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Skills developed from the Mathematical Modeling Student Problem Situation Activity

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***Abstract*:** This study discusses the formation of skills aimed at solving problem situations in the field of Mathematical Modeling, proposed in tasks for Mathematics undergraduates at the Federal University of Amazonas. The aim of this article is to present the analysis of the contributions of the Mathematical Modeling Student Problem Situation Activity (MM SPSA), based on the Historical-Cultural Theory from the perspective of Galperin, Talízina and Majmutov, developed with the undergraduates to the formation of the skills required in Mathematical Modeling. The research is qualitative, using the following tools to gather information: diagnostic tests, observation and questionnaires. From the analyses, it was possible to see that the MM SPSA enabled a positive advance in the development of the students' abilities to formulate the student's problem, build the conceptual and procedural core, solve the student's problem and analyze the solution when solving Mathematical Modeling tasks.
***Keywords*:** Activity Theory. Galperin's Theory. Modeling Tasks.

Habilidades desarrolladas a partir de la Actividad de Situaciones Problemática Docente de Modelaje Matemático

***Resumen*:** El presente estudio discute la formación de habilidades orientadas a la resolución de situaciones problemáticas en el campo de la Modelación Matemática, propuestas en tareas para estudiantes de Matemáticas de la Universidad Federal de Amazonas. El objetivo de este artículo es presentar los análisis de las contribuciones de la Actividad de Situaciones Problemas Docentes de Modelación Matemática (ASPD de MM), basada en la Teoría Histórico-Cultural desde la perspectiva de Galperin, Talízina y Majmutov, desarrollada con estudiantes de la formación de profesores para formar las habilidades necesarias en modelaje matemático. La investigación es mixta, con énfasis cualitativo. Los instrumentos para producir información fueron pruebas diagnósticas, observación y cuestionarios. A partir de los análisis, se pudo verificar que el ASPD de MM permitió un avance positivo en el desarrollo de las habilidades de los estudiantes para formular el problema docente, construir el núcleo conceptual y procedimental, resolver el problema docente y analizar la solución al resolver tareas de modelación matemática.
Palabras clave: Teoría de la Actividad. Teoría de Galperin. Tareas de Modelaje.

Habilidades desenvolvidas a partir da Atividade de Situações Problema Discente de Modelagem Matemática

Resumo: O presente estudo disserta sobre a formação de habilidades direcionadas à resolução de situações problemas no campo da Modelagem Matemática, propostas

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em tarefas aos licenciandos de Matemática da Universidade Federal do Amazonas. O objetivo deste artigo é apresentar as análises das contribuições da Atividade de Situações Problemas Discente de Modelagem Matemática (ASPD de MM), fundamentada na Teoria Histórico-Cultural na perspectiva de Galperin, Talízina e Majmutov, desenvolvida com os licenciandos para a formação das habilidades requeridas em Modelagem Matemática. A pesquisa é qualitativa, valendo-se dos instrumentos de produção das informações: provas diagnósticas, observação e questionários. Das análises, foi possível constatar que a ASPD de MM possibilitou um avanço positivo no desenvolvimento das habilidades dos estudantes de formular o problema discente, construir o núcleo conceitual e procedimental, solucionar o problema discente e analisar a solução na resolução de tarefas de Modelagem Matemática.
Palavras-chave: Teoria da Atividade. Teoria de Galperin. Tarefas de Modelagem.

1 Introduction

This article is an excerpt of the main results of a doctoral research project focused on the formation of skills required in the Mathematical Modeling process, aimed at Mathematics undergraduates at the Federal University of Amazonas, located in the city of Manaus[3].

During their academic training, mathematics undergraduates are required to develop competencies and skills (Brasil, 2002), including “understanding, criticizing and using new ideas and technologies to solve problems [...]; identifying, formulating and solving problems in their area of application [...]; establishing relationships between mathematics and other areas of knowledge” (p. 3-4).

According to D'Ambrosio, “mathematical modeling is mathematics par excellence. The origins of the central ideas of mathematics are the result of a process that seeks to understand and explain facts and phenomena observed in reality” (Bassanezi, 2009, p. 11).[4] This justifies the fact that training skills in modeling is also training skills in mathematics. Therefore, students should have the opportunity to develop mathematical skills and broaden their experience for future use in the classroom.

It is not enough to assume that Mathematical Modeling is a favorable way of developing mathematical skills in undergraduates, since we are facing a problem, with a view to understanding how the formation of skills occurs and the planning and

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[4] Foreword by Ubiratan D'Ambrosio.

execution of the necessary actions in the classroom. We also need to consider what the skills are and the system of tasks to make them possible.

The aim of this article is to present the contributions and reflections on the Student Problem Situation Activity as a learning methodology, from the perspective of Galperin's step-by-step formation of mental actions, in the direction of Talízina's study activity and Majmutov's problematizing teaching. The research was designed to build the skills of Mathematics undergraduate students at the Federal University of Amazonas in carrying out the actions involved in building mathematical models.[5]

The next sections of this article explain the theory behind the Mathematical Modeling Student Problem Situation Activity (MM SPSA), the methodological elements of the research and the analysis of the results of the intervention in the formation of the skills of the students participating in the research in the modeling process.

2 Mathematical Modeling Student Problem Situation Activity

To understand the definition of the Mathematical Modeling Student Problem Situation Activity, we will begin with the concept of activity and its relationship to the formation of skills in the cultural-historical context.

