Educação & realidade

Statistical Literacy and Thinking in Future Mathematics Teachers

Francisco Rodríguez-Alveal[I]
Maitere Aguerrea[II]

[I]Universidad del Bío-Bío, Chillán – Chile
[II]Universidad Católica del Maule, Talca – Chile

ABSTRACT – Statistical Literacy and Thinking in Future Mathematics Teachers. The objective of this research was to evaluate the literacy and statistical thinking achieved by mathematics teachers in training. Through a qualitative study, based on a case study design. 36 future teachers participated, who carried out a statistical project. Among the findings, conceptual errors related to selecting graphic representations relevant to the type of data stand out, as well as when providing arguments, evidencing weaknesses in the development of skills related to literacy and statistical thinking. Which allows us to conclude that they have not developed statistical literacy or thinking to conduct learning in school statistics.
Keywords: **Statistical Literacy. Statistical Thinking. Statistical Project. Mathematics Teachers.**

RESUMEN – Alfabetización y Pensamiento Estadístico en Futuros Docentes de Matemática. El objetivo de esta investigación fue evaluar la alfabetización y pensamiento estadístico alcanzado por profesores de matemática en formación. Mediante un estudio cualitativo, sustentado en un diseño de estudio de caso. Participaron 36 futuros profesores, quienes realizaron un proyecto estadístico. Entre los hallazgos se destacan errores conceptuales relacionados con seleccionar representaciones gráficas pertinentes al tipo de dato, como así también al proporcionar argumentaciones, evidenciando debilidades en el desarrollo de habilidades afines a la alfabetización y pensamiento estadístico. Lo que permite concluir que no han desarrollado una alfabetización ni pensamiento estadístico para conducir aprendizajes en estadística escolar.
Palabras-clave: **Alfabetización Estadística. Pensamiento Estadístico. Proyecto Estadístico. Profesores de Matemática.**

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Introduction

The widespread access to information and communication technologies (ICTs) through mobile phones and computers has enabled the continuous monitoring and updating of social, economic, and health-related data, among others. For instance, the numerical and graphical data reported during the COVID-19 pandemic – both globally and in Chile – regarding the number of active, recovered, and deceased cases, PCR testing rates (C-reactive protein tests), contagion rates, the evolution of infections, and economic impact, among other aspects, are examples of how statistical information is disseminated. According to Best (2008, p. 127), such statistics are intended "not only to inform people but also to shape their attitudes and behaviors". Moreover, engaging with statistical practices allows citizens to develop a deeper understanding of an increasingly quantitative world (Jolly, 2012), where data serve as a foundation for making objective decisions (Wild; Utts; Horton, 2017). In this context, it is crucial for individuals not only to read reports or news articles presented by the media but also to comprehend, interpret, evaluate, and critically analyze them based on the statistical data provided, whether in written or visual form. This goal aligns with the development of statistical literacy (Gal, 2002; Garfield, 2002), which should be fostered from primary education through secondary and higher education.

In the Chilean context, university-level instruction in statistics and probability has traditionally followed an algorithmic approach. This has led to a limited conceptual understanding and weak argumentative skills among pre-service teachers, hindering their ability to construct, evaluate, and communicate statistical knowledge (Estrella, 2017; Rodríguez-Alveal; Díaz-Levicoy, 2021).

Similarly, when interpreting statistical summaries involving measures of central tendency and variability, pre-service mathematics teachers often encounter difficulties in understanding, interpreting, and selecting the mean as a representative measure of a dataset, as well as in interpreting standard deviation within problem contexts (Gea et al., 2016). Furthermore, studies indicate that these future teachers tend to focus more on interpretation than on description or evaluation of observed phenomena or data (Shin, 2021), suggesting that they may lack the necessary resources to effectively teach statistical reasoning to their students.

In summary, mathematics teacher education faces significant challenges in addressing one of the key objectives of statistical education: ensuring that students develop statistical literacy and reasoning as essential life skills. As Watson (2006, p. 271) states, "Teachers are the next major frontier in bringing statistical literacy to all students to prepare them for life beyond school". In this regard, the present study aims to assess the statistical literacy and reasoning skills acquired by pre-service mathematics teachers during their initial training.

