Recebido: 17/03/2022 ■ Aprovado: 21/06/2023 ■ Publicado: 01/10/2023

eISSN 2179-426X
10.26843/rencima.v14n4a09

Revista de Ensino de Ciências e Matemática

Problems of common concentration of solutions and the multiplicative conceptual field: an analysis in chemistry textbooks

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Abstract: The present investigation aimed to analyze and classify problem situations of Common Concentration of Solutions present in Chemistry textbooks, according to the theory of Conceptual Fields. For the analyses, the six Chemistry works approved by the PNLD 2018 were considered and the volume of the 2nd year of each work was selected, as they include the content of Common Concentration of Solutions. The research analyses, carried out in the light of the Conceptual Fields theory, showed that situations of Common Concentration are associated with the Multiplicative Conceptual Field and are classified as Simple Proportion. It was also identified that situations may vary in subclasses: Partition, One-to-Many Multiplication, Quota and Fourth Proportional. Therefore, the results showed that the problem situations of Common Concentration of Solutions are presented in the textbooks analyzed through contexts close to the reality of the students and that mathematical notions such as multiplication, division, operations with decimal numbers, fractions, proportion, volume and conversion of measurement units, as well as different mathematical representations, such as algebraic, numerical, tabular, graphic and natural language are essential for solving Common Concentration problem situations in Chemistry.

Keywords*:* Chemistry. Math. Multiplicative Structure. Simple proportion.

Problemas de concentración común de soluciones y le campo conceptual multiplicativo: un análisis en los libros de texto de química

Resumen**:** La presente investigación tuvo como objetivo analizar y clasificar situaciones problema de Concentración Común de Soluciones presentes en los libros de texto de Química, según la teoría de los Campos Conceptuales. Para los análisis se consideraron los seis trabajos de Química aprobados por el PNLD 2018 y se seleccionó el volumen del 2º año de cada trabajo, ya que incluyen el contenido de Concentración Común de Soluciones. Los análisis de la investigación, realizados a la luz de la teoría de los Campos Conceptuales, mostraron que las situaciones de Concentración Común están asociadas al Campo Conceptual Multiplicativo y son clasificadas como de Simple Proporción. También se identificó que las situaciones pueden variar en las subclases: Partición, Multiplicación Uno a Muchos, Cuota y Cuarto Proporcional. Por lo tanto, los resultados mostraron que las situaciones problema de Concentración Común de Soluciones se presentan en los libros de texto analizados a través de contextos cercanos a la realidad de los estudiantes y que nociones matemáticas como multiplicación, división, operaciones con números decimales, fracciones, proporción, volumen y la conversión de unidades de medida,

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así como diferentes representaciones matemáticas, como el lenguaje algebraico, numérico, tabular, gráfico y natural, son esenciales para resolver situaciones de problemas de Concentración Común en Química.

Palabras clave: Química. Matemáticas. Estructura multiplicativa. Proporción simple.

Problemas de concentração comum de soluções e o campo conceitual multiplicativo: uma análise em livros didáticos de química

Resumo: A presente investigação teve como objetivo analisar e classificar situações-problema de Concentração Comum de Soluções presentes em livros didáticos de Química, de acordo com teoria dos Campos Conceituais. Para as análises, foram consideradas as seis obras de Química aprovadas pelo PNLD 2018 e selecionados o volume do 2º ano de cada obra, pois contemplam o conteúdo de Concentração Comum de Soluções. As análises da pesquisa, realizadas à luz da teoria dos Campos Conceituais, mostraram que as situações de Concentração Comum estão associadas ao Campo Conceitual Multiplicativo e são classificadas como Proporção Simples. Também foi identificado que as situações podem sofrer variações em subclasses: Partição, Multiplicação Um para Muitos, Cota e Quarta Proporcional. Por conseguinte, os resultados mostraram que as situações-problema de Concentração Comum de Soluções são apresentadas nos livros didáticos analisados por meio de contextos próximos à realidade dos estudantes e que noções matemáticas como multiplicação, divisão, operações com números decimais, frações, proporção, volume e conversão de unidades de medida, assim como diferentes representações matemáticas, tais como algébrica, numérica, tabular, gráfica e a língua natural são essenciais para a resolução de situações-problema de Concentração Comum na disciplina de Química.

Palavras-Chave**:** Química. Matemática. Estrutura Multiplicativa. Proporção simples.

1 Introduction

In Brazil, the subject of Chemistry provides for the study of the content of Common Concentration of Solutions in the second year in the High School. The teaching of basic mathematics concepts, such as addition, subtraction, multiplication and division operations, conversion between measurement units, volume, decimal numbers, among others associated with this content, is taught during Elementary School (PARANÁ, 2008).

According to Scott (2012), mathematical skills are intrinsically related to students' performance in chemical calculations. For the author, the main reason for difficulties in solving chemistry tasks is the lack of understanding of basic mathematical notions, such as division, multiplication, conversion of decimal numbers and units of measurement, especially when used in conjunction with fractions or proportions. Thus, if the students' math skills are not well established, this can make it difficult for them to understand other subjects, such as Chemistry.

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For Gungstone and Champagne (1990), students who have difficulty understanding basic concepts of mathematics transfer to chemistry the idea that it is a complicated subject, since mathematical operations, for example, multiplication and division, are used in the development of the chemical content of Common Concentration of Solutions. In the classroom, this content is usually taught with an emphasis on calculations and applications of formulas, without connection to everyday life. That is, the focus is on the quantitative aspects, related to mathematical formulas and calculations (NIEZER *et al.*, 2015; 2016).