A. N. Leontiev considers human activity to be the process that mediates the relationship between the human being (subject) and the reality to be transformed by him (object of activity). Activity is the way in which man relates to the world, it is the result of all social influences, it is always transformative, as it aims to make real or imaginary changes to its object, which becomes the product of that same activity (Leontiev, 2014).

From this perspective, the learning activity is also an activity, made up of two objects: the content, as the object of knowledge, and the student themselves, as the object of the internal development of their personality. For this type of activity, the student is both the subject and the object of the activity (Núñez, 2009).

On a psychological level, activity is a system that has a structure, internal steps, in other words, a development. The activity of study has “a conscious character, oriented towards a defined objective that has to be made conscious by the student as the subject of the activity, in order to be able to interact and transform the object and

[5] Also referred to as modeling skills.

transform themselves" (Núñez, 2009, p. 66). Action also stands out as the unit of analysis of activity, i.e. "knowledge is found in unity with mental actions and is assimilated in the process of activity; it does not exist outside of it" (Núñez, 2009, p. 66).

Learning is considered in terms of the actions that the student carries out with the object of study to transform it into a product, in a social context. In this way, the content assimilated by the student is the product of a system of actions, initially carried out from external actions (means of image formation), until its representations are found in the act of thought, in the form of mental actions. This is the central methodological principle of dialectical materialist psychology on the unity of activity and the psyche, i.e. internal psychic activity represents internalized external and material activity (Leontiev, 2014). According to Galperin, the link that exists between activity and new knowledge and skills

> is that during the process of activity, isolated actions are transformed into skills and, at the same time, as a consequence of actions with objects, representations and concepts of said objects are integrated (2013d, p. 478).

In this sense, it is important to "select the necessary action and organize its execution in such a way as to guarantee the training of knowledge and skills with the required quality" (Galperin, 2013d, p. 478).

In this study, learning is an activity, structured by a system of actions that the student must carry out in order to assimilate the concepts, organized in suitable conditions for the formation of skills, referred to here as mathematical modelling activity. A skill is equivalent to a mental action, because skills are actions and constitute ways of acting that allow us to operate with knowledge (Talízina, 2001).

Following the theoretical assumptions, to form the skills required in the modeling process (modeling activity), it is important to determine the appropriate action, represent the action and develop the method for transforming the external action into an internal one.

Galperin sees learning as a type of activity, the objectives are formulated in the language of the actions that the student carries out with the object of study, in the zone of proximal development, for its construction and conscious assimilation. This theory considers the assimilation of a new skill, or just its updating, to be a process that follows stages, initially in material form, then verbal, and finally mental.

Galperin (2013a, 2013d) pays special attention to orientation in learning activity and states that learning results in the formation of new concepts or skills when the student elaborates or re-elaborates a general orienting basis as a mental model of action that allows planning, the solution of problem situations, within the limits of a defined generalization, as well as the control and regulation of the process.

The anticipated representation of the task, as well as the guiding systems needed to fulfill it, form the plan for future action, called by Galperin (2013b, p. 442) the "orienting basis of the action" (OBA). His studies show three types of orientation, each type corresponding to a particular process of action formation and the quality of its final product, related to the types of learning it promotes. The third type of orientation is highlighted by Galperin (2013a) as a real possibility for the planned formation of theoretical concepts. In the third type (OBA III),

> learning is based on the method of analyzing objects and allows the necessary orientation to be established in order to solve any situation in a given class within the established generalization limits. Through analysis, the student distinguishes the essential characteristics and parameters of the material under study, i.e. those correctly necessary to carry out the action (Núñez, Ramalho and Oliveira, 2020, p. 117-118).

Based on Galperin's theory, we understand that the orientation developed by the undergraduate student about the actions they should take in the modeling activity plays an important role, since when faced with a problem situation, a person's first act is to orient themselves in search of answers which, in many cases, requires a new action and new knowledge, or even the articulation of the action or knowledge they already have, applied in a new way.

Therefore, before carrying out the modeling activity with the students, it is necessary to prepare the guiding action to be formed in advance. To meet this need, it is up to the teacher to outline the type III guiding action in a Scheme of the Complete Orienting Basis of Action (SCOBA) which serves to guide him/herself and the students. Thus,

> [...] the actions developed and supported in the SCOBAs make it possible to consciously contrast the students' mental schemas and their orientation bases

> with the schemas that represent the pedagogical and didactic model that should be the orientation reference (Pereira and Núñez, 2017, p. 81).

Later, we will return to this component, which we call the Mathematical Modelling SCOBA (MM SCOBA), as a materialized representation of OBA III, a complete and generalized composition that expresses the actions and operations that the trainee needs to carry out consciously applied to a set of modelling tasks.

The action can be assimilated by the subject with different essential indicators, according to its qualitative characteristics (Talízina, 2009). The primary (independent) characteristics of action refer to: the form, the degree of generalization, the detailed or reduced nature of the action, the degree of assimilation and the degree of independence. The secondary characteristics depend on these: rational character, degree of awareness and degree of solidity of actions. Establishing qualitative indicators for the formation of skills is an important "action parameter" (Galperin, 2013b, p. 441), which differentiates the level of appropriation of the action by the student during the modeling activity.