Theoretical Framework

Statistical Literacy and Statistical Thinking

The incorporation of statistics and probability into school curricula worldwide is relatively recent (Alsina et al., 2020). The *National Council of Teachers of Mathematics* (NCTM) was a pioneer in this regard, introducing "Data and Chance" as a thematic area in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). This inclusion was driven by the increasing interaction of citizens with large volumes of data in their daily lives and the need to interpret, analyze, and computationally process such data, while also determining the most appropriate statistical methods. Within the field of statistical education, these aspects are associated with concepts such as statistical literacy and statistical thinking.

At the international level, different perspectives exist regarding the definition of statistical literacy (Rodríguez-Alveal, 2017). Specifically, Gal (2002) defines statistical literacy as the ability to interpret and critically evaluate statistical information, encompassing dispositional capacities such as attitudes, beliefs, and critical thinking. Similarly, Ben-Zvi and Garfield (2004) describe statistical literacy as a set of basic and fundamental skills that enable individuals to understand statistical information or research findings. These skills include organizing data, creating and explaining tables, and working with various data representations. Statistical literacy also involves understanding concepts, acquiring statistical vocabulary, and interpreting the symbols used in statistics. On the other hand, Forbes et al. (2011) define statistical literacy as the ability to verbally communicate the insights conveyed by numerical and graphical summaries. In sum, Yuniawatika (2018, p. 170) describes statistical literacy as:

> [...] the ability to understand, interpret, evaluate, and communicate statistical data through various media in a critical manner. This includes the capacity to comprehend symbols, terms, statistical language, and data presentation in graphs or tables, as well as the ability to establish connections between statistical data. Interpretation and evaluation refer to an individual's ability to read and analyze data based on their existing statistical knowledge. Additionally, it encompasses the ability to predict trends from a graph and draw conclusions from existing data.

Similarly, Martadiputra (2010) identifies five key components of statistical literacy: (1) data reasoning, (2) understanding basic statistical concepts and terms, (3) data collection and processing in descriptive statistics, (4) basic ability to translate data, and (5) fundamental skills for communicating data and research findings. These aspects align with the ideas proposed by Garfield (2002), DelMas (2002), and Chance (2002). Furthermore, based on the syntheses by Yuniawatika (2018) and Martadiputra (2010), a fundamental conceptual framework for decoding graphical representations is Curcio's taxonomy (1989), which defines four categories or levels related to reading, interpreting, predicting, and critically analyzing tables and graphs (Estrella; Olfos, 2012).

Meanwhile, statistical thinking involves the development of higher-order skills, such as understanding the theories and assumptions underlying statistical methods, simulating random phenomena, and applying statistical concepts to real-world and contextualized problems (Garfield; Ben-Zvi, 2007). In this regard, the Guidelines for Assessment and Instruction in Statistics Education (GAISE, 2016) emphasize the competencies that students should develop, including critical thinking about statistical issues and recognizing the necessity of data and the omnipresence of variability. This underscores the importance of teacher training programs that go beyond numerical calculations and incorporate projects that foster these essential skills (Green; Blankenship, 2013).

ICT in Statistical Literacy and Statistical Thinking

Information and Communication Technologies (ICT) play a fundamental role in developing competencies for data processing and management. According to Perdomo and Rojas (2019), the introduction of ICT has been one of the most significant revolutions in teaching and pedagogy, despite originating outside educational contexts (Cabero-Almenara, 2005), and it helps reduce cognitive load (Baglin, 2013). In the same vein, the GAISE Report (2016, p. 19) states that "technology has changed the practice of statistics, and therefore, it should change what and how we teach". In particular, the use of ICT in statistics education enhances learning and motivates students by facilitating the understanding of complex or abstract concepts (Fernández-Morales, 2017). Additionally, it enables the graphical and numerical summarization of data, improving interpretation and decision-making (Capilla, 2014), as well as supporting statistical argumentation (Berndt et al., 2021).

In summary, the automation of statistical processes related to changes in data representation (transnumeration) allows both students and teachers to focus on higher-order concepts and statistical problem-solving (Ben-Zvi, 2000), ultimately fostering statistical literacy and thinking (Garfield, 2002; Garfield; Ben-Zvi, 2008). Along these lines, Table 1 provides a framework for identifying computational actions in students' statistical work (Woodard, 2016).

Table 1 – Framework for Computational Actions in Student Statistical Work

Statistical Computation Action	Students should be able to perform the following actions
Automation of Computational Procedures	a) Use technology to efficiently create graphs or statistical summaries to make sense of data without overloading cognitive processes. b) Use technology to perform statistical calculations and utilize the results to make appropriate decisions without overloading cognitive processes.
Pattern Recognition and Decision-Making	c) Identify patterns in the results of a statistical analysis and use this information to determine the next step in the process. d) Recognize patterns in how data are coded and use these patterns to help solve problems.

