OTHER THEMES

Connecting Gestures and Images with Concepts in Geometry

José Carlos Pinto Leivas[I]
Carmen Vieira Mathias[II]
José Anchieta da Silva[II]

[I]Universidade Franciscana (UFN), Santa Maria/RS – Brazil
[II]Universidade Federal de Santa Maria (UFSM), Santa Maria/RS – Brazil

ABSTRACT – Connecting Gestures and Images with Concepts in Geometry. The research aimed to analyze how students identified concepts of Euclidean geometric entities, their properties, and relations, through gestures with the use of a game. The investigation is characterized, regarding technical procedures, as participant research with a qualitative approach and was carried out with high school students in a private school. The analyses performed, via categorization of the gestures used, indicated that students produce gestures more efficiently when they have a mental image of the geometric entity to be represented. The research also showed that those who play the role of observers in the game, identify the gesture produced when they have knowledge of the concept attached to it.
Keywords: **Geometry. Gestures. Games.**

RESUMO – Conectando Gestos e Imagens com Conceitos em Geometria. A pesquisa teve como objetivo analisar como os estudantes identificaram conceitos, propriedades e relações de entes geométricos euclidianos, por meio de gestos com o uso de um jogo. A investigação caracteriza-se, quanto aos procedimentos técnicos, como uma pesquisa participante com abordagem qualitativa e foi realizada com alunos do Ensino Médio em uma escola privada. As análises realizadas, via categorização dos gestos utilizados, indicaram que os alunos produzem gestos de forma mais eficiente quando possuem uma imagem mental do ente geométrico a ser representado. A pesquisa também evidenciou que, aqueles que desempenham o papel de observadores no jogo, identificam o gesto produzido quando possuem conhecimento do conceito atrelado a ele.
Palavras-chave: **Geometria. Gestos. Jogos.**

Introduction

This text starts by reviewing the development of Geometry over centuries, even millennia, to reach the current stage of civilization. It is necessary, therefore, to evoke Euclid and his formalization of this precious area of knowledge for humankind. The preface of the direct translation from Hebrew by Irineu Bicudo into Portuguese (Bicudo, 2009) indicates the guiding spirit of the work: "In fact, Euclid's practice often contemplates concision - for example, instead of 'the square on AB (i.e., of side AB)' he most often says 'the on AB': and, 'the by AB, CD', instead of 'the rectangle contained by AB, CD (i.e., of sides AB, CD)'" (p. 12). This is to exemplify the language employed at the time, very hermetic.

This form of language gradually changed to meet new ways of reading, reflecting, and apprehending the mathematical meanings thought by Euclid. The need to understand the space in which we live brings out new questions, theories/conceptions, in this case, for Geometry. In this direction, Mlodinow (2010, p. 10) raises several questions about the nature of space, among which we extract: [...] "Can space be curved? How many dimensions are there? How does geometry explain the natural order and unity of the Cosmos?" For this author, such questions "lie behind the five geometric revolutions in world history" (Mlodinow, 2010, p. 10).

Descartes' idea of organizing Analytic Geometry, assigning coordinates to points, allowed him to play with the places and geometric shapes until then arising from the Euclidean system, mixing numbers, letters, and Geometry itself. With this, Differential and Integral Calculus, for example, was developed, until reaching the technological modernisms.

Differential Geometry involves Geometry and Calculus, and we start formulating Non-Euclidean Geometries: Hyperbolic and Elliptic, which can be observed from the curvature of a surface. In this direction, the surfaces of constant Gaussian curvature have the spherical surface, of positive curvature, the pseudosphere, of negative curvature and the Euclidean plane, whose curvature is zero, as prototype. In these geometric places, new conceptions of lines arise, for example, the parallel lines. One of the foundations of Euclidean Plane Geometry arises from the Axiom defined by Euclid, stating that through a line and a point outside of it, there is a single line that passes parallel to it. This will no longer occur in Spherical Geometry, where there is no such parallelism, because the lines are the maximum circumferences of the spherical surface, called geodesics. In Hyperbolic Geometry, there may be more than one parallel through a point not belonging to a line, models such as Klein's and Poincaré's illustrate such a property.