During the internalization process, changes occur in each of the characteristics of the action. The form of the action is the measure of the internalization of the action (Talízina, 1988). Three fundamental forms are distinguished in this process: material, verbal and mental. Galperin (2013b, 2013d) and his collaborators aligned qualities of actions as indicators of the formation of mental actions in five stages: elaboration of the guiding basis of the new action, material or materialized formation of the action, formation of the action in external language, formation of the action in external language for oneself, and formation of the mental action. Motivation is also highlighted as an initial and constant stage in the formation of skills (Talízina, 1988, 2001).

Based on Galperin's theory, each stage in the formation of mental actions is mentioned in this article in line with the object of research, in the form of planning, already incorporated with the appropriate characteristics, as we will see below.

3 Stages of planned formation of mental actions in modeling activity

Continuing with the foundations that support the SPSA of MM, the first stage in the formation of the skills required in the mathematical modeling process refers to the representation of the guiding actions in materialized form (external OBA). In this representation, the essential properties for determining mathematical models as a way of solving a given problem situation are reproduced.

In the OBA III stage of the modeling activity, a problem situation can be proposed by the teacher, and the students must look for possible ways to solve the problem, collaboratively and under the guidance of the teacher. After this, the teacher can conduct a negotiation with the students, bringing to light their perceptions of the actions they have taken in constructing a mathematical model, from the start of the investigation to the critical analysis of the possible results generated by such a model.

As well as becoming aware of the actions and operations they must carry out, along with the conditions that allow the modeling task to be guided, it is important that students record the negotiated GOOD on a guidance card. The external (materialized) form of the action is an essential tool for future consultation, regulation and control of the skill formation process in the next stage.

In the stage of materialized formation of the action, the student actually begins to carry out the action, in an unfolded, detailed way, consciously performing all the operations established in the previous stage. The trainees must carry out the actions, in groups or pairs, under the mediation of the teacher to correctly fulfill the operations in the proposed tasks.

In the stage of forming external verbal action, "the action is freed from direct dependence on objects [...], it is an account of the action without any type of material execution" (Galperin, 2013b, p. 444). This means that the trainee must verbalize (orally or in writing) the actions taken that made it possible to construct the mathematical models, in conjunction with the pre-existing knowledge needed to complete the tasks. This presupposes the beginning of the generalized nature of the actions. In addition, the actions become less detailed and the reduced form of the action begins, omitting some operations.

The forms of language change from "language-information for someone else to language-information for oneself" (Galperin, 2013c, p. 457). In the stage of forming an external language for oneself, the trainee should be offered challenging modeling tasks with different types of problems. At this stage, "the process of solving the problem takes place in the form of internal language, as an individual process that does not require the participation of other people" (Talízina, 2009, p. 174).

This is the moment when the trainee is given the opportunity "[...] for reflection and 'speaking to oneself' becomes a means of transforming oneself into an object for better analysis" (Galperin, 2013b, p. 445, emphasis added). Thus, the actions and operations materialized in the guidance card, going through the stages of training, become a fact of thought, skills revealed only in the result of the activity, in a totally independent way.

When the corresponding degree of assimilation is reached, control by operations is removed and only control by the overall result remains, and this is when the fifth stage appears. In the fifth stage, action rapidly shortens and becomes automatic (Galperin, 2013d). In the final stage, guidance in solving problems is a mental action by the student and is "integrated into their cognitive structures, part of scientific thinking" (Núñez, Ramalho and Oliveira, 2020, p. 127). The orientation of actions during the execution of tasks goes through different stages

Initially, orientation is carried out with external help materialized in the SCOBA, moving on to the form with the help of external language, oral and/or written, and, in the end, to the fully mental form, in which the mediators of the previous stages are not needed to successfully solve the tasks (Núñez, Ramalho and Oliveira, 2020, p. 124).

Since Galperin's theory is "an adequate psychological basis for the scientific direction of teaching processes" (Talízina, 2001, p. 9), it becomes pertinent to determine the actual direction of the study activity. In this scenario, the effective direction of the teaching-educational process is possible when a system of requirements is developed, centered on the interaction between the object and the student, paying attention to the transformation of the external actions of this object into internal ones (Talízina, 1988).

The first direction is to define the teaching objective of the cognitive activity (D1) and the second is knowledge of the starting level (D2), the students' diagnosis. To be able to move from actual knowledge (starting level) to potential knowledge (objective), the teacher must guarantee, within the student's zone of proximal development, the necessary resources for the different stages in the formation of mental actions to take place. In this case, the third requirement is met, moving through the different stages of the assimilation process (D3) (Talízina, 1988).

The fourth refers to process feedback (D4), also called systematic feedback, linked to the controlling function of the activity, in which the teacher must identify the errors revealed during the assimilation process and analyze the origin of their occurrence for subsequent correction. The fifth is the correction of the assimilation process (D5), in which the teacher evaluates the errors made by the students throughout the process and can thus redirect the teaching plan to achieve the objective of the activity (Talízina, 1988).

Under this psychological and methodological basis for the formation of mathematical skills in undergraduates, we corroborate Talízina (2009, p. 174), for whom "skills cannot be developed without solving problems, performing specific tasks and the process of valid and complete assimilation of knowledge".

To conclude the definition of the Mathematical Modeling Student Problem Situation Activity, it is necessary to discuss the nature of the actions and the contradictions inherent in problem situations. Below is the task system for carrying out the modeling activity, in which the actions and operations set out in the MM SCOBA are carried out.

