathematics initial teacher education for pre-service early childhood teachers.

Future studies should further delve into the possible effects of Mathematics Teaching Capability in pre-service teachers as it concerns to children's learning.

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Appendix A – Examples from the *Knowledge Test on Number Teaching for Future Early Childhood Teachers*.

Component	Items
CK	Two children put their dolls to bed. They place one doll with hair, another one with no hair; one with hair, and another one with no hair. Teresa, a pre-service early-childhood teacher, thinks the children are putting a mathematical notion into practice. What do you think? Mark one of the responses.
PCK-T	Mercedes, an early-childhood teacher, attended an advanced training course and learned that it is important for children to become familiar with different representations of quantity. She wants to buy some materials. Can you help her decide which materials are the best options? Mark one of the responses.
PCK-MTEC	Mercedes, an early childhood teacher, observes a child that does not state the total when counting objects. She asks herself, "What counting principle should I work on with this child?" What do you think? Mark one of the responses.

CK: content knowledge; PCK-T: pedagogical content knowledge for teaching; PCK-MTEC: pedagogical content knowledge on mathematics thinking during early childhood.

Appendix B – Assessment rubric applied to Concept Map.

Component	Evaluated criteria	Achieved (2 points)	Somewhat achieved (1 point)	Not achieved (0 points)
CK	Principal concepts of logic and number	Presents number and logic as principal concepts	Presents number or logic as principal concepts	Does not present number or logic
	Subordinate concepts of logic	Presents classification, seriation, patterning, one-to-one correspondence. (3–4 notions)	Presents classification, seriation, patterning, one-to-one correspondence. (2–4 notions)	Presents classification, seriation, patterning, one-to-one correspondence. (0–1 notions)
	Subordinate concepts of number	Presents oral series, quantifiers, numerals, ordinal numbers, cardinality, counting, conservation, sum/difference. (5 to 8 notions)	Presents oral series, quantifiers, numerals, ordinal numbers, cardinality, counting, conservation, sum/difference. (3 to 4 notions)	Presents oral series, quantifiers, numerals, ordinal numbers, cardinality, counting, conservation, sum/difference. (0 to 2 notions)
	Concepts of number representations	Presents pictographic, iconic, and symbolic representations	Presents pictographic-symbolic or iconic-symbolic representations	No, or only pictographic, representations

Continue...

Appendix B – Continuation.

Component	Evaluated criteria	Achieved (2 points)	Somewhat achieved (1 point)	Not achieved (0 points)
CK	Grouping of concepts	Groups and adequately relates concepts	Groups, but does not adequately relate, concepts	Concepts not grouped
	Connections between concepts	Uses arrows to adequately relate concepts	Uses arrows, but does not adequately relate concepts	Concepts not related
PCK-T	Subordinate concepts of teaching	Presents tasks or scenarios linked to problem-solving	Presents tasks or scenarios generally linked to problem-solving	Tasks or scenarios not presented
	Grouping of concepts or ideas	Groups and adequately relates concepts	Groups, but does not adequately relate, concepts	Concepts not grouped
	Connections between concepts or ideas	Uses arrows to adequately relate concepts	Uses arrows, but does not adequately relate concepts	Concepts not related
PCK-MTEC	Subordinate concepts of cognitive development	Presents tasks or scenarios linked to cognitive development	Presents tasks or scenarios generally linked to cognitive development	Not linked to cognitive development
	Grouping of concepts or ideas	Groups and adequately relates concepts	Groups, but does not adequately relate, concepts	Concepts not grouped
	Connections between concepts or ideas	Uses arrows to adequately relate concepts	Uses arrows, but does not adequately relate concepts	Concepts not related
	Concept map structure	Fully organised and easy to understand	Simple organisation and hard to understand	Linear organisation or illegible structure

CK: content knowledge; PCK-T: pedagogical content knowledge for teaching; PCK-MTEC: pedagogical content knowledge on mathematics thinking during early childhood.

Appendix C – Classroom Observation Form.

Component	During an initial student teaching opportunity, the trainee teacher:
CK	Uses mathematical language correctly
	Uses more than one representation of number (pictographic, iconic, and symbolic representations)
PCK-T	Presents a problem-solving situation to children
	Asks a central question that poses a task to be performed by children
	Confirms comprehension of the task
	Organises children individually, or in groups of two to four, and as an entire class to complete the given task
	Synthesises findings, achievements, and/or mathematical notions present during the activity
	Presents an activity with expected learning goals, in line with the educational programs of respective student levels
	Promotes exposure of resolution strategies between children
	Uses manipulative materials and representations
	Presents an activity that will allow children to judge the results of their actions for themselves
PCK-MTEC	Asks about prior knowledge
	Provides feedback, responses, or actions to children's questions without giving away the strategy and/or answer

CK: content knowledge; PCK-T: pedagogical content knowledge for teaching; PCK-MTEC: pedagogical content knowledge on mathematics thinking during early childhood.
Scoring: 1 = Present; 0 = Absent; NA = Not Applicable.