With Cartesianism, the question of the number of coordinates of a point is solved, but its interpretation does not go beyond the real world in three dimensions, even if they are given by integers. Even a philosopher speculated about the existence of the fourth dimension, the *habi-*

tat of ghosts, which would see 3D beings, while the opposite could not happen. By analogy, the beings that inhabit the 2D world would not perceive the 3D beings, while the latter would see them flattened, similar to what happens on movie screens in 3D movies.

But beyond whole dimensions, even larger than three, the fractal dimension has emerged, giving rise to the so-called Fractal Geometry. Although many express these new geometries as discoveries, here we understand them as creations, since they are based with their mathematical logic, with axioms, theorems, and well-defined relations. One could go on and on about human creations in the field of Geometry, which is not the subject of this article. The intention is to reach what is discussed today, in terms of Mathematics Education and teaching in general, as inclusion, since gestures will be explored in the teaching of Geometry, by means of a game that involves geometric concepts.

Both in initial and continuing education, teachers need to be prepared to encounter students in their classroom, with hearing impairment or without any possibility of hearing them, for example. Therefore, they need to use other teaching resources to be able to communicate beyond the Brazilian Sign Language (LIBRAS), rarely found in their initial training, although there is a compulsory discipline of LIBRAS in the current curricula of undergraduate courses, in general, this is insufficient. The question is, how to make this communication in Geometry effective, with rare symbols that can help them understand this area?

According to Thompson and Rosch (1991), a new paradigm has emerged in recent decades, challenging Cartesian assumptions of early cognitive science. The paradigm in question recognizes the embodied nature of cognition, involving the sensorimotor as a key element in.

> [...] Embedded implies the following: (1) cognition dependent on the kinds of experience resulting from a body with sensory-motor capacities; and (2) individual sensory-motor capacities that are themselves embedded in a broader biological and cultural context [...] sensory and motor processes, perception, and action, are fundamentally, inseparable in experienced cognition and not merely contingently linked as input/output pairs (Varela, 1999, p. 12).

It is believed that, based on this idea, it is possible to combine new possibilities for teaching and learning Geometry, especially in inclusive classrooms. In this sense, the XXX group has been conducting research on the theme of gestures in Geometry. In the year 2019, the group focused on the game *Imagem & Ação*[1] (Image and Action), creating the so-called *Geometria & Ação* (Geometry and Action), which will be described below, as well as its validation with students of various grade levels.

Thus, the research addressed in this article is justified, and aimed to analyze how high school students identified concepts, properties, and relationships of Euclidean geometric entities through gestures with the use of the game Geometry in Action.

Literature review

McNeill (2005) classifies gestures as hand and arm movements with symbolic characteristics, intricately linked or even produced simultaneously with spoken language, called co-speech gestures. Depending on the situation, context, and counterpart, a gesture can be understood as deliberately expressive. In this case, it is distinguished according to the interpreter, making the act of gesturing a social endeavor (Kendon, 2004). In either case, gestures are dissociated from self-touch, such as nose scratching, object manipulation, expressions of affection, and sign languages.

According to Goldin-Meadow (2006), there are occasions when speech is impossible, such as when noise makes it hard to hear; rules require silence, or no one uses ordinary spoken language. Under such circumstances, speakers use their hands to develop sign systems. Thus, gestures can fulfill different purposes in a person's utterance. They can: replace verbal expressions; be used as a support in a communication process; or be independent carriers of meaning, eventually interacting with verbal utterances (Clement, 2008).

According to Kita and Oëzyuërek (2003), gestures arise from mental spatial or motor representations that are interconnected with the mental representations underlying speech. This perspective has recently been presented by Hostetter and Alibali (2008, p.721) as "Gesture as Simulated Action" (GSA). This framework holds that both gesture and speech are based on simulated actions and perceptual states, activating premotor and motor areas of the brain. Sometimes, individuals are unable to inhibit this motor activation and, as a result, produce gestures (Hostetter and Alibali, 2008).

From this perspective, Alibali *et al.* (2014, p. 69) state that

> [...] spekears produce gesture as a natural part of their efforts to communicate intended meanings that have perceptual and action-based elements. Simulated actions and perceptual states play a central role in the GSA framework. This view is aligned with the embodied cognition perspective, which holds that human cognitive processes have their roots in action and perception.