4 Mathematical modeling tasks and SCOBA according to the principles of the student problem

Following the perspective of Mathematics Education, the concept of modeling is adopted as a teaching-learning strategy (Bassanezi, 2009). In terms of didactics, theoretical and practical activities based on problems awaken creative thinking in students, since their knowledge is confronted in search of a solution.

In this process, the dean is the contradiction of assimilation, which, according to M. I. Majmutov (1983, p. 63), "is interrelated with the dialectical contradiction that exists between previous knowledge and new facts, for which the explanation of previous knowledge, past experience, is insufficient".

The teacher plans a task considering an objective contradiction between the known and the unknown as an external cognitive activity to be posed to the student. However, when the student recognizes the contradiction (what is known to them is insufficient to determine the unknown), the task becomes subjective and, at this point, the student's problem arises (Mendoza and Delgado, 2020). This means that "the task, after receiving new content in the student's consciousness, is transformed into a completely new phenomenon, the student problem” (Majmutov, 1983, p. 132).

The modeling tasks proposed to the participating undergraduates were designed/adapted with an objective and potential contradiction in becoming a problem for the students, in other words, a student problem. Based on Majmutov's (1983) studies of mental activity in the search for a solution to the student's problem, the system of tasks applied to the students takes into account the following types of contradictions that can be presented in problem situations: recognizing previous schemes and/or facing new practices; analytical and/or heuristic procedures in the resolutions; and the solution paths according to the student's experience with the modeling practice.

The tasks progress according to how the students overcome the contradictions inherent in the student's problem, sequenced as type 1, type 2 and type 3 tasks (Wakiyama and Mendoza, 2021). Under this system, at each stage of the orientation formation process, the tasks are structured according to the qualitative indicators of the activity (Talízina, 1988). It is worth noting that

> [...] the concepts of task, problem situation, student problem, activity and the orienting basis of the action, based on Dialectical Materialism and Activity Theory, are different. The task is presented to the student as an objective contradiction between known and unknown knowledge; however, when the student assumes the objective contradiction, it becomes subjective and, at this point, the problem situation arises, i.e. their knowledge is insufficient to respond to the proposed task. The student's problem is when the student determines the difficulty that does not allow them to solve the proposed task. The activity is a system of student actions and operations to solve the task combined with their motives and needs. orienting basis of the action (OBA) the student's real (subjective) orientation towards solving the task (Wakiyama and Mendoza, 2021, p. 5).

To align the tasks with the actions needed to deal with the contradictions, the MM SCOBA is inspired by the problem-solving SCOBA proposed by Mendoza and Delgado (2020), who suggest that resolutions should be approached from the same perspective. In addition, the MM SCOBA incorporates into its operations the characteristics of the stages in the development of modeling according to different authors who have studied the subject in the field of Mathematics Education (Wakiyama, Mendoza and Delgado; 2021).

Based on 4 invariant actions and their respective operations, the MM SCOBA covers the modeling process represented in the 3 types of tasks. For the sake of illustration, we will simplistically show the resolution of task type 1, according to the guiding schema, which requires little experience on the part of the student in the practice of modeling and requires recognition of old schemas.

Problem situation *Ideal purchase of a fleet of buses*[6]: "An entrepreneur in the transport industry wants to invest R$10 million in the purchase of a fleet of buses. The entrepreneur is interested in a budget, in which a bus capable of carrying 15 passengers (small) costs R$120,000; a bus capable of carrying 30 passengers (medium) costs R$200,000 and a bus capable of carrying 40 passengers (large) costs R$280,000. The businessman found that, in terms of his company's demand, the number of small buses should be equal to the sum of the number of medium and large buses".

Resolution guided by MM's SCOBA: 1st Action to formulate the student problem: Operation 1 - Identify the known elements: the known elements are the value of the investment and the values of the three buses according to their capacity. There is an imposed condition, in which the number of small buses is equal to the sum of the number of medium and large buses; Operation 2 - Identify the unknown elements: neither the number to be purchased nor the investment to be made in each type of bus is informed; Operation 3 - Recognize the contradiction: questioning how to invest R$10 million in the purchase of the bus fleet if the quantity to be purchased is unknown; Operation 4 - Determine the knowledge to be sought: according to the conditions, how many buses of each type can be purchased so that the investor gets more benefit or more profit?

2nd Action of building the conceptual and procedural core: Operation 5 - Selecting the known concepts and procedures needed to solve the student's problem: this operation is marked by the transposition of information into mathematical language: Considering the number of small buses x, medium buses y and large buses z, the first relationship that can be established is the monetary $120{,}000x+200{,}000y+280{,}000z=10{,}0000{,}000$ (I). Another relationship concerns the passenger capacity p per bus $15x+30y+40z=p$ (II). The third relationship concerns the

[6] Task with fictitious data, but which simulates a common investment situation in small and medium-sized companies.

sum of the number of buses x=y+z (III); Operation 6 - Update other known concepts and procedures that may be linked to the unknown ones: check whether there is a need for a brief review or update of the concepts or procedures for the resolution. Operation 7 - Find a strategy for connecting the known and unknown concepts and procedures: the system formed by equations I, II and III has three equations made up of four unknowns. This question must be raised in order to then determine the strategy and carry out the appropriate analysis of the model.2nd Action to build the conceptual and procedural core: Operation 5 - Select the known concepts and procedures needed to solve the student's problem: this operation is marked by the transposition of information into mathematical language: Considering x number of small buses, y number of medium buses and z number of large buses.