Source: Prepared by the authors.

Active Learning Environments

To foster statistical literacy and develop probabilistic thinking among students and pre-service teachers, scholars such as Azcárate and Cardeñoso (2011, p. 792) emphasize the need to promote "active learning environments," where students act as both consumers and generators of data. In these settings, they engage with real-world problem situations, immersing themselves in authentic research and discussion activities related to statistics. This approach is based on the idea that statistical knowledge cannot be understood in isolation from real-world contexts (Cobb; Moore, 1997). In this regard, project-based learning enables students to contextualize statistical concepts and techniques using real data (MacGillivray; Pereira-Mendoza, 2011), allowing them to analyze, interpret, and communicate the information conveyed by the available data (Azcárate; Cardeñoso, 2011).

Building on this perspective, Guven et al. (2021) propose integrating Wild and Pfannkuch (1999) investigative cycle into the learning process as a tool for developing statistical and probabilistic literacy. Table 2 outlines this five-phase cycle.

Table 2 – Phases of Wild and Pfannkuch's Investigative Cycle

Phase of the cycle	Description
1	Problem formulation;
2	Identification of variables, specifying how they will be measured and what type of sampling design will be used (plan);
3	Data collection and selection of software for processing the data (data);
4	Summarizing and analyzing data using graphical and numerical representations (analysis);
5	Interpretation and conclusions.

Source: Prepared by the authors.

In this regard, researchers such as Özmen and Baki (2017) and Watson et al. (2017) have documented that incorporating Wild and Pfannkuch (1999) investigative cycle enhances statistical literacy. Moreover, its structured approach supports the development of statistical thinking and promotes student interaction, especially when projects are carried out collaboratively in group settings.

Methodology

A qualitative methodological approach was adopted for this research, based on a case study design (Pérez-Serrano, 1994).

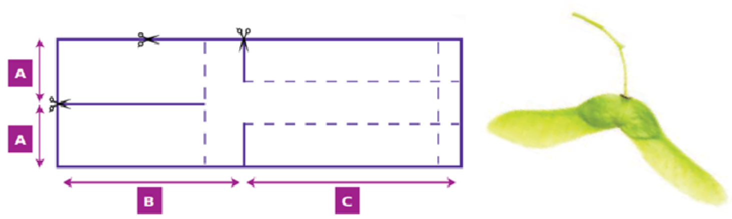
Context and Participants

The study participants consisted of 36 pre-service mathematics teachers who had successfully completed coursework in statistics and probability as part of their training program. At the time of the study, they were enrolled in the Didactics of Statistics course, which was conducted through remote teaching via the Zoom platform, in accordance with institutional COVID-19 confinement policies. Participant selection followed a non-probabilistic purposive sampling approach (McMillan; Schumacher, 2011). The students were divided into 11 working groups, each responsible for responding to the proposed activities. To ensure participant anonymity, the groups were assigned alphanumeric codes, using the format [Group No.] and [Pre-service Teachers]. For instance, [GPF1] refers to Group No. 1 of pre-service teachers.

Data Collection Instrument and Analysis Procedure

For this study, a problem-based task was selected from the *STEM Learning* website (https://www.stem.org.uk) after being translated and adapted to the Chilean context. The task simulated the seed dispersal process of the sycamore tree (*Ficus sycomorus*), a species native to Central Africa. To investigate the effect of wind on seed dispersal, the problem required modeling the seed structure, as illustrated in Figure 1. In this model, **A** represents the width of the seed wing, **B** denotes the length of the seed wing, and **C** refers to the length of the seed tail, following the "helicopter method".

Figure 1 – Helicopter Seed Model



Source: Prepared by the authors.

The participants engaged in three activities. In the first activity, they were asked to drop the "Helicopter" seed ten times from a standard height of one meter, marking the impact point on the ground and recording the distances from the center to the point of impact. This process actively involved the students in data collection. Then, based on the collected data, they were required to perform a graphical and numerical statistical analysis of the distances to describe the behavior of the seeds. In the second activity, participants were asked to modify parameters (**A**), (**B**), and (**C**) of the "Helicopter" seed, repeat the experiment (indicating which parameters were altered), and analyze the new data obtained. Finally, in the last activity, they were asked to compare the statistical results from the control and experimental designs, observing the relationship between numerical and graphical measurements in both experiments. These activities were guided by the following questions:

1. What information do numerical and graphical measurements provide in the experiments concerning the proposed activities?