Pier *et al.* (2014), for example, describe an individual who produced a gesture using both hands to simulate two gears moving in opposite directions. From his gestures, it is possible to infer that this individual mentally simulated the action of turning a gear.

The same authors state that several research papers highlight ways in which mathematical cognition is embodied or formulated through perception and action. Such research specifically identifies gesture as an important cognitive resource for mathematics learning.

In mathematics classrooms, gestures are used spontaneously to connect disparate mathematical representations and ideas (Nathan *et al.*, 2013). Or, to promote sharing common ground with students and foster mathematical fundamentals among the class as a whole (Alibali *et al.*, 2019).

According to Novack and Goldin-Meadow (2015, p. 405)

> [...] a learner's gestures can index moments of conceptual instability, and teachers can make use of those gestures to gain access into a student's thinking. Learners can also discover novel ideas from the gestures they produce during a lesson or from the gestures they see their teachers produce. Gestures thus has the power not only to reflect a learner's understanding of a problem but also to change that understanding.

Thinking about teaching and learning in mathematics, gestures can be seen as the embodied movements that create contours of geometric shapes or portray concepts such as tilts or angles through representations of the hands or body (Melcer; Isbister, 2016).

In this same direction, Congdon *et al.* (2013) point to evidence that students retain and generalize what they learn in a mathematics lesson better when they receive instructions that contain simultaneous speech and gesture than when they receive instructions that contain sequential speech and gesture. Thus, they notice that gesture synchronized with speech has the ability to promote learning that endures and can be generalized.

Walkington *et al.* (2014) point out an important difference between two types of depictive gesture. According to the authors,

> [...] in static depictive gestures, problem-solvers represent an object (like a triangle or line segment), but do not attempt to directly act upon that object. The gesture shows a static representation of a single object that is not interacting with other objects. In dynamic depictive gestures, problem-solvers first represent an object, and then engage in fluid transformations of that object using the affordances of their body. For example, a problem-solver might 'collapse' a triangle formed with their hands into two line segments on top of each other, or create a rectangle with their hands that 'grows' as move their hands outwards (Walkington *et al.* 2014, p. 480).

Also, according to Walkington *et al.* (2014, p. 481), "[...] the physical action of gesturing results in and initiates cognitive states. Therefore, performing dynamic gestures with the body can be a by-product of reasoning processes and give rise to new ideas".

Such reasoning processes are particularly important in the ways of teaching and learning. In particular, with regard to Geometry, the Common National Curricular Base (BNCC), points out that it involves a large set of concepts and procedures. These are fundamental for solving physical problems in various areas of knowledge. In addition, "[...] studying position and displacements in space, shapes, and relationships between elements of plane and spatial figures can develop students' geometric thinking [...]" (Brasil, 2017, p. 271).

As for High School education, students in all three years are expected to develop the following general competence:

> [...] use different languages - verbal (oral or visual-motor, such as libras, and written), corporal, visual, sound, and digital - as well as knowledge of artistic, mathematical, and scientific languages, to express themselves and share information, experiences, ideas, and feelings in different contexts and produce meanings that lead to mutual understanding (Brasil, 2017, p. 9).

Regarding spatial perception skills, Del Grande (1994) indicates categories formulated by Frostig and Horne (1964) and Hoffer[2] (1977), among which are: visual-motor coordination, visual discrimination, and visual memory. It is understood that these abilities can be observed, and even developed, by means of 'gestures'. For these authors, the first skill is defined as "coordinating vision with body movement" (Del Grande, 1994, p. 158), which directly refers to the act of gesturing certain concepts, as will be seen in the analysis ahead in this article.

As for visual discrimination, this is defined as "the ability to distinguish similarities between objects" (Hoffer, 1977 apud Del Grande, 1994, p. 159). In this sense, the gesture to discover the concept involved allows the game "*Geometria em Ação*" to verify the connection between what is gestured and what is verbalized.

Finally, the same author defines visual memory as the ability to "accurately remember an object that is no longer in sight and relate its characteristics to other objects, whether they are in sight or not" (Del Grande, 1994, p. 159). This occurs in the mentioned game, from the moment students evoke concepts that were studied in previous years.