3rd Action to solve the student's problem: Operation 8 - Build the mathematical model: make the successive substitutions, fixing one of the unknowns. Each unknown is positive or zero to determine the model:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ z \geq 0 \\ p \geq 0 \end{cases} \Rightarrow \begin{cases} x \geq 0 \\ y = 5(x-25) \geq 0 \\ z = 125 - 4x \geq 0 \\ p = 5(x+250) \geq 0 \end{cases} \quad (IV)$$

Operation 9 - Correctly applying the solution strategy(ies): with the model in hand, a table of possible values of and can be set up to obtain the values of and; Operation 10 - Determining the knowledge sought: in the solution set, there are seven possible outcomes. According to model (IV), the largest number of passengers that can be transported is 1405, when buying 31 small buses, 30 medium-sized buses and 1 large bus.

4th Action of analyzing the solution to the student's problem: Operation 11 - Check that the solution corresponds to the objective, data and conditions of the student's problem: the calculations made in the substitution method and the mathematical relationships established to build the model must be checked. Operation 12 - Reflect on the results obtained and predict the best decisions in the face of the unknown: time to analyze any misjudgments made during the process, as well as a critical analysis of the model found.

In short, the MM SPSA is a methodological proposal for developing skills in the mathematical modeling process, made up of the Galperin-Talízina-Majmutov didactic system. It is an activity psychologically guided by the cognitive stages of Galperin's theory, to be carried out in a progressive system of tasks based on Majmutov's student problem, in the student's zone of proximal development, under the didactic direction of Talízina's study activity for modeling practice in the classroom.

5 Methodological procedures

The MM SPSA took place over three academic semesters of the Mathematics degree course at the Institute of Exact Sciences at UFAM, in the city of Manaus. This article only reports the results of the MM SPSA developed with the students who, in addition to the initial diagnosis, took part in the entire modeling activity.

The 6 research participants enrolled in Mathematics Teaching Instrumentation I (IEM I) in the 1st semester, known as Group 1 (G1), continued as participants in IEM II in the 2nd semester. Of these 6, three students enrolled in IEM III in the 3rd semester. In this last class, 7 new students formed Group 2 (G2) of the research, totaling 13 undergraduate participants. Under the supervision of the class teacher, the researcher implemented the MM SPSA, teaching the classes as a research teacher[7].

The G1 participants are indicated by P1, P2, P3, P4, P5 and P6. The G2 participants are indicated by Q1, P5, Q3, Q4, Q5, Q6, Q7 and Q8. It's worth noting that participant P5 from G1, who continued in the research, started the activity again in G2 when the need for correction/reinforcement in the assimilation process was verified, while P3 and P6 continued to progress through the stages of mental actions.

The research is qualitative, but it is also aided by elements of quantitative research to guide the qualitative analysis. The instruments used to gather information were: a diagnostic test, modeling tasks, two pedagogical tests, observation guides and questionnaires.

The results were evaluated according to the criteria of correct execution of the MM SCOBA actions, where each action is converted into a variable and each of the operations into an indicator. The range of correct executions of each action is set at [1,5], and can reach a total of 4 actions in the following range categories: [4;8), [8;12), [12;16) and [16;20]. Other criteria are the primary and secondary qualitative indicators

[7] Research Ethics Committee (CEP) - Opinion Number: 4.637.17

of the actions carried out by the student participants, also recorded in the observation guides.

Having determined the learning objective in the model of the actions materialized in the MM SCOBA as the orientation for forming the skills required in modeling (Talízina's D1), the research proceeded in four stages.

The first stage was to determine the students' starting level (Talízina's D2) by solving an initial diagnostic test, made up of 4 type 1 tasks, in the first semester. We found flaws in the guiding basis and the need for appropriate (re)elaboration. As a result, it was possible to design the teaching plan according to a system of tasks made up of contradictions, considering the students' starting level and the stages of assimilation.

After the implementation of the MM SPSA (D3, D4 and D5 of Talízina), the results were analyzed by triangulating the levels of performance, qualitative indicators and student responses recorded in questionnaires. The triangulation helped to estimate the mental stages reached by the students in the development of the MM SPSA, as we'll see below.

6 Results and analysis

The results of the initial diagnostic test (IDP) showed that the students in both groups encountered obstacles when carrying out the actions. Table 1 shows the group averages for each action performed correctly in the IDP. The action of formulating the student problem (1stA) had the highest average, but not understanding some of the operations in this action compromised the rest of the resolution. The action of building the conceptual and procedural core (2ndA), as well as solving the student problem (3rdA), were equally unsatisfactory.

Table 1- Group 1 and Group 2 share averages

Group	1ªA	2ªA	3ªA	4ªA	Total
Average G1	3,38	3,16	3,16	1,25	10,96
Average G2	3,57	3,03	2,90	1,00	10,51

Source: Research Data

There was no analysis of the solution to the student's problem (4thA), an important and always requested verification, validation and interpretation of the model found. This fact is also highlighted by Gonçalves (2020, p. 123), in which “none of the students showed, in the initial diagnosis, that they were aware of how to carry out the control action when solving mathematical problems", showing the absence of problem solution analysis.

Knowing the students' starting level allowed the MM SPSA to be planned, with each stage of assimilation being assigned tasks that move from one type to another according to the degree of independence, generalization and explanation of the students' actions.