Vergnaud (2009a), a French researcher, defends, from a psychological point of view, that a concept cannot be studied in isolation, as several other concepts, properties, symbolic representations, associated with different situations, are interconnected in what he calls the Conceptual Field. In the direction of the study of a concept from the conceptual field and considering the need for representation and mathematical concepts for the understanding of chemistry contents, we will look at the problem situations of Common Concentration of Solutions present in Brazilian High School Chemistry textbooks, approved by the National Book and Textbook Program – PNLD (2018).

The Common Concentration of Solutions is expressed by a ratio between the mass of the solute and the volume of a solution, which allows an association with the Multiplicative Conceptual Field. Vergnaud (2009a) establishes the Multiplicative Conceptual Field as a set of situations, associated with the concept of multiplication, which for their resolution require several other concepts, procedures, and symbolic representations, which are connected to each other.

This work is part of a master's research that has the goal of analyzing and classifying problem situations of Common Concentration of Solutions present in Chemistry textbooks, based on the theory of Conceptual Fields. Thus, this article presents, at first, the theoretical framework that supports the analysis, and then the general result of the analysis developed.

2 The Conceptual Field of the Multiplicative Structure

The Theory of Conceptual Fields was proposed by Vergnaud (2009a), having as its genesis Piaget's theory and the legacy of Vygotsky. This fact is noticeable in the

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importance attributed to social interaction, language, and symbolization in the progressive understanding of a conceptual field by students.

The TCF is interested in the different problem situations associated with a concept, gradually developed in the school process, which, consequently, involve other concepts and schemes that are required of the student to adequately solve the situations being addressed, as well as words and symbols that can represent concepts effectively (VERGNAUD, 1996b).

Although it is not exactly a didactic theory, it has contributions and implications for didactics. Vergnaud (1996a) assumes that the essence of cognitive development is conceptualization. Therefore, attention should be paid to the conceptual aspects of the schemes and to the analysis of the situations for which students develop their schemes, in or out of school. Vergnaud (1996a) understands schemes as the invariant form of how tasks are organized by subjects.

Vergnaud (1993) mentions that students' cognitive functioning involves operations that are progressively automatized from different experiences and situations they go through during the schooling process. Thus, the subject builds, in action, new schemes, which can be re-elaborated in each situation experienced.

For Vergnaud (1990), cognitive development occurs mainly through a vast repertoire of schemes, which are developed by subjects when facing and mastering the various situations that are presented to them.

The TCF offers teachers a new way of selecting and organizing situations that enable students to learn, while allowing an analysis of the development of subjects in a given conceptual field (VERGNAUD, 2009b). Thus, the teacher is understood as a mediator, who organizes situations to be experienced by the students.

Conceptual Fields are fruitful units of study, capable of making sense of problems and observations made in relation to conceptualization. Thus, a conceptual field means "[...] an informal and heterogeneous set of problems, situations, concepts, relationships, contents, and operations of thought, connected to each other and, probably, interconnected during the acquisition process" (VERGNAUD, 1982, p. 40, our own translation).

Concepts are so essential for the TCF that they are given a definition from a psychological point of view. Vergnaud (1982; 1990; 1993) defines concept as a trio of three sets, C=(S, I, R), where S is a set of situations that give meaning to the concept; I is a set of invariants (objects, properties and relations) on which the operability of the concept rests, or the set of invariants that can be recognized and used by subjects to analyze and grasp the situations of the first set; and R is a set of mathematical representations (natural language, graphs, diagrams, formal sentences, etc.) that can be used to indicate and represent these invariants and, consequently, represent the situations and the procedures to deal with them.

Vergnaud (1996b) argues that a subject learns and develops throughout their schooling process, based on different situations experienced. In order to differentiate situations, the researcher understands that it is not enough to vary the context and/or its representations, but it is necessary to diversify the structure of situations, as they enable new resolution schemes by students, that is, diversifying the structure of situations provides new ways of organizing tasks and new lessons for students.

Considering the importance of a diversity of situations associated with the same concept, Vergnaud established two conceptual fields – that of additive structures and that of multiplicative structures, which have been studied in depth in the academic community. Each of these conceptual fields consists of classes of well-defined situations, which demand from students their own schemes for their resolutions. For each class, Vergnaud (2009a) presents a sagittal scheme that represents the structure of the situation and helps identifying its classification.

Thus, it is important to propose different classes of problems to students, related to the same concept, during their schooling process, so that they can develop new schemes and new operative invariants.

The present study is interested in the conceptual field of multiplicative structures, as it is associated with the concept of common concentration of solutions. The problems of Multiplicative Structures are established as the set of situations that involve one (or more) multiplication and/or division. These situations are well-defined and consist of two major classes – Isomorphism of Measurement and Product of Measurement (VERGNAUD, 2009b).

The Isomorphism of Measurement is a quaternary relationship that occurs between four quantities, related two by two, of the same magnitude, in which three quantities are given and the fourth is sought. Isomorphism of measurement is formed by three classes: single proportion, double proportion, and multiple proportion (VERGNAUD, 2009a).

The Multiplicative Conceptual Field is vast and among the classes of situations involved in this field, we will focus on quaternary relations, of the *simple proportion* class, because our analyses have shown that these relations are associated with the problems of Common Concentration of Chemical Solutions.

The class of Simple Proportion belongs to the quaternary relation, as it involves a relationship between four quantities, two of one type and two of another type, or even a simple direct proportion between two variables, like: people and objects, goods and costs, time, and distance, among others.