Thus, it is understood that such skills can be perceived among the interlocutors of a game that explores gestures involving geometric concepts. These gestures are produced by an individual to be interpreted by a group of competitors and reproduced orally by another individual who interprets them.

Blanco's (2014) research with future mathematics teachers sought to diagnose, in some geometric activities, visualization skills to obtain problem solutions. For example, when cutting a cube through a plane that intersects it at the three edges that compete at a vertex, the author asks how many vertices the new solid would have. The author indicates there is the content task of the problem itself and that of cutting the solid at a given distance from the vertex. She states that, although it is not necessary to identify the section produced, because the image is already presented in the figure of the statement itself, it constitutes a visual discrimination skill. On the other hand, it is possible that the action of counting the vertices of the new figure requires a mental image of the resulting body, to be mobilized during the solving process. Next, the methodological procedures of the research are presented.

Methodological Procedures

According to Severino (2016, p. 23), "The University, in its deepest sense, should be understood as an entity that, as a knowledge worker, is intended to provide service to society in the context in which it is located". Based on this contextualization advocated by the author, this work was carried out by a group of studies and research in Geometry teaching in a Graduate Program in Science and Mathematics Teaching, involving undergraduates in Mathematics, teachers working in basic education, masters and masters' students, doctors and doctoral students. This strengthens what the author indicates about the role that the University should play in the community in which it is inserted.

Thus, as the group goes to the school, experiments its findings, and investigates its results, qualitative research is involved in the process. Qualitative research proposes the direct and prolonged contact of researchers with the environment and the investigated situation, with data produced mostly in a descriptive way (Lüdke; André, 1986).

From this point on, a first contact with the school was established through a high school mathematics teacher, presenting the research proposal. The teacher presented the project to the pedagogical coordination of the institution. The research group, together with the pedagogical coordinator and the high school teachers, found out that there was a possibility of working with two third grade classes.

As the class teacher is part of the research group, this investigation is characterized, in terms of its technical procedures, as participant research. This type of research "[...] is one in which to carry out the observation of phenomena, the researcher shares the experience of the researched subjects, participating in a systematic and permanent way, throughout the research time of their activities" (Severino, 2016, p. 126).

This research meets the author's indications, as far as it applies the game Geometria & Ação, created by the group, in two third-year high school classes. The material was implemented on the eve of the National High School Exam (ENEM), in the same school, located in a city in the central region of Rio Grande do Sul.

The game that will be described next required the researchers to control the groups during their dispute, to dictate the rules, write down scores and correct answers. The process was recorded in photos and the gestures produced by the students were filmed in an attempt to get one of the group members to verbalize the concept gestured, which corresponded to a card that the gesturer would draw from the pile, without the others noticing.

Therefore, the participation of the researchers involved needs to be collaborative, for each one must pay attention to a detail or group during the game. For this reason, the activity was carried out by the three authors of the article. Furthermore, in the first class, 32 students participated, which were divided into four groups, while in the second, there were 17 participants, divided into two groups.

Records were made by filming the participants, and a questionnaire consisting of ten open questions was applied. This questionnaire was printed and distributed to the students, in order to understand the positioning of these participants before the experience and to evaluate the potential of the game.

Filming is necessary because it clarifies to the researcher some dubious details in their observations, especially due to their involvement during the use of the game, which may cause a movement or 'indiscipline' in the classroom. Regarding this term, Parrad-Dayan (2008) indicates that violence and indiscipline are not synonymous. The author points out at least three types of indiscipline in the school environment, among them, "[...] the third type of indiscipline is a protest against the rules and ways of working. There is an implicit contract that operates in a class without the students' opinions being considered and should be pointed out" (Parrad-Dayan, 2008, p. 27).

In this sense, play, as a classroom activity, is not within what the traditional school points to as "an ordered and controlled space, where the different activities are ritualized. Movement, the act of taking the floor, studying, etc. respond to structured routines" (Parrad-Dayan, p. 28). In contrast, "the new school allows democratic school coexistence, which does not exclude the possibility of the emergence of conflict situations" (Parrad-Dayan, 2008, p. 28).