In the first stage of skills training, the negotiation of the materialized elaboration and/or re-elaboration of the OBA took place in two classes lasting a total of 4 hours for each group of participating students. One lesson was devoted to solving the type 1 task *Ideal purchase of the bus fleet* in the form of executing the actions. The next class brought up the actions carried out previously and their sequencing in written form, with the product being the modeling guidance card for future reference.

The students' performance in solving the student problem and composing the real-time guidance actions allowed the teacher-researcher to observe, analyze and arrive at some results. Regarding the presence of contradiction for the development of learning (an aspect of Majmutov's student problem), the students overcame the conflict between the means of action required by the problem and the means of action they could carry out.

Collaboratively, the students determined the guiding base in its external material form by solving a detailed modeling problem. At times, the teacher-researcher induced the process and collaborated with the students' construction of the OBA, adjusting and correcting mistakes (D4 of Talízina's study direction):

> It's not a question of the student discovering the essence of the theoretical concept, the appropriate action and the necessary conditions that make up the type III orientation on their own, but of working it out with the help and collaboration of the teacher and peers in the context of a given Zone of Proximal Development (ZPD) (Núñez, Ramalho and Oliveira, 2020, p. 118).

The students' actions are still shared, dependent and poorly assimilated. Therefore, given these results, it can be said that the participants reached Galperin's first stage, with the understanding of the actions to solve the student's modeling problem in its practical and then reflective form.

Once the stage of understanding *what to do* had begun, the SPSA of MM proceeded to the next stage, the formation of the action in its materialized form, that is, *the doing*. The materialized action is associated with a hectic time in MM's SPSA, searching for elements favorable to the solution, consulting, discarding hypotheses, making mistakes, manipulating technological resources, experimenting, observing, correcting, making suggestions, among other aspects.

The class solving a type II task, *Planting potatoes*[8], was the initial milestone in the conscious and controlled execution of actions. The teacher-researcher chose to demonstrate the use of the guidance card so that when the students received future tasks, they would use this action and control scheme. Collaboratively, the actions were carried out in detail.

In the following lesson, the teacher-researcher distributed to the class selected type 1 tasks for the *start of* the guiding activity, in pairs, to be handed in written form and defended (orally) in the following lesson. With the support of the guidance card, each student was assigned to solve a task and to help check that their colleague's actions had been complied with when solving another task, and vice versa. This allowed the students to work individually and at the same time together. Regarding the first three actions, students P1, P3, P6, P2, Q1, Q6, Q4 and Q8 showed correct execution and explanation of these actions. Students P4, P5, Q3, Q5 and Q7 failed two operations, requiring the intervention of the teacher-researcher at the time of the commented resolution. In addition, 11 students were able to comment on and improve their answers, extracting elements from the solution that contributes to effective decision-making. This was evidence of the last operation of the solution analysis action, which had not been carried out in any of the tasks answered by the students previously.

It's worth noting that carrying out the operations set out on the guidance card caused concern among the participants, who at that moment were faced with a new (or updated) organization and guidance for solving the students' problem. Given Talízina's direction of study, mistakes were corrected to move on to the next stages. In terms of analyzing the qualities of the actions, the first task gave the students

[8] Adapted from BIEMBENGUT, M. S. Modelagem na Educação Matemática e na Ciência. São Paulo: Livraria da Física, 2016.

awareness, *how to do things* by experimenting, *how to do things* in a shared collaborative way.

For the next tasks, the formation of the materialized action and the formation of the action in external verbal language were also aligned as in the previous task, as recommended by Núñez and Pacheco (1998, p. 104), "the material or materialized form is combined, from the beginning, with the verbal one; the students formulate in external language everything they do materially". Therefore, at the end of each task in the materialized stage, the students verbalized their resolutions in written form.

Next, the type 2 task performed by the undergraduates was T1 - *The Frog's Leap*[9]. Considering all the participants, six students (43%) achieved the sum of the actions in the category of [16, 20] and obtained a satisfactory level of performance. The other part of the students (43%) is in the [12, 16] category and the remaining 12% represent those who didn't do the task (P2 and P5).

From this, it is possible to make some observations regarding the qualitative indicators: P1, P3, P6, P4 from G1, correctly perform the first three actions and are already trying to validate the model using other confirmatory methods; each student has their own resolution which differs from that of their classmates; they explain all the operations.

In addition, students Q1, Q4 and Q8 show independence, initiative and clarity in the arguments used in their solutions. On the other hand, Q1 still has flaws in the operations relating to the 3rd action and students Q3, Q5, Q6 and Q7 have similar writing, sharing the conceptual core of the other colleagues. In addition, all the participants in G1 and G2 consulted or even wrote down the actions on the guidance card at each moment of execution.

Despite the partial results, Group 1 was directed towards tasks with contradictions arising from investigation, experiments, simulations, confirmatory bias and critical analysis of the solution, elements characteristic of type 3 tasks. On the other hand, in Group 2, in order to reduce sharing, the tasks were individual and the verbal action was written down.

[9] Adapted from ALVES, F. J. C.; SÁ, P. F. Modelagem matemática no ensino de funções. Belém: UDUEPA, 2018.

In general, to comply with operation 6 of the SCOBA to update other known concepts and procedures that may be linked to the unknown, the student was allowed to consult the teacher or books, virtual and/or physical, to recall some concept already studied. As encouraged from the start of the MM SPSA, the participant could use mathematical *software* to help build the models. Throughout the activity, the teacher was attentive to the qualitative indicators of the students' actions.