Below, Chart 1 presents problem situations of common concentration of solutions, their respective sagittal scheme, and their classification.

Chart 1 - Simple proportion subclasses and situations of common concentration of solutions

Problem situation	**Sagittal Scheme**	**Class/subclass**
What is the mass of naphazoline for a bottle of naphazoline hydrochloride (0.25mg/ml) containing 15 ml?	C = 0,25mg/ml Volume (ml) | Massa (mg) 1 | 0,25 15 | x	Simple proportion – One-to-many multiplication
Calculate the concentration of a drug solution at a volume of 20ml, which contains 300mg of diclofenac potassium.	C = desconhecida Volume (ml) | Massa (mg) 1 | x 20 | 300	Simple proportion – Partition
Calculate the volume of a solution that contains 100 g of a salt at a concentration of 2 g/ml.	C = 2g/ml Volume (ml) | Massa (g) 1 | 2 x | 100	Simple proportion – Quotation
Milk is rich in calcium. Knowing that in 200 ml of milk there are 335 mg of calcium, determine the amount of calcium ingested, in mg, by someone who consumes 600 ml of milk every day.	C = 1,675mg/ml Volume (ml) | Massa (mg) 200 | 335 600 | x	Simple proportion – Fourth proportional

Source: The authors, based on Cieslak (2021)

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The four examples presented in Chart 1 show that Common Solution Concentration problems can be classified within the Multiplicative Field classes (VERGNAUD, 2009a). More specifically, the present research shows that problems of Common Concentration of Solution are of the simple proportion type, including its four subclasses (GITIRANA *et al*., 2014): one-to-many, partition, quotation, and fourth proportional.

It should also be noted that the sagittal schemes presented in Chart 01 are different, since the position of the unknown 'x' is different in each of these schemes. Consequently, for each of the subclasses presented, the mathematical calculations to be performed by students in search of the value of 'x' will be different, and the very interpretation of the statement demands different reasonings from students for each situation. That is, different classes and/or subclasses provide students with the ability to develop new schemes, and therefore enable new learning.

In this sense, considering the importance of a diversity of situations related to the same concept; that problems of common concentration of solutions can be classified according to simple proportion subclasses; and that, as shown by Costa and Allevato, 2010, the textbook has the function of contributing to learning, constituting a support material for the pedagogical practices of teachers, we justify the development of this research, that intends to *analyze problem-situations of Common Concentration of Solutions present in chemistry textbooks*.

3 Methodological Procedures

This is a qualitative study, and the data was analyzed using the subjective approach, trying to understand the facts raised to then associate them with the object of study. Qualitative studies respond to particular questions, which deals, in the Human Sciences, with a level of reality that cannot or should not be quantified, addressing a broad universe of meanings, motives, aspirations, beliefs, values and attitudes (DESLANDES *et al*., 2009).

Focusing on the study of problem situations of Common Concentration of Solutions, we used as documental sources Chemistry textbooks approved by the PNLD in 2018, aiming to extract from them the necessary information to answer the following research question: *What are the characteristics of problem situations of*

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Common Solution Concentration in High School Chemistry textbooks? In Chart 2 we present the works selected for analysis.

Chart 2 – List of Chemistry textbooks

No.	Code	Collection	Authors
1	0020P18123	Química	FONSECA, M. R. M.
2	0153P18123	Vivá – Química	NOVAIS, V. L. D.; ANTUNES, M. T.
3	0185P18123	Química	CISCATO, C. A. M.; PEREIRA, L. F.; CHEMELLO, E.; PROTTI P. B.
4	0206P18123	Química Cidadã	SANTOS, W. L. P. MOL, G.
5	0041P18123	Química	MORTIMER, E. F.; MACHADO, A. H.
6	0074P18123	Ser Protagonista	LISBOA, J. C. F.

Source: Research data (2021)

In order to present an overview of the types of problem situations of Common Concentration of Solutions proposed in High School Chemistry textbooks, we chose to investigate the six works approved by PNLD in 2018, specifically volume 2, intended for the 2nd year of high school in each collection, as these are the volumes that address concepts pertinent to Common Concentration of Solutions.

After a qualitative analysis of the textbooks listed in Chart 2, we found that the problems on Common Concentration of Solutions involved mathematical notions belonging to the Multiplicative Conceptual Field and classified as Simple Proportion and their respective subclasses.

For this study, we analyzed the structure of each problem of common concentration of solutions solved and proposed, presenting its sagittal scheme, classification, representations in the statements, contexts involved in the statement, ideas and mathematical concepts present in the resolutions of situations.

When selecting these problems from the different textbooks for presenting them in this article, we chose to list those that represent situations that are part of different classifications of Simple Proportion.

In other words, for each analyzed book, we present the analysis of a problem of a certain class, in order to avoid repetitions of classes in the analyses. The problems presented vary between solved examples and problems proposed to the students, with the sagittal scheme for each proposed problem.

4 Data analysis

The analyses and discussions presented in this section are organized from each of the selected works indicated in Table 2, which, according to the aim of this study, proposes to analyze and classify problem situations of Common Concentration of Solutions present in Chemistry textbooks, according to the Conceptual Fields theory.

Book 1: Química no Ensino Médio

The first work analyzed is by Martha Reis Marques da Fonseca. The content of Common Concentration of Solutions is covered in Unit 2 – Water Pollution, Chapter 4 – Study of Solutions, spanning pages 63 to 97. The specific content of Common Concentration of Solutions is present in the section *Relationships between Solute and Solutions*.