2. Which parameters of the "Helicopter" seed allow it to fall farther from the central point (tree)?

3. What is the relationship between the height from which the "Helicopter" seeds are dropped and the distance to the central point (tree)?

For the analysis procedure, a data matrix was created in Excel, containing the activity questions and the participants' responses. The information was analyzed based on the categories associated with the phases of the investigative cycle proposed by Wild and Pfannkuch (1999), as presented in Table 2, from the perspective of statistical literacy and thinking. Additionally, it was examined in conjunction with the framework for identifying statistical computing actions outlined in Table 1. Furthermore, following the steps proposed by Mayring (2000), a content analysis was conducted by selecting segments from the transcriptions of participants' responses according to the aforementioned categories. This approach allowed for the identification of emerging codes aligned with the research objectives, revealing the convergences and divergences in the questions that guided the activities.

Analysis and Results

The following section presents the results of the activities corresponding to the categories described in Tables 1 and 2, as well as the objectives of this research.

Results of Activity 1: Graphical and Numerical Statistical Analysis of the First Activity

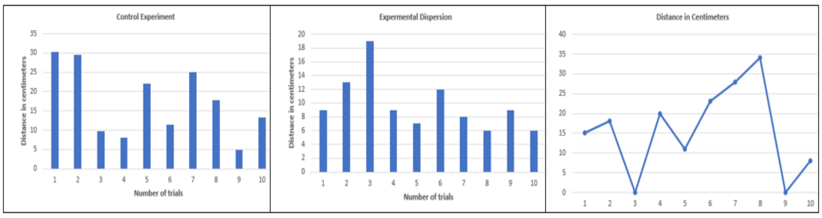
In general, the student groups, aligned with the data generation phase of the investigative cycle proposed by Wild and Pfannkuch (1999), conducted the experiment by specifying the wing width (**A**), wing length (**B**), or the tail mass (**C**) of the helicopter, following the model described in Figure 1. Consistent with this process, Figure 2 displays images illustrating the data collection procedure by simulating the helicopter's descent after its construction.

Figure 2 – Images of the execution of experiment 1 by pre-service teachers

Source: Prepared by the authors.

The following section, considering the data analysis phase of the investigative cycle proposed by Wild and Pfannkuch (1999) and aligned with statistical thinking, requires pre-service teachers to select appropriate models and procedures for analysis based on the nature of the data (Garfield; Ben-Zvi, 2008). In this regard, participants from 4 out of the 11 groups summarized the behavior of the variable under study – the distance in centimeters at which the seed helicopters landed (Figure 3) – using simple bar charts, while only 3 groups utilized line graphs.

Figure 3 – Examples of graphical representations created by participants



Source: Prepared by the authors.

In these responses, an inadequate use of graphical representations is observed, as they fail to summarize or visually capture the variability of the data for the variable of interest, considering its quantitative nature. This is because the graphs created by the groups of pre-service teachers do not demonstrate a connection between the nature of the data and the proposed model—an essential skill for developing statistical thinking (Garfield; Ben-Zvi, 2008). These findings are similar to those reported by Rivas, Godino, and Arteaga (2018), which indicate that pre-service teachers struggle to select appropriate graphs to represent their data. Additionally, the line graph contains errors in its construction, as it does not reference the variables represented on the axes or the measurement units used, similar to the issues identified by Arteaga et al. (2016).

Furthermore, all graphical representations were created using Excel, where, as in Arteaga et al.'s study (2016, p. 27), pre-service teachers "incorrectly choose the data to be represented, treating variable values (which should be labels in Excel) in the same way as frequencies".

Regarding the transnumeration phase, which involves conveying or communicating the meaning of the data (Pfannkuch; Rubick, 2002), members of one of the groups commented:

In the graph, the numerical data from the helicopter seed drops vary even when they are released from the same height (GPF1).

The graph shows the distance values of each sample observation in relation to the central point (GPF6).

These statements reflect a literal reading of the summarized data in the graph, consistent with Curcio's taxonomy (1987). This aligns with the findings of Rodríguez-Alveal and Díaz-Levicoy (2019, p. 100), who state that "in general, both pre-service and in-service teachers struggle with reading and interpreting graphical representations and are not always able to discern the key insights summarized within them". Similarly, Salcedo et al. (2021) indicate that pre-service teachers tend to operate at the basic reading level according to Aoyama's taxonomy (2007).