Although they didn't carry out a task with as many contradictions as G1, Group 2 solved a final diagnostic test (PDF) specific to each student's zone of proximal development. During the test, there was no consultation or use of the guidance card, precisely to check that the student would internalize the actions. As a result, they would face the contradictions of a new student problem.

The results are presented separately by group. For a specific look directly at the skills required, Table 2 refers to the averages of the actions carried out by G1 and G2 students in the MM SPSA. The results show that the action of formulating the student problem was carried out correctly by the students who took part in all the tasks.

Table 2 - Averages of the actions of G1 and G2 in the SPSA of MM

STUDENTS	A1ªA	A2ªA	A3ªA	A4ªA	Total
P1	5	5	5	2	17
P3	5	5	5	3,25	18,25
P6	5	5	5	3,25	18,25
P2	4	4	4	1,75	13,75
P4	4,75	5	3,75	1,75	15,25
STUDENTS	A1ªA	A2ªA	A3ªA	A4ªA	T.A
Q1	4,4	4,4	2,8	3	14,6
P5	5	4,4	4,4	1,8	15,6
Q3	5	3,8	3,6	2,4	14,8
Q4	5	4,5	4,33	2,83	16,66
Q5	5	4,6	3	1,4	14
Q6	5	3,6	3	2,6	14,2
Q7	5	3	2,8	1,2	12
Q8	5	4,4	4,2	3	16,6

A1ªA: Average of the first action; A2ªA: Average of the second action; A3ªA: Average of the third action; A4ªA: Average of the fourth action; TA: Total of the averages.

Source: Research data

Fulfillment of the 1stA reflects positively on the selection of concepts, procedures and choice of strategy for solving the student's problem, i.e. the 2ndA. Group 1 had an excellent performance up to the 3rdA. It is worth remembering that participant P2 did not hand in task T1 and P4 encountered obstacles when solving the task in pairs. Student participants Q3, Q6 and Q7 found it more difficult to associate the concepts and procedures with what they were trying to achieve, compromising the rest of the resolution.

Failure to check the solution effectively is still a hindrance to the correct construction of models. Some students have found a model, but the lack of error investigation makes validation impossible, i.e. the model found does not fit the conditions and data of the problem.

To help with the analysis, a quantitative comparison was made of the results obtained in the initial diagnosis with the results of MM's SPSA. Table 3 shows the sum and overall average of the levels achieved in the four tasks of the diagnostic test (M.G.PDI), as well as all the tasks proposed (M.G.T) in the MM SPSA, including the final pedagogical test in the case of G2. The positive variations of all the students are noted, the averages achieved at the end of the MM SPSA are higher than the averages of the initial diagnostic test. The only case of negative variation in both groups is P2, due to not performing some control operations and not handing in the T1 task.

Table 3 - Averages of PDI and MM SPSA tasks per student in G1 and G2

STUDENTS	T.M.PDI	M.G.PDI	T.M	M.G.T
P1	10,5	2,625	17	4,25
P3	10,8	2,7	18,25	4,56
P6	10	2,5	18,25	4,56
P2	16	4	13,75	3,43
P4	11,5	2,875	15,25	3,81
STUDENTS	**T.M.PDI**	**M.G.PDI**	**T.M**	**M.G.T**
Q1	8,5	2,1	14,6	3,65
P5	10,5	2,6	15,6	3,9
Q3	10,6	2,7	14,8	3,7
Q4	9,8	2,5	16,66	4,16
Q5	10,8	2,7	14	3,5
Q6	11,1	2,8	14,2	3,55
Q7	7,0	1,8	12	3
Q8	15,8	4,0	16,6	4,15

T.M.PDI: Total average of the tasks in the initial diagnostic test, M.G.PDI: Overall average of the tasks in the initial diagnostic test, T.M: Total average of the tasks in the SPSA, M.G.T: Overall average of the tasks in the SPSA.

Source: Research data

The information filled in by the teacher-researcher in the observation guides, lesson by lesson, helped to verify the qualitative indicators of the participants' actions in MM's SPSA. These indicators consist of independence, generality, explanation, assimilation and form.

In this context, it should be noted that these indicators alone do not express great significance. According to Núñez, Ramalho and Albino (2013), qualitative indicators should be understood as a subsystem, in which different elements are integrated to gain dimensions that they do not have separately and express qualitative transformations in the process of skill formation. By cross-referencing this information, the shape of the orientation linked to the mental stage reached by each student was estimated.

Table 4 - Form of action internalized by G1 and G2 students

STUDENTS	I	G	E	A	F
P1	Partial	Partial	Not much	Partial	External verbal
P3	Total	Total	Not much	Total	Verbal external to you
P6	Total	Total	Not much	Total	Verbal external to you
P2	Total	Partial/Total	Partial/not much	Partial/Total	External verbal
P4	Partial	Partial	not much	Partial	External verbal
STUDENTS	**I**	**G**	**E**	**A**	**F**
Q1	Partial	Partial	Total/Partial	Partial	External verbal
P5	Partial	Partial	Partially	Partial	External verbal
Q3	Partial	Partial	Partially	Partial	External verbal
Q4	Total	Total	not much	Total	Verbal external to you
Q5	Partial	not much /Partial	not much /Partial	not much /Partial	Materialized
Q6	Partial	not much /Partial	Total/Partial	not much /Partial	Materialized
Q7	not much	not much	Total/Partial	not much	Materialized
Q8	Total	Partial	not much	Partial	External verbal

I: Independence; G: Generalization; E: Explanation; A: Assimilation; F: Form

Source: Research data

To get as close as possible to the level of internalization of the actions materialized in the modelling SCOBA by the students, it is estimated that P2's guiding base is moving towards the external language stage for him because the 3rd and 4th actions have not yet been assimilated effectively. Students P1 and P4 are at the independent stage of materialized OBA, but it is crucial to achieve the generality of SCOBA, which is why they are at the external verbal stage.