When approaching the content of Common Concentration of Solutions, Fonseca (2016) starts the section with the concept of Concentration and contextualizes it with information regarding the chemical composition of mineral water of a specific brand. Then, the author presents a solved problem, guiding the solution through the formula of Common Concentration of Solutions. The problem is classified as *One-to-Many Multiplication*.

In addition to the problem solved, the author proposes six tasks for students to solve, which have both natural and decimal numbers, three situations require a unit conversion of capacity measurements and a unit conversion of mass measurements, four of the situations are presented in the form of statements in natural language, and two of them are accompanied by visual support in the form of a table. In Chart 3 we present one of the problems proposed.

Chart 3 – Problem P1

Homemade saline solution consists of an aqueous solution of sodium chloride (3.5 g/L) and sucrose (11 g/L). The masses of sodium chloride and sucrose needed to prepare 500 mL of saline solution are, respectively: a) 17.5 g and 55 g. b) 175 g and 550 g. c) 1 750 mg and 5 500 mg. d) 17.5 mg and 55 mg. e) 175 mg and 550 mg.

Source: Fonseca (2016, p. 72).

The problem presented in Chart 3 raises two questions, two different situations in a context related to medication. The data are presented in natural language, and the numerical values of concentration and volume are presented in natural and decimal numbers, and it is necessary to convert the capacity measurement unit from L to mL and, at the end, the mass measurement unit conversion.

The sagittal scheme of a simple proportion problem involves matching two distinct measurement types. In the case of Problem P1, the quantities involved are volume and mass. Specifically for problems involving Common Concentration of Solutions, based on Vergnaud (2009a), our studies show that the Common Concentration of Solutions, given by $\mathrm{C} = \mathrm{m}/\mathrm{V}$, is the operator-function that allows moving from one category to another (VERGNAUD, 2009a). Therefore, the concentration C (grams/liter) allows moving from Volume (L) to Mass (g).

The sagittal scheme for the Sodium Chloride – P1 problem situation is:

C = 3.5g/L

Volume (L)	Mass (g)
1	3.5
0.5	x

The common concentration given in the problem represents the operator-function $\mathrm{C} = 3{,}5\mathrm{g}/\mathrm{L}$, that allows identifying the mass (grams) in 1 L of volume of any concentration. That is, $\mathrm{C} = 3.5\mathrm{g}/\mathrm{L}$ allows identifying the mass corresponding to the volume in 1 L of solution, as in 0.5L of this same solution.

From the sagittal scheme, we have the following relationships and mathematical calculations to identify the mass needed for a concentration of 0.5L:

$$1L \rightarrow 3.5g$$
$$0.5L \rightarrow x$$

That is,

$$x \times 1L = 3.5g \times 0.5L$$
$$x = 3.5g \times \frac{0.5L}{1L} = 1.75g$$

Thus, we observe that the mass of Sodium Chloride for a solution at a volume of 0.5L and a concentration of 3.5g/L corresponds to $1.75\mathrm{g}$ of Sodium Chloride, answering the first question of the problem.

Regarding the problem situation of Sucrose – P1, the sagittal scheme is given by:

C = 11g/L

Volume (L)	Mass (g)
1	11
0.5	x

Based on the sagittal schemes presented for Sodium Chloride and Sucrose, it can be identified that the two Situations of Problem P1 are of Simple Proportion One-to-Many Multiplication.

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Under this situation, the common concentration is known, and is represented by the operator-function $\mathrm{C} = 11\mathrm{g/L}$, which is also used to identify the mass (grams) in 1 L of volume of any concentration. Thus, $\mathrm{C} = 11\mathrm{g/L}$ allows identifying the mass corresponding to the volume in 1L of solution, as well as in 0.5L of this solution.

From the sagittal scheme, there is the following relationships and mathematical calculations to identify the mass needed for a concentration of 0.5L:

$$1L \rightarrow 11g$$
$$0.5L \rightarrow x$$

That is,

$$x \times 1L = 11g \times 0.5L$$
$$x = 11g \times \frac{0.5L}{1L} = 5.5g$$

Therefore, we observe that the mass for a solution at a volume of 0.5L and a concentration of 11g/L corresponds to $5.5\mathrm{g}$ of Sucrose, answering the second question of the Problem.

Book 2: Vivá Química

Book 2, authored by Vera Lucia Duarte de Novais and Murilo Tissoni Antunes, presents the content of Common Concentration of Solutions in chapter two, entitled Concentration Units. The Solutions content starts on page 36 and goes through to page 53 of the book. The Concentration in grams per liter section addresses the specific Common Solutions Concentration content. In the section, two pages focus on the presentation of the content in question.

Novais and Antunes (2016) start the section with the concept of Concentration, presenting the generalization of the formula and proposing seven contextualized problems to be solved, involving natural and decimal numbers, as shown in Chart 4, below.

Chart 4: Problem P2

Exercises 1 and 2 are based on this information: certain vinegar contains 3% by mass of acetic acid; its density is 1g/ml. Calculate the concentration of this vinegar in g/L.

Source: Novais and Antunes (2016, p. 53)

The problem in Chart 4 is the first to be proposed to students, and its context approaches the concentration of acetic acid.

The magnitudes involved in problem 1 are Volume and Mass. In this problem, the ratio of Volume to Mass (x = 3/100) is the number 0.03.