On the other hand, the numerical analysis performed by participants involved calculating measures of central tendency, such as the arithmetic mean, as well as measures of variability, including variance, standard deviation, and range. The following excerpts illustrate how pre-service teachers interpreted these statistical measures within the context of the problem:

It can be observed that the distance from the point of release to where the helicopter lands ranges from zero to 34 centimeters. Although the height and the release point remain the same, the helicopters land at different distances, with an average of 15.7 centimeters (GPF4).

The shortest observed distance from the central point was 4.4 cm, and the longest was 16.9 cm, revealing a considerable range of 12.5 cm. The sample mean is 10.31 cm. Similarly, the sample standard deviation is not excessively large, as it is smaller than the sample mean. It is important to note that this statistic is meaningful because it is measured in the same units as the sample observations (GPF6).

In these responses, members of GPF4 refer literally to the dispersion of the data and implicitly treat the range as a positional measure (an interval). Meanwhile, pre-service teachers in GPF6 explicitly refer to the range as a measure to quantify the variability in the distances at which the seed (helicopters) landed. However, both groups mention the mean without interpreting it in the context of the problem, instead treating it purely as an arithmetic calculation.

Additionally, members of GPF11 provided values for the mean, range, standard deviation, and variance to quantify variability and describe the numerical behavior of the data, commenting:

Mean: 9.5, Variance: 24.8, Standard deviation: approximately 4.9. Range: 17.5. Based on these results, we can infer that the seeds exhibit a high dispersion index, as the results of each drop are not uniform, allowing for greater seed spreading, which benefits agriculture (GPF11).

In the GPF11 explanation, participants use and relate concepts such as dispersion, uniformity, and spreading to describe the concept of variability in the observations. This indicates that pre-service teachers in this group do not associate the terms dispersion and homogeneous with the concept of variability, which are essential elements of statistical literacy (Garfield, 2002; DelMas, 2002).

Conversely, GPF6 comments on the presence of multiple factors influencing the results, intuitively recognizing that the experiment involves a “quite significant” experimental error. However, this reflects an inappropriate use of statistical language, as the term significant is more relevant to statistical inference. Furthermore, in their explanation, group members acknowledge that certain factors could influence controlling variability in the data (Wild; Pfannkuch, 1999).

In summary, participants primarily calculated the arithmetic mean, range, and standard deviation using Excel, demonstrating technological proficiency but limiting their statistical analysis to a literal interpretation of these measures. They did not establish meaningful connections between the statistics and the context of the problem, preventing them from analyzing and explaining the behavior of the variables based on the information provided by the statistical measures. Moreover, it is evident that pre-service teachers generally exhibit a limited conceptual understanding of statistical concepts such as significance and variability measures. This is particularly evident in their misinterpretation of the standard deviation as a significant statistic simply because it shares the same unit as the variable of interest. This conceptual gap may hinder the development of statistical thinking, as described by Garfield and Ben-Zvi (2007), since participants fail to effectively convey or communicate the data’s behavior within the context of the experiment.

Results of Activity 2: Graphical and Numerical Statistical Analysis of the Second Activity

In the second activity, participants were asked to repeat the experiment while modifying the parameters associated with the "Helicopter", placing them in a scenario of induced variability. Table 3 presents data on the number of simulations conducted by the groups of pre-service teachers, which generally fluctuated between five and six trials.

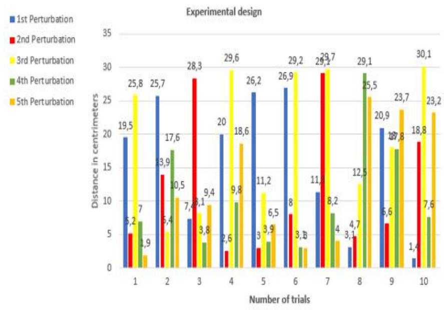
Table 3 – Examples of simulations conducted by participants with altered parameters