The activity carried out follows the guidelines of the second kind of school, since the students spontaneously participated and were active in carrying out the activities. Initially, some were a bit inhibited, in particular the student who had been chosen to remove the card and make the gesture that would translate the concept involved in it, for discovery by his group mate. As the moves progressed, the participants felt comfortable to be 'unruly' in the game.

The game is a powerful resource for providing such 'indiscipline' within the classroom. However, many discussions emerge among educators about the role of the game as well as the manual, since both refer to "the lack of clarity in the explanation, which is understandable because the author often presumes an obviousness in some respects that are not obvious to some readers. Therefore, the elaborate description of each game is detailed in a way that is sometimes even exhaustive. The ultimate goal is good understanding on the part of the reader" (Jurado; Nieta, 2006, p. 13). Therefore, besides paying attention to the issue of 'indiscipline', this research also explores the game adapted for a good understanding of Geometry, in order to remedy any deficiencies found, contributing to a satisfactory geometric learning.

Regarding the data collection for the subsequent analysis, especially because of the exuberance of the individuals' manifestations during the game, the use of current resources, such as the video camera and the cell phone itself, became important.

> Today, with professional video recorders or even cell phones and tablets, the ability to video record, the unveiling of sounds and images of a phenomenon, moment by moment, has become a broad and powerful tool of the Mathematics Education research community (Powell; Silva, 2015, p. 19).

In fact, in the case of this research, this resource allows the researcher to monitor the gestures produced by an individual, so that their group partner can provide visual aids that allow them to identify what concept the person is trying to communicate. In this sense, images can indeed be significant for the desired analysis and indication of the data related to research (Powell; Silva, 2015).

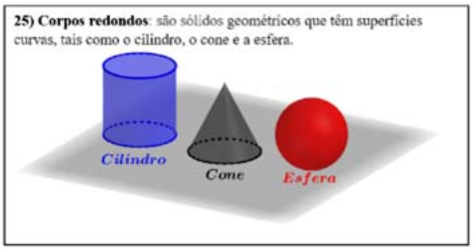
The game developed by the group, called Geometria & Ação, consists of a board, pawns, dice, 102 cards containing concepts, properties and important results involving geometric topics that are part of the High School programs. The game also has two glossaries, one of which only contains definitions, and the second contains the definition and image of the concept. Figure 1 illustrates the board and some cards.

Figure 1 – Board and some cards

Source: Research Group File.

Figure 2 illustrates a glossary item, consisting of the definition and the image. The difference between the two is that when consulting the glossary of definitions, the team that correctly guesses the gesture performed by the person who gesticulates loses one point from the number that is rolled on the dice (and represents the number of spaces they should advance), if they consult the glossary of definitions, and two points if they consult the glossary of images.

Figure 2 – A Picture Glossary Item



Source: Research Group File.

The analysis of the gestures made by the high school students during the game in this article is limited to the concepts that were drawn and that resulted in a need by the student to resort to the glossaries.

Results and Discussions

As already announced, the research was conducted right before ENEM-2019, with two Senior High School classes, in a private school in the central region of Rio Grande do Sul. The first had 14 participants, while the second had 17. At first, the researchers observed a certain shyness from the first group, while those in the second group became more agitated and excited as the competition became more intense. Figure 3 illustrates some of the situations in the development of the game.

Figure 3 – Disputes and Gestures

Source: Research Data.

The importance of the moment of application of the research is considered, especially in the verification of what is recommended by the BNCC, both for Elementary and High School Education. In the first year of High School, the BNCC brings the following object of knowledge for the thematic unit Geometry: "Spatial geometric figures: recognition and relations with familiar objects in the physical world". It also indicated the following skill required in this object: "Relate spatial geometric figures (cones, cylinders, spheres, and rectangular blocks) to familiar objects of the physical world" (Brasil, 2017, p. 279). Thus, the fact that the game Geometria em Ação can provide students at the end of high school a resumption of these contents is considered relevant. Moreover, a searching in one's memory for elements that can be gesticulated, so that the concept involved can be verbalized, indicates the relevance of the game for geometric learning.