Ratio = 0,03

Volume (mL)	Mass (g)

100	3
1000	x

It is possible to notice, in the sagittal scheme, that the ratio was calculated by dividing the Mass by the Volume. Thus, we have the following relationships and mathematical calculations:

$$100mL \rightarrow 3g$$
$$1000mL \rightarrow x$$

That is,

$$x \times 100mL = 3g \times 1000mL$$
$$x = \frac{3g \times 1000mL}{100mL}$$
$$x = 30\text{g}$$

From the sagittal scheme, we observe that Problem P2 can be classified as a Simple Proportion – Fourth Proportional.

Thus, we found that the concentration of this vinegar is 30g/1000mL, that is, 30g/L.

Book 3: Química 2 – Ensino Médio

The book is authored by Carlos Alberto Mattoso Ciscato, Luis Fernando Pereira, Emiliano Chemello and Patrícia Barrientos Proai. The content of Common Concentration of Solutions is found in Chapter 1, called Drinking Water: physical and chemical properties and processes for obtaining it, and is part of topic 02 – The main ways to express the concentrations of Solutes in Solutions. The content starts on page 25 and ends on page 36 of the book. The specific content of Common Solution Concentration is in the Expressing Concentrations in Grams Per Liter and in Amount of Matter section, which has a page dedicated to it.

When beginning to talk about the content of Common Concentration of Solutions, the authors bring up the example of mineral water. Table 5 presents problem P3, proposed by the authors to be solved by the students. The situation comprises three items for resolution and presents the information as a table.

Chart 5: Problem P3

Calculate the concentration of a solution that has a volume of 2.0 L and contains 10.0 g of solute.

Source: Ciscato *et al.* (2016, p. 35)

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The problem, shown in Chart 5, is the fourth problem proposed in the book, and its context is the concentration of a solution that is not defined. The quantities involved are mass and volume, requiring resolution by dividing the mass by the volume to find the concentration.

Considering the information from Problem P3, the following sagittal scheme can be constructed:

C = unknown.

Volume (L)	Mass (g)
1	x
2.0	10.0

From the sagittal scheme, it becomes clear that the problem can be classified as Simple Proportion – Partition.

For this problem, from the horizontal analysis, we identified the value of the operator-function. However, we need to look at the scalar 2 which, within the same scale, changes the volume from 2 L to 1 L. Similarly, if we look at the quantity mass and apply the same scalar 2, we get $x = 5g$.

And we find the following relationships and mathematical calculations:

$$1L \rightarrow x\ g$$
$$2.0L \rightarrow 10.0g$$

That is,

$$x \times 2.0L = 10.0g \times 1L$$
$$x = \frac{10.0g \times 1L}{2.0L}$$
$$x = 5gx = 5g/L$$

Book 4: Química Cidadã

The fourth book is by Wildson Luiz Pereira dos Santos and Gerson Mol. The content of Common Concentration of Solutions is found in volume two of the collection and is part of Chapter 2, called Chemical Calculations – Stoichiometry and Solutions, presented on pages 41 to 98 of the book. The specific content of Common Concentration of Solutions can be found in the Concentration and Composition section - item Mass Concentration, which is discussed in one page.

At the beginning of the section, the author presents the concept of mass concentration and exposes its respective formula. Afterwards, Santos and Mol (2016)

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demonstrate a solved problem on the mass concentration of sodium chloride in saline. Below, we bring one of the proposed problems, which has a single question, a situation that can be classified as Simple Proportion – Quotation, as can be seen in Chart 6.

Chart 6: Problem P4

A painkiller, given in drops, should be administered in the amount of 3 mg per kilogram of body weight, not exceeding 200 mg per dose. Each drop contains 5 mg of pain reliever. How many drops should be given to an 80 kg patient?

Source: Santos and Mol (2016, p. 75)

The problem shown in Chart 6 corresponds to the third problem proposed in the textbook by Santos and Mol (2016). The context of the situation has to do with the dosage of medication, according to body mass. The numerical values presented are in natural numbers.

Resolution:

m (painkiller) = (3 mg painkiller/kg) x (80kg) = 240mg – that is, above the maximum dosage of 200mg. Therefore, the maximum dosage to be administered will be 200mg.

Considering the information given in the problem, the following sagittal scheme can be drawn.

Ratio = 5mg/drop

Volume (drops)	Mass (mg)
1	5
x	200

From the sagittal scheme, we observe that Problem P4 can be classified as a Simple Proportion – Dimension, with the following relationships and mathematical calculations:

$$5x = 200$$
$$x = 40$$

The value of 40 drops can be found.

When carrying out the analysis of the variety of classes listed in Book 4, we noticed that the authors do not present problem situations that cover all classes, as we did not identify any situation representing One-to-many Multiplication. Only two problem situations of Quotation and three of Fourth proportional were identified, and the ones that appear in greater number are Partition situations, with six problem situations, totaling eleven problem situations on the content of Common Concentration of Solutions.

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The problem situations of Common Concentration of Solutions proposed by Santos and Mol (2016) present ideas and mathematical concepts such as Multiplication, Division, Units of measurement of mass and capacity, Decimal numbers, Conversion of units of measurement of capacity, and interpretation of tables.

Book 5: Química – Ensino Médio

The Book Química – Ensino Médio is authored by Eduardo Fleury Mortimer. The Common Concentration of Solutions content is part of Chapter 1 – Solutions and Solubility, which spans pages 10-46 of the book. The Common Solution Concentration content is part of the Studying Solution Concentration section, part B – Expressing Concentrations. The author does not present a solver problem of Common Concentration of Solutions, he only shows a contextualization of mineral water and proposes a reflection, asking a question to represent the concentration in g/L in the 3rd item, which is classified as partition, as stated in Figure 1.