Group	Distance in centimeters										Parameter values		
GPF1	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**A**	**B**	**C**
	19.5	25.7	7.4	20.0	26.2	26.9	11.3	3.1	29.9	1.4	3.5	6.8	8.2
	5.2	13.9	28.3	2.6	3.0	8.0	29.1	4.7	6.6	18.8	5.0	8.0	7.0
	25.8	5.4	8.1	29.6	11.2	29.2	29.7	12.5	18.0	30.1	4.8	7.0	6.5
	7.0	17.6	3.8	9.8	3.9	3.1	8.2	29.1	17.8	7.6	4.7	5.0	4.0
	1.9	10.5	9.4	18.6	6.5	3.0	4.0	25.5	23.7	23.2	3.9	4.4	5.8
GPF2	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**A**	**B**	**C**
	9.0	13.0	19.0	9.0	7.0	12.0	8.0	6.0	9.0	6.0	10.0	8.0	8.0
	15.0	25.0	38.0	36.0	9.0	13.0	22.0	5.0	17.0	13.0	4.0	8.0	8.0
	25.0	27.0	25.0	28.0	19.0	17.0	7.0	5.0	12.0	14.0	6.0	8.0	8.0
	4.0	2.5	2.0	1.5	3.0	5.0	4.0	2.0	1.5	9.0	4.0	6.0	8.0
	33.0	8.0	27.0	13.0	6.0	6.0	7.0	7.0	3.0	18.0	4.0	8.0	6.0
GPF5	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**A**	**B**	**C**
	19.0	6.0	12.0	21.0	13.0	7.0	21.0	16.0	22.0	11.0	3.0	3.0	3.5
	15.0	21.0	18.0	14.0	11.0	8.0	16.0	11.0	7.0	8.0	6.0	4.0	3.0
	22.0	21.0	13.0	7.0	18.0	19.0	15.0	9.0	13.0	9.0	5.0	4.5	3.5
	10.0	20.0	12.0	5.0	12.0	5.0	12.0	6.0	12.0	4.0	4.5	4.0	2.5
	14.0	11.0	7.0	16.0	14.0	9.0	6.0	3.0	7.0	16.0	8.0	6.0	4.0

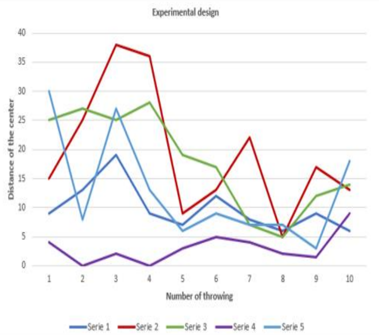
Source: Prepared by the authors.

As an example, Figure 4 presents some of the graphical representations created by participants to communicate the behavior of the collected data.

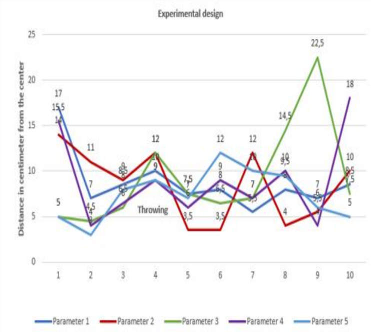
Figure 4 – Examples of graphical representations created by participants with altered parameters



GPF1



GPF2



GPF6

Source: Prepared by the authors.

The figure illustrates the use of grouped bar charts to present the variation in data across different simulations. This choice of representation indicates a limited understanding of the most appropriate graphical method for displaying the variability of quantitative data, as grouped bar charts are typically used for summarizing data related to qualitative variables. This reflects a lack of conceptual knowledge in the transnumeration process (Wild; Pfannkuch, 1999). Similarly, line graphs are used to represent the trends in results when altering the parameters of the "Helicopter" seed model. However, these representations are not suitable for modeling data variability, suggesting that participants have not yet developed the necessary skills associated with statistical thinking.

The following are some excerpts from the participants commenting on the observations made regarding the graphical representations shown in Figure 4:

When altering the parameters of the helicopter seed, we can conclude that the distance variables change depending on whether the parameter values are high or low (GPF1).

We can see in the graph that the series that deviates the most from the center is Series 2, which was dropped from a greater height, while the closest one is Series 4, which has smaller B values (GPF2).

The parameter that shows the greatest difference is Parameter 3, while the most stable parameter is Parameter 5. Parameters 1 and 4 are similar. Altering the parameters significantly changes the experiment in Part 1 (GPF6).

Despite the inappropriate graphical representations used to visualize quantitative data behavior, participants still demonstrated limited statistical thinking (Garfield; Ben-Zvi, 2007). Their reflections suggest an attempt to explain data behavior using non-explicit information, such as parameter modifications in the "Helicopter" seed model (GPF2) and the height from which the helicopters were dropped (GPF6). According to Curcio taxonomy (1989), this corresponds to the level of "reading within the data". However, participants did not explore questions related to the data distribution, such as commenting on asymmetry in response to variations in the "Helicopter" seed parameters.