Allied to what is suggested for the Elementary and High School Level, the following general competence is established in the BNCC:

> […] use different languages - verbal (oral or visual-motor, such as libras, and written), body, visual, sound, and digital - as well as knowledge of artistic, mathematical, and scientific languages, to express themselves and share information, experiences, ideas, and feelings in different contexts and produce meanings that lead to mutual understanding (Brasil, 2017, p. 541).

To fulfill such an objective, the following skill is also highlighted from the same document:

> To use the notions of isometric transformations (their translation, reflection, rotation, and compositions) and homothetic transformations to build figures and analyze elements of nature and different human productions (fractals, civil constructions, works of art, among others) (Brasil, 2017, p. 533).

Based on these two highlighted indications, we highlight the importance of analyzing how the game is promoted, in order to realize its relevance for the moment and for the individuals participating in the activity (the game was applied a week before ENEM).

One group was filmed at a time for the analysis of the activity and, thus, it was not possible to effectively monitor all the concepts drawn and, consequently, the gestures performed.

Table 1 summarizes the gestures that were analyzed from the videos, for which gestures there was or there was not a request for the glossary by the participants who would perform them, and whether the gestures were (or were not) identified by the group.

Table 1 – Synthesis of the gestures drawn

Drawn concept	Glossary of concept with image	Gesture identified by the group
Angle	No	Yes
Supplementary angle	No	Yes
Area	No	Yes
Barycenter	Yes	Yes
Circle	No	Yes
String	No	Yes
Round bodies	No	Yes
Generatrix	Yes	Yes
Grade	No	Yes
Hypotenuse	No	Yes
Laterality	No	Yes
Mediatrix	Yes	No
Square base pyramid	No	Yes
Inscribed polygon	Yes	No
Spot	No	Yes
Midpoint	No	Yes
Hexagonal based prism	No	Yes
Translation	Yes	No
Triangle	Yes	Yes
Cone drome	No	No
Vector	No	No

Source: The Authors.

For the analysis of the gestures, we chose to investigate those in which students requested the glossary. Thus, we will consider those corresponding to the concepts of translation, mediatrix, barycenter, inscribed polygon and generatrix. We categorized the selected gestures that represented the drawn concepts as static or dynamic gesture, according to the definition given in Walkington *et al.* (2014).

The first selected concept that students requested the glossary for was that of translation. It is noted that this is a term used in other areas of knowledge, such as physics, for example. Even though it is a common term, there was an insecurity factor, which made the student resort to the glossary to remember it. Figure 4 illustrates a sequence of gestures made by the participant.

Figure 4 – Gesture representing the concept translation

Source: Research Data.

In this case, it is possible to observe that there was the idea of movement, meaning, a dynamic gesture occurred. The set of images in Figure 4 only illustrates some of the gestures made by the participant, who moved her hands to produce a certain gesture in the air, using her body to move and repeating the same gesture with her hands. Since the concept was not known by the participant, it can be considered complex for her to reproduce. In this context, Melinger and Kita (2007) claim that speakers produce more gestures at times of high conceptual load (e.g., when they were asked to describe an image with greater complexity) than for those of low conceptual load (when they were asked to represent an image with less complexity).

According to Kita (2000), the production of representational gestures helps speakers organize visuospatial information and occur more frequently with descriptions of increasing complexity. This is exactly what happened with the representation of the concept of the perpendicular bisector. When performing the gestural representation of this concept, so that the classmates could understand it, the student first expressed the idea of segment, then of half, and finally the idea of perpendicularity, as shown in Figure 5.

Figure 5 – Gesture representing the concept of the perpendicular bisector

Source: Research Data.

Despite the movement made, this gesture can be considered static, since it fragments the concepts. This may be the reason, even with all the student's effort, her classmates were unable to identify the concept suggested by the gesture. The same occurred with the concept related to the barycenter. Figure 6 illustrates part of the gestures produced by the student.

Figure 6 – Gesture representing the concept barycenter

Source: Research Data.

In this representation, the student drew a triangle in the air, pointing to its center. The classmates began to risk possible associated concepts, such as triangle, midpoint, circumcenter, incenter, among others. Almost immediately, someone said barycenter, correctly saying the concept that was being represented through the dynamic gesture produced by the colleague. It is observed that the gesture, in the specific case of the concept of barycenter, was an incorporated movement, in the concept of Melcer and Isbister (2016). This enabled the creation of the outline of the triangle and the notable point of this geometric entity, portraying not only a single concept, but several.