Figure 1: Problem P5 Simple Proportion - Partition

INVESTIGAÇÃO

A seguir, reproduzimos as informações de um rótulo de embalagem de água mineral. As perguntas das partes A, B e C referem-se a essas informações e devem ser respondidas em grupo.

RÓTULO DE ÁGUA MINERAL

Água mineral gaseificad[a] artificialmente com gás natural.

Composição química provável (em mg/L):
Sulfato de bário: 0,61; sulfato de estrôncio: 0,10; sulfato de cálcio: 3,42; bicarbonato de cálcio: 106,5; bicarbonato de magnésio: 53,49; bicarbonato de potássio: 58,86; bicarbonato de sódio: 92,78; nitrato de sódio: 1,37; cloreto de sódio: 0,33; fluoreto de sódio: 0,04; óxido de ferro: 0,07; óxido de manganês: 0,24; óxido de alumínio: 0,18; óxido de silício: 16,20; gás carbônico: 2 554,20.

Características físico-químicas:
Aspecto *in natura* e após fervura: límpido e incolor; odor a frio e a quente: nenhum; temperatura da água na fonte: 20 °C; pH a 25 °C = 5,9; condutividade a 25 °C = $3{,}95 \times 10^{-4}$ mohm/cm (miliohm por centímetro); resíduo de evaporação a 180 °C = 220,0 mg/L.

PARTE A – Analisando a composição química

>1º Sob que forma as diversas substâncias se encontram na água mineral?
>2º Desenhem um modelo que represente a constituição da água mineral.
>3º As concentrações aqui são expressas em mg/L. Por que optou-se por essas unidades? Seria possível expressá-las de outra forma? Escolham algumas das concentrações e representem-nas em g/L e em % *p/V*.

Source: Mortimer and Machado (2016, p. 30)

The sagittal scheme for this Partition situation is the same as the one presented for Problem P3 in the analysis of book 3. The problem situation is explained only by the statement given in natural language, proposing that the students indicate some of the concentrations in g/L that the author presents, but in mg/L, requiring the conversion of mass measurement unit.

Afterward, Mortimer, and Machado (2016) present a new contextualization about the preparation of a potassium dichromate solution, and then presents a list of six problems to be solved by students.

Book 6: *Ser Protagonista: Química*

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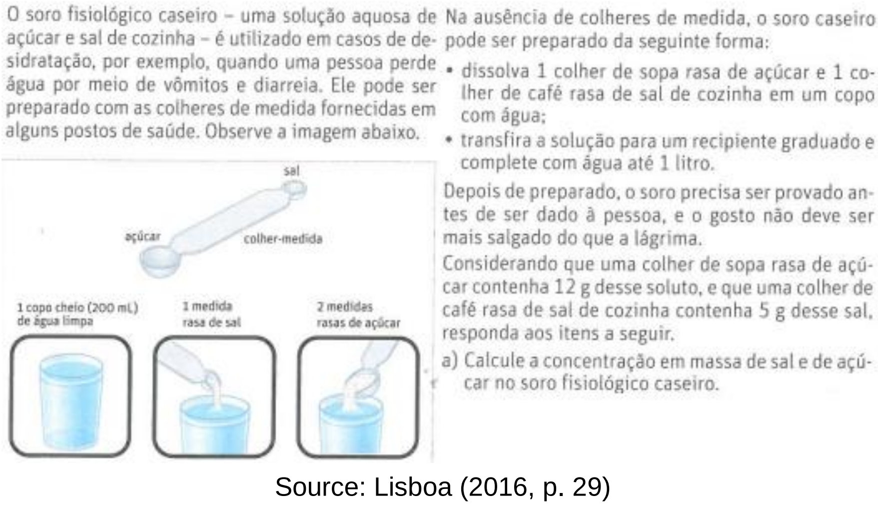
Book 6, by Julio Cesar Foschini Lisboa, addresses the content of Common Concentration of Solutions in Unit 1, called Solutions. Chapter 1, called Dispersions, Colloids, Suspensions and Solutions, runs from pages 10 to 31 of the volume.

When presenting the Common Concentration of Solutions, the author states that the relationship between the amount of solute and the amount of solution determines the concentration. Then he brings an example of preparing a juice, indicating the relationship between the amount of sugar and the amount of juice: if more solute is added, which in this case would be sugar, the solution will be more concentrated, that is, sweeter.

Then, Lisboa (2016) presents the concept of Mass Concentration, demonstrating the formula and a solved problem. In addition to this solved example, nine problems are proposed in the book for students to solve. All problems, whether solved or proposed, are contextualized. They are proposed in natural language, involving natural and decimal numbers, followed by their respective units of measurement. Two problem situations present numerical data through a table, and one situation uses an image as visual support, as shown in Figure 2.

Figure 2: Problem P6: Simple Proportion – Partition


O soro fisiológico caseiro – uma solução aquosa de açúcar e sal de cozinha – é utilizado em casos de desidratação, por exemplo, quando uma pessoa perde água por meio de vômitos e diarreia. Ele pode ser preparado com as colheres de medida fornecidas em alguns postos de saúde. Observe a imagem abaixo.

Na ausência de colheres de medida, o soro caseiro pode ser preparado da seguinte forma:

- dissolva 1 colher de sopa rasa de açúcar e 1 colher de café rasa de sal de cozinha em um copo com água;
- transfira a solução para um recipiente graduado e complete com água até 1 litro.

Depois de preparado, o soro precisa ser provado antes de ser dado à pessoa, e o gosto não deve ser mais salgado do que a lágrima.