Regarding numerical analysis (Table 4), only three out of eleven groups synthesized their numerical results into a table, summarizing the calculated measures of central tendency and variability in the context of the simulated data.

Table 4 – Examples of Numerical Representations Created by Participants

GPF6	**Statistical Analysis**		**Statistical Analysis**		**Statistical Analysis**	
	Minimum	4.3	Minimum	1.0	Minimum	3.6
	Maximum	13.0	Maximum	8.6	Maximum	11.3
	Range	8.7	Range	7.6	Range	7.7
	Mean	8.76	Mean	5.13	Mean	7.09
	Standard D.	2.87	Standard D.	3.07	Standard D.	2.28
	Variance	8.25	Variance	9.40	Variance	5.21
GPF5	**Parameter**	**Simulation 3**	**Simulation 4**	**Simulation 5**	**Simulation 6**	**Simulation 7**
	A	3.0	6.0	5.0	4.5	8.0
	B	3.0	4.0	4.5	4.0	6.0
	C	3.5	3.0	3.5	2.5	4.0
	Mean	3.1	4.3	4.3	3.6	6.0
	Mean Distance	14.8	12.9	14.6	9.8	10.3
GP11	**Tail Altered**		**Wing Width Altered**		**Wings Altered**	
	Mean	5.75	Media	6.6	Mean	6.9
	Variance	7.76	Varianza	9.99	Variance	3.39
	Standard D.	2.78	D. Estándar	1.84	Standard D.	1.84
	Range	9.0	Rango	6.0	Range	6.0

Source: Prepared by the authors.

In this situation, one can see that the three groups determine the average distance at which the "Seed Helicopters" fell, giving this statistic an important role in describing the data's behavior. However, GPF6 and GPF11 refer to measures of variability such as the range, standard deviation, and variance, which allows them to delve deeper into interpreting the behavior of quantitative data, given that distributions with the same average can have different variability. This indicates a basic level of procedural statistical knowledge (Garfield; Ben-Zvi, 2008). Additionally, it is observed that GPF5 also calculated the averages of the corresponding "Seed Helicopter" parameters, revealing a lack of understanding of the problem situation, since the resulting figure has no clear meaning or usefulness for solving the problem. On this point, some of the participants' comments were:

When comparing the statistical results with the measurements of the experimental design parameters, we can see that, in the case of those parameters, the measurements range on average from 3 to 7 centimeters, while in the case of distances, they range from 9.8 to 14.8 centimeters. This indicates that if the parameter measurements become larger, the distances from the center become smaller, and conversely, when the parameter measurements are smaller, the distances from the tree are greater (GPF5).

Regarding the statistical analysis, we see that the smallest distance observed from the central point was 4.3 cm, and the largest was 13 cm, revealing a range of 8.7 cm, which is considerably smaller than the range in the control experiment. The sample mean is 8.76 cm; similarly, the sample standard deviation is not large because it is much smaller than the sample mean (GPF6).

In these accounts, it can also be seen that GPF5 and GPF6 analyze the behavior of the data based on the range of the distances obtained to explain the effect of parameter perturbations in the "Seed Helicopter", without using the variability of the data to describe how the seeds are dispersed.

In summary, although the preservice teachers calculate the necessary statistical measures to study the data's behavior, they fail to decode the message these measures convey in the context of the problem, which demonstrates a weak development of statistical literacy and statistical thinking (Garfield, 2002; Garfield; Ben-Zvi, 2007).

Results of Activity 3: Comparison of Results from the First and Second Activity

Finally, in Activity 3, the preservice teachers were required to compare the results of the first simulation—referred to as the control experiment—with the perturbations applied to the parameters of the "Seed Helicopters" (experimental design). In this regard, the preservice teachers provided arguments and reasoning of different natures. The following excerpt serves as an example:

Variations in the studied statistics are observed in relation to the numerical analyses obtained for the control experiment and the experimental designs. This is due to the perturbations made to the conditions of the "Seed Helicopters". It can be seen that the standard deviation and variance of the control experiment are higher than those of all the other experimental designs. This is because the control model is more aerodynamic, which results in greater dispersion of the seeds. The range analysis shows that the control experiment is the most variable, as it has the largest range of observations in the samples. Regarding the graphs, we can also conclude that the control experiment exhibits the greatest variability in its results, as it produces a more diverse frequency polygon in relation to the mean. (GPF6)