One concept drawn at two separate times was that of an inscribed polygon. On one occasion, students had to resort to the glossary, while on the other they did not. When analyzing the gestures produced by the students for this concept, the researchers' attention was drawn by the fact that, in both opportunities, the classmates (who were not the protagonists) also gestured, as illustrated in Figure 7.

Figure 7 – Gesture representing the concept inscribed polygon

Source: Research Data.

This action of gesturing together is corroborated by Goldin-Meadow (2006, p. 143) when it is stated that “gesture is also meaning”. The authors indicate that “gestures not only evoke visual images, but can evoke motor images, thus expanding the possibilities of representation” (Goldin-Meadow, 2006, p. 143). The same fact occurred when representing the concept of generatrix, as illustrated in Figure 8.

Figure 8 – Gesture representing the concept generatrix

Source: Research Data.

In this case, the student began by representing a segment, then gave the idea of being a cone, making the representation of a triangle in the air and then showing the number three. In this phase, the classmates began to risk guessing concepts of geometric entities related to the cone. In these last two analyses, it can be seen that the students built motor representations not only as gesticulators, but also when they observed each other’s gestures. This factor expands the set of representational tools available. This means that, in addition to visual formats, the apprehension of information also occurred by motor means.

After the game, the teacher responsible for the class, who corresponds to the third author of this article, proposed a questionnaire, as described above. The following is a synthesized analysis of the questioned items.

The first question requested the students in the classes to identify themselves. Of the respondents, fourteen students were from class 231 and twenty-two from class. The second question focused on the difficulties in understanding the concepts related to Geometry. Most students claimed that Geometry was difficult to understand. This answer was expected, due to the amount of glossary consultations that were necessary during the game, since the students did not remember or were not aware of the concepts covered.

The next question was related to the motivation to participate in the game. In response, the following loose expressions were mentioned: competition, participation, socializing, learning, having fun with friends, winning with the group, review for ENEM, working with colleagues, the method, practicing in a different, interesting way, learning in a playful way, among others. In this sense, it is clear that the goal of bringing the game into the classroom was achieved. Whereas,

> [...] using games as pedagogical material aims to create a relaxed environment that enables learning through observation, creativity, logical thinking, problem solving, articulation with different knowledge and inter-relationship with classmates (Teixeira; Apresentação, 2014, p. 305).

The other five questions addressed aspects related to play. They were asked if they performed gestures for their groupmates to figure out what it was about. Eleven (11) students answered no, while twenty-five (25) were in front of the group. Among the 25 (twenty-five) who made gestures, the words they remembered were: area, side, midpoint, triangle, supplementary angles, barycenter, circle, string, orthocenter, angle, hypotenuse, point, measure, parallelepiped, cube, line, cylinder, base, pyramid, and generatrix.

In this question, they were also asked if any of their groupmates guessed the concept they represented by gestures correctly, which led 23 (twenty-three) to answer positively, while another 2 (two) said they did not. Thirty-two (32) said they were able to discover the concept their colleague represented, and only three (3) said they could not get it right. Also questioned about the use of the glossary, nineteen students answered positively.

The last two questions dealt with the evaluation of the activity and changes to be made in the game. The students were very suggestive, indicating changes to the rules, ways of organizing the game, and the number of people needed to manage the response time. Such considerations will be considered in new versions that should be organized by the research group.

Conclusion

The research aimed to analyze how high school students identified concepts, properties, and relations of Euclidean geometric entities with the use of the game Geometria em Ação. Initially, one of the interests of the researchers, participants of a study and research group in Geometry, was to present the game to students of this level, giving them the opportunity to learn about the existence of mathematical activities related to visualization, gesture representation, symbolization, and reasoning.

The surprise was in terms of acceptance and participation of the students of this grade level, who, at first, did not understand that this was an opportunity to collaboratively produce, without the constraints imposed by the school routine. However, after this adaptation phase, they realized that the space given by the game was favorable to explore Geometry and recapitulate concepts seen in other moments, which would lead them to a review for the ENEM that was approaching.