Considerando que uma colher de sopa rasa de açúcar contenha 12 g desse soluto, e que uma colher de café rasa de sal de cozinha contenha 5 g desse sal, responda aos itens a seguir.

a) Calcule a concentração em massa de sal e de açúcar no soro fisiológico caseiro.

Source: Lisboa (2016, p. 29)

We do not present Vergnaud's sagittal scheme for the problem situation presented in Figure 2, because for this situation we highlight the visual support given by the book. The sagittal scheme for the Partition subclass is presented in the analysis of Book 3 for Problem P3.

I should be pointed out that the visual support image shown in Figure 2 helps in the interpretation of the problem situation, as it highlights the information, relating it to everyday objects (cup and measuring spoon). Visual aids like this help in the interpretation of the situation and in the development of strategies and use of concepts. Also, regarding the statements of the problems proposed by the author, three problem situations require conversion of measurement units, two of them for volume and one for mass and volume.

General analysis of the books

After analyzing the textbooks, we noticed that the problem situations are varied in relation to their classification within the TCF, but Partition largely predominated among the problems. In addition to Partition being the class that appeared in greater quantity in the analyzed situations, it is also the type that appears in greater quantity among the six analyzed textbooks. Of the forty-one problems solved and proposed, which total sixty-nine distinct problem-situations of Common Concentration of Solutions, thirteen are classified as One-to-Many Multiplication, eight as Quotation, fifteen as Fourth Proportional, and thirty-three are classified as Partition, as described in Table 1.

Table 1: Problem situations in each textbook according to the classification of the Multiplicative Conceptual Field – Simple Proportion

	One-to-many multiplication	Partition	Quotation	Fourth proportional	Total
Fonseca	3	6	1	1	11
Mortimer and Machado	1	8	-	-	9
Lisboa	6	7	2	9	24
Novais	1	4	1	2	8
Ciscato *et al*.	2	2	2	-	6
Santos and Mol	-	6	2	3	11
	13	33	8	15	69

Source: The author (2021)

We observed that the problem situations classified as *One-to-many Multiplication* are those that present the values of concentration, units of mass measurement for a unit of measurement of capacity and of the volume that is given from a certain value of units of capacity, and that *search for the mass for that respective volume*. The volume obtained by multiplying the concentration and volume fits into this type of classification. Of the sixty-nine Common Concentration of Solutions problem situations, thirteen are Isomorphism of Measurement – One-to-many Multiplication problem situations.

Isomorphism of Measurement problem situations of the Partition type aim to find a unit value. In their statement, this type of problem situation presents an initial quantity and the number of parts into which the initial quantity must be divided, undertaking the

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search for the value of each of the parts. In relation to the analysis carried out in our research, we found that the problem situations of Common Concentration of Solutions that present the value of mass and volume and that seek the value of concentration, obtained through the quotient of the values of mass and volume, are of this type, being, therefore, the ones that prevail in relation to the number of problem situations in the textbooks under analysis. This is because, out of a total of sixty-nine situations, thirty-three are Partition-type Isomorphism of Measurement problem situations.

Isomorphism of Measurement problem situations of the quotation type consider the quotient, seeking the number of parts into which the whole was divided; the whole represents the dividend, and the divisor refers to the size of the parts (quotas). This type of division involves a direct inversion of the function operator. We observed that in Common Concentration of Solutions problem situations, those that are classified as this are the ones that present the value of mass and concentration and *seek the value of volume*. Through the concentration given, we know the mass in one unit of measure of capacity, and search for the unknown value of how many units of measure of capacity are needed for a certain value of units of mass. In our study, we noticed that of the sixty-nine problem situations of Common Concentration of Solutions, only eight of them are Isomorphism of Measurement of the Quotation type.

Vergnaud (1985) refers to difficulties in understanding division because there is a need to perform different relational calculations: look for and obtain the extension of the part (unit value of the same measure) according to the scalar value indicated (in division by partition); or search and obtain the number of parts (the quota) as indicated (in the division by quota).

The class of problem situations of Isomorphism of Measurement of the Fourth proportional type consists of comparing two equivalent ratios. These are quaternary relationship problem situations that do not have any value equal to the unitary value. The process of solving fourth proportional problems, especially those whose measures of the same quantity are known and are not multiples of each other, is more complex than that of simple proportion One-to-many Multiplication problems, demanding greater cognitive effort from students. Analyzing the textbooks, we found that of the sixty-nine problem situations of Common Concentration of Solutions, only fifteen are problem situations of Isomorphism of Measurements of the Fourth proportional type.

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The different classes of problem situations of Common Concentration of Solutions allow students to attribute meaning to the concept, offering the opportunity of meaning to the Multiplicative Conceptual Field. After all, a single concept needs a variety of situations to become meaningful (VERGNAUD, 1990).

In relation to the forms of *mathematical representations* analyzed in the forty-one problems on Common Concentration of Solutions solved and proposed in the textbooks, thirty-two of them only presented statements in natural language, that is, most of the statements of these problems do not require any visual support for their resolution, such as images, graphs, or tables. In the analysis, seven problems are accompanied by a table and only one has visual support of an image, and another of a Cartesian graph. Although a diversity of mathematical representations being essential for students to learn (BRASIL, 2018), our study found that this fact is little explored by the authors of the textbooks, regarding the concept of common concentration of solutions.

In turn, in relation to the forms of mathematical representations requested in the problems, of the forty-one solved and proposed problems, fifteen of them require the resolution of calculations with numerical values in the form of natural numbers, and twenty-six of them require the resolution of calculations with numeric with values in the form of natural and decimal numbers. We also point out that thirteen problems require conversion of units of measurement of capacity, five require conversion of units of measurement of mass, three require conversion of units of measurement of capacity and mass, and twenty of them do not require conversion of units of measurement.