For P3 and P6, confirmation of the mental stage was reinforced in the tasks carried out in the following academic term. The students in question used the previously assimilated schema to direct their actions in solving the two proposed problem situations independently. There are strong indications that these students' orientation reached the mental stage, forming part of their thinking, as an ability to solve mathematical modeling student problems, within the limits established.

In the case of Q5 and Q6, they are still in the process of materializing the actions, depending on the guidance and sharing card. However, it was possible to observe some abbreviations and the assimilation of the 1st and 2nd actions, so they are moving towards external verbal action. Participant Q8 showed the potential to form an external language for himself, but needs to achieve a greater degree of generalization.

Overcoming the contradictions of the type 2 tasks indicates assimilation of the actions of the modeling SCOBA, but does not cover the full range. In other words, the conscious actions of students Q1, P5 and Q3 were correctly expressed in external verbal language. With a high degree of dependence and little assimilation, Q7's guidance took place with materialized external help.

At the start of the internalization process, participant Q4 had orientation failures like all the students in the group, even with the support of the materialized OBA. However, over the course of the Galperin stages, he showed signs of assimilating the four actions of the SCOBA when performing the last task (T4) and, in the final diagnostic test, he showed generalized orientation in the face of a new problem situation. It is therefore estimated that Q4's orientation lies in the external language for himself.

7 Contributions and points for improvement

Looking at the mental action stage they were in at the end of the MM SPSA, 23% of the students, Q5, Q6 and Q7 had reached the materialized form of the actions, i.e. the part of the action that remains related to their representations. The starting point for continuing to build skills has been consolidated, since they have fully experienced the stage of *doing* and *carrying out* the tasks with problem situations with some of the contradictions expected for modeling practice, even with the support of their classmates and the teacher-researcher, remaining in the zone of proximal development.

Most of the research participants (54%): P1, P2, P4, P5, Q1, Q3 and Q8 have developed external verbal language. There is no longer a special movement from one material part to another, there is the *ability to explain* the actions carried out. In terms of the actual level of development, the students in question are able to start a new stage materialized with type 3 modeling tasks. In addition, the students have assimilated the action of formulating the student's problem based on new data and new well-defined conditions in the task, which presupposes fewer investigative and exploratory traits, but they are already showing gradual autonomy and effective assimilation.

Participants Q4, P3 and P6 (23% of the students), whose actions acquired the deepest generalization, shortened and reached the stage of forming the action into an external language for themselves. The cognitive activity of these students encompasses all the stages of the process of solving a problem, which they have already formulated, through independent analysis of the problem situations.

Therefore, due to the awareness shown in the explanation, the extent of the generalization and the solidity, the three students have appropriated the new type 3 orientation, which is now represented as an act of scientific thinking and as a skill in solving a Mathematical Modelling student problem.

In relation to the SCOBA actions, the fourth action proved to be the most challenging and one of the possible reasons is the addition of an action to the resolution scheme with no links to previous experiences, unlike the other actions, which seem more familiar to the students. Difficulty in analyzing the solution is not an isolated case in this research. At the end of his study on the process of learning to orient the control action in solving mathematical problems, Gonçalves (2020, p. 185) stresses that “the gaps in students' training or their inexperience with conscious control actions, based on systematized guidelines, indicate the urgent need to introduce learning control from the earliest levels of education”.

Despite the tasks, the development of the type 3 task requires time, knowledge of the functionality of computer resources and considerable experience in type 2 tasks. This led to a possible misunderstanding in the indication of the T6 task by students P1 and P4 in the IEM II class.

With regard to the dialectical relationship between teaching and learning in which the SPSA of MM is constituted, we agree with Núñez, Ramalho and Oliveira (2020), despite Galperin's theory, that this process does not have a linear format as many studies suggest, but “in a form that interweaves different stages marked by contradictions and conflicts, reaching a higher plane, an indicator of human development” (p. 128). These contradictions are not equally subjective for all students. However, according to the students, the discrepancies in learning could be greater if the tasks were more challenging than necessary.

Thus, it can be concluded that the MM SPSA developed in the mathematics undergraduate classes contributed to the formation of the skills required in the mathematical modeling process. In any case, it indicates an improvement in the formation of the skills of formulating the student's problem, building the conceptual and procedural core, solving the student's problem and analyzing the solution.

8 Considerations

This study, based on psychological and didactic foundations, articulated Galperin's conductive psychology in the formation of mental actions in stages with Talízina's didactics of directing the activity of study and the contradictions imposed by Majmutov's student problem.

The undergraduates consciously experienced the practice *of* modeling. Three of them have acquired the skills, internalizing the actions of the MM SCOBA and being able to improve their guiding bases with each new problem situation they face. The others can continue to solve the tasks, under the guidance of the materialized OBA, until the desired skills are achieved.

From the above, the study carried out can serve as a model to be implemented by educators whose interest is generalized and complete guidance for their students, and consequently develop independence and skills in the modeling process, with the appropriate adaptations.

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