As for the gestures performed, it was possible to notice that the more complex the concept was for the student (with consultation of the glossary), the more difficult it was to make the associated gestural representation understood. This finding can be explained by theories that describe a precise relationship between gesture and spoken expression, such as the one proposed by Morsella and Krauss (1992)[3] apud Goldin-Meadow (2003). The hypothesis is that the gesture prepares the word, and the effect of this preparation is faster for familiar words than for unfamiliar ones.

In the case of the game Geometria & Ação, gestures are an expression of geometric concepts. From this perspective, gestures are effective when students have correct mental images and are able to organize them efficiently just by using their hands and body. In addition, such concepts must be known by those observing them. If observers do not remember the concept, they are unable to recognize the gesture, no matter how accurate it is. In sum, this research suggests that individuals produce gestures more efficiently when they have a mental image of the geometric entity, making themselves understood through motor representation, when observers also know the concept.

As for the answers given by the students to the questions in the questionnaire, the students' involvement in the proposed activity stands out, because when asked about changes to be made, several aspects were listed. They suggested that there could be some kind of prize for the winning teams and that the dynamic in the participation of the groups could be changed, so that no group would be left too long without playing. One proposed change in the game that is being considered by the research group for a new version is to divide the concepts into levels (from the easiest to the hardest). The group believes that this suggestion is especially important and needs to be incorporated, since it came from subjects who tested the game and made serious and critical contributions.

Considering that the application of the activity occurred on the eve of ENEM, there was curiosity to know if the game promoted any benefits due to this exam. Twenty-six students responded positively to this question, which is considered a positive contribution of the research group for the audience involved. In addition, the students were asked to provide a short evaluation of the game. The answers were given through typical expressions for the students' age group, such as "cool activity," "lighter and fun class," "knowledge with fun," "uses competition to instigate learning," "great to review content," among others.

This evaluation was received in a positive way, since GepGeo aims to create and suggest activities in which Geometry is explored in the classroom, so that students can understand the concepts involved.

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Notes

1 *Imagem & Ação*, launched in 1986 by Grow, is a mimicry and guessing board game with more than 2,400 words or expressions subdivided into 6 categories. In each round, the teams must guess what the mimic of one of their members wants to express in order to advance the spaces on the board. Whoever gets to the end first wins (Muniz *et al*., 2016).

2 HOFFER, Alan. *Mathematics Resource Projec*t: Geometry and Visualization. Palo Alto, Calif.: Creative Publications, 1977.

3 MORSELLA, Ezequiel; KRAUSS, Robert. The role of gestures in spatial working memory and speech. *The American Journal Of Psychology*, v. 117, n. 3, p. 411-424, 2004.

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José Carlos Pinto Leivas has a degree from the Catholic University of Pelotas (UCPel), specialization from the Federal University of Pelotas (UFPel), from the Federal University of Santa Catarina (UFSC), in Mathematics. His doctorate was in Education, in the Research Line in Mathematics Education, from the Federal University of Paraná (UFPR). He is a retired full professor at the Federal University of Rio Grande (FURG).
ORCID: https://orcid.org/0000-0001-6876-1461
E-mail: leivasjc@ufn.edu.br

Carmen Vieira Mathias is a professor of mathematics at the Federal University of Santa Maria (UFSM). He holds a degree from the UFSM, and a master's and doctorate degree from the Federal University of Rio Grande do Sul, in Mathematics. She has been working as a volunteer since 2014 in an Extension Project entitled GeoGebra Course, in which more than 5000 teachers have already been trained to use the software that entitles the project in the teaching of Mathematics.
ORCID: https://orcid.org/0000-0001-5667-159X
Email: carmenmathias@gmail.com

José Anchieta da Silva is a mathematics teacher at Basic Education. He works in private schools and preparatory courses in Santa Maria. He has a degree from the Faculty of Teaching and Research of Itajubá and is currently a student of the Professional Master's Degree in Mathematics in National Network (PROFMAT) at the Federal University of Santa Maria (UFSM).
ORCID: https://orcid.org/0000-0003-1517-0654
Email: prof.anchieta@gmail.com

Editor in charge: Carla Karnoppi Vasques