The analysis found that twenty-six Common Concentration of Solutions problems in the textbooks involved calculations with decimal numbers. Esteves and Souza (2012) explain that the difficulties faced by students when learning decimal numbers have to do with two topics mentioned by most teachers: the lack of understanding of what decimal numbers represent and how to divide them. The authors mention the comparison that students make of decimal numbers with natural numbers, as they see decimal numbers as natural ones separated by a comma. Thus, they find it difficult to understand decimal numbers as a number that belongs to another number set. For the authors, the predominant factor is that, perhaps, the lack of conceptual understanding and the decimal positional system is the cause of the mistakes students make in relation to decimal numbers.

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It is also possible to infer that all problem-situations of Common Concentration of Solutions involve units of measurement of capacity and mass, and of the forty-one problems analyzed, twenty-one of them require conversion of some type of unit of measurement – mass, capacity, or even mass and capacity. Costa, Vilaça and Melo (2020) point out that Quantities and Measures can be found in several real-world situations, but they are not always clearly perceived. That said, the authors highlight how necessary it is for students to notice the presence of this mathematical knowledge in social practices, in the connection with other areas of knowledge, as well as with other fields of mathematics itself, highlighting the importance of this field of mathematics for the exercise of citizenship.

The analyses show the presence of mathematical ideas and concepts, such as Multiplication, Division, Mass and Capacity Measurement Units, Decimal Numbers, Capacity Measurement Unit Conversion, and interpretation of Tables and Graphs involved in common solution problem situations. That is, mathematical ideas, concepts and representations are needed for solving problems of common concentration of solutions. This shows the interlocution between the subjects of Mathematics and Chemistry, and corroborates Scott (2012), who found that the difficulty students face when solving chemistry tasks stems from misunderstandings of basic mathematical operations, such as division and multiplication, especially when used in set with fractions or proportions, which in turn are associated with the concept of common concentration of solutions, the focus of this paper.

Regarding the contextualization that is given for the problems of Common Concentration of Solutions analyzed in this research, we can say the contextualization is used as an exemplification and description of facts or situations of everyday life, with the aim of addressing social issues, with a view to developing attitudes and values in relation, for example, to the reading and interpretation of information about Common Concentration of Solutions present on the labels of food products, medicines, resulting in a healthier life.

Still in relation to the contextualization of the statements, of the sixty-nine problem situations analyzed, fifteen of them presented scenarios referring to chemical solutions of NaOH, HCl, NaNO3, among others. However, from this total of situations investigated, we noticed fifty-four of them had to do with contexts of the students' reality, such as Medicines, Food, Drinks, among others. This fact is considered

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positive, as it meets the considerations and guidelines of the educational protocols (BRASIL, 2018; PARANÁ, 2008), with regard to contextualization.

5 Final remarks

The main objective of this study was to analyze and classify problem situations of Common Concentration of Solutions present in Chemistry textbooks, based on the Theory of Conceptual Fields. The analyses found that these situations belong to the Multiplicative Conceptual Field, and to the *Simple Proportion* classification. These situations were part of different subclasses such as Partition, One-to-Many Multiplication, Quotation, and Fourth Proportional.

Of the sixty-nine questions analyzed, thirty-three of them are Partition-type Simple Proportion problems, which are configured as the gateway to the formalization of the mathematical concept of division. The least contemplated class in the analyzed textbooks is the Quotation class. We also emphasize that Santos and Mol (2016) does not include the One-to-many Multiplication class; Mortimer and Machado (2016) does not include the Quotation and Fourth proportional classes; and Ciscato *et al*. (2016) do not include the Fourth proportional class. That is, the entire variety of situations are not always contemplated in the textbooks under analysis here.

In this sense, we suggest that teachers consider a variety of situations of Common Concentration of Solutions to be proposed to their students, according to the information and problem situations presented in this research, since, based on Vergnaud (1993), it is this variety of situations that can promote effective learning in students.

We believe that it is necessary to provide students with a variety of situations, regarding their classification and a diversity of contexts, representations and mathematical ideas involved. Thus, the problem-situations of Common Concentration of Solutions proposed will enable students to expand and develop new schemes, associated with mathematical concepts.

The analysis shows that the concept of common concentration is linked to the Multiplicative Conceptual Field and that the diversity of situations involving this concept, in addition to learning Chemistry, enables students to improve ideas, concepts and mathematical representations, including division, multiplication, proportion, decimal numbers, fractions, functions, volume, conversion of measurement units, Cartesian graphs, interpretation of tables, solving algebraic equations etc.

Regarding the forms of representation presented in the forty-one problems solved and proposed in the textbooks under analysis, thirty-two of them presented only natural language statements, and most of them were not enriched with visual supports, like images, graphs, or tables.

In this sense, problem situations used in Chemistry classes, specifically those of Common Concentration of Solutions analyzed in this study, represent opportunities to aid in the use and improvement of Mathematics contents. This is in line with the approach cited by the DCE (PARANÁ, 2008) of Mathematics and Chemistry, which encourage both disciplines to work together, since their articulated concepts, theories and practices can enrich the students' understanding.

Acknowledgements

We would like to thank the Graduate Program in Mathematics Education (PRPGEM) of the State University of Paraná (Unespar) and the Coordination for the Improvement of Higher Education Personnel (CAPES) for funding the grammar and spelling review and translation of this article.

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