Here, the variability of the data is justified based on the perturbations applied to the Seed Helicopters, which is reflected in the comparison of the standard deviation between the control and experimental conditions, demonstrating statistical thinking. Similarly, members of GPF7 and GPF11 commented that the dispersion of the data is related to the perturbation of the parameters, although they did not explicitly reference the standard deviation statistic:

It could be said that the distances when launching the helicopter are more uniform in Experiment 1, whereas in Experiment 2, the data obtained are more dispersed, which depends on the modified parameters, even though the experiments were conducted under the same study conditions. (GPF7)

In general, based on the obtained results, we infer that unaltered seeds have greater dispersion than altered seeds. (GPF11)

Only one of the eleven groups, in addition to perturbing the parameters of the Seed Helicopter, conducted launches from a height of 2 meters. As a result, the Seed Helicopters tended to fall farther from the center, as described by the preservice teachers in the following excerpt:

When comparing the statistical results of the control experiment with the results of Perturbation B, which was launched from a height of 2 meters, we can observe that the helicopter spins more before falling. For this reason, it tends to move farther from the central point. (GPF10)

In summary, the participants mentioned statistics related to quantifying variability but did not connect them with the mean in a way that would allow them to make informed decisions about the behavior of the seeds in the context of the problem situation.

Conclusions

This theoretical-empirical article presents background information on the importance of statistical projects as innovative teaching and assessment methodologies for concepts and statistical skills acquired by preservice mathematics teachers in the statistics and probability courses of their training programs. The aim is to develop statistically literate teachers with well-developed statistical thinking.

In this regard, data summarization through graphical representations and numerical summaries, as well as the interpretation of statistical results, are key learning objectives in statistics education. Their significance is emphasized both in the GAISE report (2005, 2016) and in Chilean curricular guidelines. However, it is noteworthy that, in general, preservice teachers primarily use graphical representations associated with qualitative variables, such as simple and grouped bar charts, rather than employing appropriate graphical tools to depict the behavior of quantitative variables. This issue represents a specific instance of transnumeration (Pfannkuch & Rubick, 2002). These findings provide evidence that the study participants have not adequately developed statistical thinking (Garfield & Ben-Zvi, 2008) or the essential skills of statistical literacy (Garfield, 2002), aligning with similar results reported by Rivas, Godino, and Arteaga (2018). For example, they do not use histograms or box plots to illustrate data distributions, despite their presence in the Chilean school curriculum. As Estrella (2016, p. 20) points out, their "inclusion [...] provides a fruitful opportunity to begin visualizing extreme values (which are very common in real-world data collection)".

Furthermore, preservice teachers tend to make literal readings of the information summarized in the graphs, which limits the depth and scope of their interpretations within the context of the assigned activity. On the other hand, in an effort to numerically reveal hidden data patterns, participants calculate the arithmetic mean, range, variance, and standard deviation. This process is associated with transnumeration (Wild; Pfannkuch, 1999), but they do not interpret these statistics within the context of the given situation. In other words, "Preservice Mathematics Teachers are cognitively anchored to algorithmic calculation processes, preventing them from going beyond numerical values or summarizing information in contextualized ways to decode the nature of what is statistically represented" (Rodríguez et al., 2016, p. 948). This observation aligns with studies conducted by Batanero (2000).

According to Chick, Pfannkuch, and Watson (2005), transnumeration should be actively promoted and integrated into the teaching and learning process to foster statistical literacy and statistical thinking. This requires the ability to read, interpret, and evaluate statistical results in contextual situations, as well as incorporating problems that involve different sources of variability (Torok; Watson, 2000)[1].

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Francisco Rodríguez-Alveal is a professor of Education in Mathematics and Master in Statistics and Doctor in Education currently working in the Department of Science of Education of the Universidad del Bío-Bío de Chillan and several lines of interest in Initial Teaching Training, Didactics of Statistics and Theory of Distribution.
ORCID: https://orcid.org/0000-0003-2169-0541
Email: frodriguez@ubiobio.cl

Maitere Aguerrea is a professor of Mathematics, Magister and Doctora in Mathematics, currently working in the Department of Mathematics, Physics and Statistics of the Faculty of Basic Sciences of the Catholic University of Maule.
ORCID: https://orcid.org/0000-0002-7513-982X
Email: maguerrea@ucm.cl

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